BOND YIELDS

MODELS AND MOMENTS

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1 Introduction

People have borrowed money ever since time immemorial. Indeed, there is evidence of money lending as far back as 1800 BC (see Homer and Sylla, 2005). Given this long history of loaning money, it is astonishing that yield modeling is still a critical issue today, and one that received a great deal of attention in September 2008. The financial crisis peaked with Lehman Brothers filing for bankruptcy.[1] Instantly, every actor in the financial industry cared about fixed income. The foundation of fixed income – and the focus of this thesis – is a bond. Fabozzi (2013, p. 11) defines a bond as:

> *A bond is a debt instrument requiring the issuer (also called the debtor or borrower) to repay to the lender/investor the amount borrowed plus interest over a specified period of time.*

Piazzesi (2003, p. 3) outlines four reasons why it is important to understand what drives bond yields. The first reason involves the ability to forecast. The price of long bonds may indicate the evolution of future short yields, at least after subtracting a risk premium. Put differently, the yield curve contains information on the future development of the economy. There is a heterogeneous clientele interested in the outcome of this forecast, including investment strategists, saving consumers, and policymakers.[2]

Monetary policy constitutes the second reason for analyzing bond yields. Central banks of developed countries can move the short end of the yield curve; however, big

[1] The historical high of over 250 points on the MOVE index (Merrill Lynch Option Volatility Estimate) indicates that this month might be referred to as the peak of the financial crisis (see Simko, 2013).

[2] Fama and Bliss (1987), Cochrane and Piazzesi (2005), among others, study yield spreads for forecasting future short yields.

investment decisions are conditioned on the long end. That is, a private consumer considering the purchase of property will base his decision, at least to some extent, on the terms of a long-term loan. Central banks seem to be less powerful in determining the long end of the yield curve; indeed, under the expectations hypothesis, it is more the expectations of market participants that influence the yields of long maturity bonds (see Cox et al., 1981).

Debt policy is the third reason. Governments that are able to issue bonds in their own currency have the power to decide the maturity of these bonds. This decision can have either direct or indirect impact on the yield curve. Indirect impact occurs via supply of and demand for new government debt. A direct impact occurs when the government influences the term structure of interest rates. For example, the Kennedy Administration actively managed the yield curve, sometimes referred to as "Operation twist," by issuing short-term debt and repurchasing long-term bonds. The government aimed to flatten, or even invert, the yield curve by manipulating the debt's maturity (see Greenwood and Vayanos, 2010).[3]

Bond and derivative pricing are the fourth reason why it is important to understand what drives bond yields. A derivative is defined as a financial instrument whose value depends on the value of other, more basic, underlying variables (see Hull, 2011). Under this definition, a bond is a derivative that depends on the underlying interest rate. If banks want to lessen the risk they face from paying short-term interest rates on deposits and receiving long-term interest rates on commercial loans, they rely on interest rate derivatives to smooth their interest rate risk exposure. The price of the derivative, which is the core of their hedge, depends crucially on the term structure of interest rates (see Duffie et al., 2000).

[3] Beginning in December 2008, the Federal Reserve tried to combat the dramatic slowdown of the U.S. economy by repurchasing long-term debt. Although similar in the action taken, this was a slightly different strategy than that taken by the U.S. government in the 1960s. The differences are discussed in Nelson (2013).

The remainder of the introduction to this thesis is organized as follows. Section 1.1 introduces bond pricing and bond risk and defines a utility function of a bond investor. This μ-σ bond investor corresponds to the Gaussian models for bond pricing of Part I of the thesis. Specifically, the corporate bond model (Subsection 1.1.1) and the international bond model (Subsection 1.1.2) are revisited in Chapters 2 and 3, respectively. Section 1.2 extends the μ-σ preference and serves as a bridge to Part II. Particularly, Chapter 4 studies higher order moments of government bond returns. Section 1.3 provides an outline of the thesis.

1.1 μ-σ preference

This section illustrates the implications of an investor's choice of a bond yield model. First, however, the bond needs to be described in more detail. A typical zero-coupon bond specifies a fixed date when the amount borrowed is due and no coupon payments take place in between that date and the bond's issuance. The date of repayment is called the maturity date. For now the borrower is assumed to be default-risk free, meaning that the investor knows with certainty which amount to expect at maturity.[4] The situation is slightly different, though, if the investor wants to sell the bond before maturity. The bond price depends on the current yield curve in the following way.

Based on Lemke (2006), let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $\mathbb{F} = \{\mathcal{F}_t : 0 \leq t \leq T\}$ a filtration of sub σ-algebras with $\mathcal{F}_s \subseteq \mathcal{F}_t \subseteq \mathcal{F}$ for $s < t$. The filtration provides the information for the subsequent model. $\mathbb{P}$ corresponds to the physical probability measure. $\mathbb{Q}$ is the equivalent risk-neutral measure. In the absence of arbitrage opportunities, the time-t price of a zero-coupon bond that matures at time $t + \tau$ is given by (see Dai and

[4] This assumption will be relaxed in Subsection 1.1.1 and Chapter 2. For other risks associated with bond investment, such as liquidity, tax, and the like, see Driessen (2005) and the references therein.

Singleton, 2000)

$$P(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} r(u)du)\right], \tag{1.1}$$

where $E_t^{\mathbb{Q}}$ denotes the expectation at time t under the risk-neutral measure $\mathbb{Q}$ and r the instantaneous short rate as defined below.

Let $X(t)$ be a d-dimensional factor process. If $X(t)$ is $\mathcal{F}$ measurable for each $t \in [0,T]$, the stochastic process is said to be adapted to $\mathbb{F}$. The instantaneous short rate $r(t)$ is unobservable and the parameters of the model can not be observed directly. A solution to this problem is to estimate the parameters implicitly, while the instantaneous short rate $r(t)$ is modeled as a *latent* variable. The instantaneous short rate can be deduct from the observable spot rates and the model parameters can be estimated implicitly (see Mayer, 2009, p. 2). The instantaneous short rate $r(t)$ is an affine function of vector $X(t)$:

$$r(t) = \delta_0 + \delta' \, X(t). \tag{1.2}$$

In Equation (1.2), δ_0 is a scalar and δ is a vector of parameters.

The source of randomness is a standard d-dimensional $\mathbb{P}$-Brownian motion $W(t) = (W(t)^1, \ldots, W(t)^d)'$. The stochastic process $X(t)$ is defined by the stochastic differential equation (SDE) (see Munk, 2011):

$$dX(t) = \mu(X(t),t)\, dt + \sigma(X(t),t)\, dW(t). \tag{1.3}$$

Duffie and Kan (1996) impose requirements on $\mu(\cdot)$ and $\sigma(\cdot)$ such that there is a solution to Equation (1.3).

Since the path of $r(t)$ is unknown ex ante, the bond price cannot be anticipated either. The bond price changes in the opposite direction of the interest rate $r(t)$ (see Equation

(1.2)). That is, the bond price is a decreasing function of the interest yield. The investor faces a loss if interest rates have been increasing and he sells prior to maturity. A risk-averse investor wants to be compensated for taking that risk. This raises two questions: (1) What is risk? (2) What is a risk aversion? Bodie et al. (2009, p. 268) characterize risk and give an answer to question (1):

> *Risk is uncertainty that "matters" because it affects people's welfare. Thus uncertainty is a necessary but not a sufficient condition for risk.*

The answer to question (2) is taken from Eeckhoudt et al. (2005, p. 7):

> *An agent is risk-averse if he or she dislikes every lottery with an expected payoff of zero.*

For determining a bond pricing model it is necessary to describe a risk-averse investor in a mathematically consistent way. Let V be a set of random prices and $U(V)$ a subjective value assigned to it (see Rubinstein, 2006, p. 30). The investor seeks to maximize his expected utility $E[U(V)]$. This is also referred to as Bernoulli's principle (see Bernoulli, 1738).

The characteristics of a set V of random outcomes can be split into two main components: location and scale. A risk-averse investor prefers a high expected return (location) and a low risk (scale). Let μ_V be the expected price of V. If the k^{th} central moment $E[(V - \mu_V)^k]$ exists $\forall\ k \in \mathbb{N}$ and is finite, the investor prefers high odd moments and low even moments. The expected utility $E[U(V)]$ of a set of random outcomes for an arbitrary investor can be computed with the Taylor expansion (see Breuer et al., 1999, p. 178):

$$\begin{aligned} E[U(V)] =& E[U(\mu_V + (V - \mu_V))] \\ =& U(\mu_V) + U'(\mu_V) \cdot E[V - \mu_V] + \frac{1}{2} \cdot U''(\mu_V) \cdot E[(V - \mu_V)^2] \\ & + \ldots + \frac{1}{k!} \cdot U^{(k)}(\mu_V) \cdot E[(V - \mu_V)^k] + \ldots \quad . \end{aligned} \tag{1.4}$$

To this point, determination of expected utility has been very generic. The analysis of the utility function starts with focusing on the first two central moments ($E[(V - \mu_V)]$ and $E[(V - \mu_V)^2]$) and the investor's risk aversion.

Academics and investment managers have expended a great deal of effort on trying to answer the question of to what extent the path of an asset price can predict future prices (see Fama, 1965). Much of this research assumes that the past behavior of asset prices contains information about future price behavior or, in other words, history repeats itself. Many of the models used in this line of work assume, explicitly or implicitly, that price changes can be described by some probability distribution.

Bachelier (1900) proposes the normal distribution for modeling asset price behavior. The normal distribution has many features that make it convenient and easy to use, one of which is that for a normally distributed variable $V \sim N(\mu_V, \sigma_V^2)$, with σ_V^2 being the variance of V, the central odd moments are zero and the central even moments can be written as (see Schmitz, 1996, p. 60):

$$E[(V - \mu_V)^{2k}] = \frac{(2k)!}{2^k \cdot k!}(\sigma_V^2)^k, k \in \mathbb{N}. \tag{1.5}$$

The central moments can be used to simplify the expected utility function of Equation (1.4). Assuming normality, the expected utility reads (see Breuer et al., 1999, p. 43):

$$\begin{aligned} E[(U(V)] &= U(\mu_V) + \frac{1}{2} \cdot U''(\mu_V) \cdot \sigma_V^2 + \ldots + \frac{1}{2^k \cdot k!} \cdot U^{(2k)}(\mu_V) \cdot (\sigma_V^2)^k + \ldots \\ &=: \Theta(\mu_V, \sigma_V^2). \end{aligned} \tag{1.6}$$

Θ is defined as the investor's preference function. Equation (1.6) reveals that the investor's utility solely depends on μ_V and σ_V^2. Hence, a determination of μ_V and σ_V^2 will sufficiently characterize the investor's preference in this scenario. It follows that every investor will act according to the $\mu - \sigma$ model.[5] Terminating the Taylor expan-

[5]The $\mu - \sigma$ model dates back to the seminal work of Markowitz (1952). It is treated in more detail in

sion after the second term and evaluating the differentials of μ_V and σ_V^2 shows that the investor's preferences result in favoring higher means and smaller standard deviations (see Rubinstein, 2006, p. 82):

$$\frac{\partial \Theta}{\partial \mu_V} = U'(\mu_V) > 0 \text{ and } \frac{\partial \Theta}{\partial \sigma_V^2} = U''(\mu_V) < 0. \tag{1.7}$$

Put differently, the investor prefers higher expected returns and lower risk. Under this assumption, an asset with the greatest expected return for a given level of variance and, simultaneously, the smallest variance for a given expected return is optimal.

Because the focus of the thesis is on modeling bond yields, this introduction demonstrates the consequences of the expected utility of Equation (1.6) for the stochastic process $X(t)$ of Equation (1.3). The Vasicek (1977) model is a Gaussian model of the instantaneous short rate $r(t)$. In what follows, the linkage between the $\mu - \sigma$ bond investor and the Vasicek model is presented, knowledge of which is a necessary precondition for understanding why the corporate and international bond models of Chapters 2 and 3 are of the Vasicek type. In the Vasicek model, the functions $\mu(\cdot)$ and $\sigma(\cdot)$ (see Equation (1.3)) reduce to $\mu(X(t), t) = K[\vartheta - X(t)]$ and $\sigma(X(t), t) = \Sigma$. The variables K, ϑ, and Σ determine the correlation of the factors, the speed of mean reversion, and the volatility of the factors, respectively. Rewriting the stochastic process $X(t)$ of Equation (1.3) gives (see Filipovic, 2009, p. 85):

$$dX(t) = K[\vartheta - X(t)]dt + \Sigma \cdot dW(t). \tag{1.8}$$

Branger and Schlag (2004) show that $X(t)$ is normally distributed when defined as in Equation (1.8), which is in line with the investor defined above, who acts based on the $\mu - \sigma$ preference. Restricting the model to this $\mu - \sigma$ preference has important

Subsection 1.1.2. For the sake of brevity, the subscripts and superscript of μ_V and σ_V^2 are dropped when speaking of the $\mu - \sigma$ model.

consequences. The following subsections illustrate these consequences for corporate bond pricing (Subsection 1.1.1) and international bond pricing (Subsection 1.1.2).

1.1.1 Corporate bonds

Corporate bonds are popular instruments for raising funds in a variety of industries, including public utilities, transportation, banks/finance, and industrials (see Fabozzi, 2013, p. 153). A *corporate bond* is a bond – as defined above – that is issued by corporations (see Berk and DeMarzo, 2011, p. 233). A bond issued by a corporation implies that repayment of the amount borrowed is uncertain. A *default* occurs when a bond issuer fails to satisfy the terms of the obligation with respect to the timely payment of interest and repayment of the amount borrowed (see Fabozzi, 2013, p. 19). Rating agencies try to quantify the issuer's ability to meet its future contract obligations and summarize the result in a single mark. The idea of a rating is the same across all big rating agencies, although the notation varies. In this thesis, S&P's (Standard & Poor's Corporation) notation is used. Ratings generally fall into two main categories: investment grade and non investment grade. The notation for credit worthiness in descending order of the investment grade category is: AAA, AA, A, and BBB. Noninvestment grade ratings are: BB, B, CCC, and C. A rating of D indicates a defaulted bond. Since investment grade bonds have the biggest market capitalization, the analysis is restricted to AAA, AA, A, and BBB rated bonds. In addition, the notation TR is introduced to denote treasury bonds, which have a credit worthiness higher than AAA.

The main challenge in modeling corporate bonds is modeling default. Models of default basically take one of two forms: structural models or intensity-based models. The focus of the present thesis is on intensity-based models.[6] Intensity-based models do not require that the short rate is explicitly observable. Leaving aside the question of what

[6] Structural models model firm value explicitly. The structural model assumes that a corporation defaults when its assets drop below the value of its liabilities. The idea dates back to Black and Scholes (1973) and Merton (1974). See Duffie and Singleton (2003) and Lando (2004) for a thorough introduction to structural models.

exactly triggers the default event allows employing the entire machinery of default-free term structure modeling (see Lando, 2004).[7] That is, the econometric specification from term structure modeling and the knowledge of pricing derivatives can be transferred to defaultable claims.

Lando (1998) and Duffie and Singleton (1999) propose intensity-based models for analyzing corporate bond yields. The affine term structure model (ATSM) is subject to restrictions imposed by the absence of arbitrage opportunities.[8] Default risk is modeled using a doubly-stochastic intensity-based framework, where risk-neutral instantaneous default loss rates are assumed to be affine functions of state variables (see Amato and Luisi, 2006).[9] In the event of default, the firm's assets are liquidated and distributed to the lenders, a procedure referred to as *recovery*. An important assumption in modeling corporate default has to do with how recovery takes place (see Lando, 2004, p. 120). Following is a brief discussion of the three most prevalent recovery assumptions.

Brennan and Schwartz (1980) propose the recovery of face value (RFV) assumption, which measures the value of recovery as a fraction of face value. This is close to what occurs in actual practice, where debt with the same priority is assigned a fractional recovery, corresponding to the notional amount outstanding and leaving coupon payments aside. The quantity is computed via a post-default market price. According to Moody's, this is 30 days after the default date. The shortcoming is that the bond price has no analytical solution.

The recovery of market value (RMV) assumption dates back to Duffie and Singleton (1999). The change in market value determines the amount recovered. The economic meaning is straightforward. The change in the bond price at the time of default is what

[7] Interest rates depend on the horizon, the *term*, of the bond. The relationship between the bond term and the interest rate is called the *term structure* (see Berk and DeMarzo, 2011, p. 137). Models that capture the movements of all interest rates for the entire term structure are referred to as *term structure models.*

[8] Affine term structure models (ATSMs) are term structure models that assume an affine relationship between the model factors and the bond price. The focus of this thesis is on ATSMs.

[9] Since the risk-free rate and the credit spread are both modeled stochastically, this approach is referred to as a *doubly-stochastic* intensity-based framework.

market participants expect the bond to lose in value. The two prices before and after the default date are the basis for evaluating the change. However, it is difficult to pin down the price right before the default and to segregate a single drop in value. The RMV offers closed-form solutions for bond prices and is convenient to use.

Schönbucher (1998) proposes the multiple default (MD) assumption. Similar to the RMV assumption, recovery is measured as the loss in terms of a price drop at default date. However, the recovery is not actually paid to the investor after default; instead, the debt is restructured. Restructuring may occur multiple times. It is convenient to assume restructuring of debt and deal with the bond's market price. To this end, recovery is measured in accordance with the MD assumption in this thesis.

Recall the time-t price of a zero-coupon bond maturing at time $t + \tau$ as provided in Equation (1.1). This zero-coupon bond and the corporate bond, soon to be defined, are from a single economy. This assumption will be relaxed in Subsection 1.1.2. The default-risk free instantaneous short rate $r(t)$ determines the price of a government bond. Assuming a scenario for the default date enables the investor to adapt the instantaneous short rate $r(t)$ to default risk. Let $h^{CB}(t)$ denote the hazard rate for default at time t and $L^{CB}(t)$ be the expected fractional loss due to all defaults up to time t. The corporate bond can then be priced using the default-adjusted short-rate process $R^{CB}(t) = r(t) + h^{CB}(t)L^{CB}(t), t \in [0, T]$. The time-$t$ price of a corporate bond with maturity τ is (see Duffie and Singleton, 1999; Schönbucher, 1998):

$$P^{CB}(t, \tau) = E_t^{\mathbb{Q}} \left[\exp(-\int_t^{t+\tau} R^{CB}(u)du) \right]. \tag{1.9}$$

The superscript CB indicates that $P^{CB}(t, \tau)$ is the price of a single corporate bond. Therefore, the time value discounted with the short-rate process is $R^{CB}(u)$. This is in contrast to the bond price $P(t, \tau)$ of Equation 1.1, where $r(u)$ constitutes the short-rate process.

Equation (1.9) specifies the price of a single corporate bond. There is evidence, however, that corporate bonds should not be viewed in isolation. Collin-Dufresne et al. (2001) investigate the determinants of credit spread changes. A principal components analysis implies that these are mostly driven by common factors. Although the authors consider several financial variables as candidate proxies, they cannot explain these common systematic components. Their suggestion is that monthly credit spread changes are principally driven by "hidden" common factors.

However, studies do show that common factors determine a large fraction of the variation in corporate bond yields (see Amato and Luisi, 2006; Mueller, 2009; Speck, 2013). In what follows, the default-adjusted short rate is defined for multiple rating classes. The proposed model captures the joint variation in the common factors and the individual variability in the credit-specific factors. This model is also referred to as a *multi-rating ATSM*. Let $R^{CR}(t) = r(t) + h^{CR}(t)L^{CR}(t), t \in [0, T]$ be the default-adjusted short-rate process defined for different rating classes $CR = TR$, AAA, AA, A, and BBB. The time-t prices of corporate bonds with different credibility CR maturing at $t + \tau$ read:

$$P^{CR}(t, \tau) = E_t^{\mathbb{Q}} \left[\exp(-\int_t^{t+\tau} R^{CR}(u)du) \right]. \tag{1.10}$$

Equation (1.10) defines a corporate bond pricing model for multiple rating classes. For nesting the different rating classes, $R^{CB}(u)$ changes to $R^{CR}(u)$ in comparison to Equation (1.9). This multi-rating ATSM is described in detail in Chapter 2.

The source of randomness of R^{CR} is assumed to follow Equation (1.8). Restricting the investor to a μ-σ preference has immediate consequences for the corporate bond pricing model. Since the d-dimensional factor process $X(t)$ is normal, there are easy closed-form solutions for the bond prices. In the empirical study of Chapter 2 of this thesis, corporate bonds of five different rating classes ($CR = TR$, AAA, AA, A, and BBB) are priced with the multi-rating ATSM.

The corporate bond pricing study reveals that common factors capture a large fraction of the corporate yield variation. In particular, two common factors are economy wide and one factor is rating-specific. Nevertheless, these common factors are the source of risk in *one* market. Can these common factors also be found in different economies? The following subsection answers this question.

1.1.2 International bonds

Uncertainty is the predominant feature of investment. Economic forces are not well enough understood to predict their trajectory free from error. In addition, noneconomic influences can impact market prices or the success of a particular asset. Moreover, it is always possible that an asset does better, or worse, than even the most optimistic, or pessimistic, investor had any right to expect (see Markowitz, 1959). In short, no investor thinks it is a good idea to rely on a single asset. This is why they hold a *portfolio*.

Averaging out of independent risks in large portfolios is referred to as *diversification* (see Berk and DeMarzo, 2011, p. 209). Diversification is an important issue in bond management. Not surprisingly, the bonds of one market crucially depend on the interest rate level of the economy in which they are originated. Therefore, the main motivation for bond investment across countries is diversification, resulting in reduced risk for the investor (see Fabozzi, 2013, p. 199). However, increasing integration of international capital markets poses a challenge to this strategy. When correlations between asset returns increase, the benefits of international diversification vanish. Because naive diversification may no longer be sufficient, identifying common risk factors in international bonds is a vital task.

Common factors in international bond markets are identified in earlier studies. For example, Driessen et al. (2003) discover common risk factors in the U.S., German, and Japanese bond markets. Juneja (2012) shows that the U.S., the U.K., and Germany share risk factors. However, these studies do not provide a model for describing the term structure of interest rates in different countries. Egorov et al. (2011) focus on modeling aspects and provide a classification of joint affine term structure models (joint ATSMs).[10]

The existence of common factors in international bond markets raises two questions. (3) What are the consequences for the specification of a joint ATSM? (4) How can an optimal portfolio model be set up that accounts for these common factors? The questions will be discussed against the background of a μ-σ investor.

Suppose that the uncertainty of two economies can be described by a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ denotes the physical measure. Let $\mathbb{Q}$ and $\mathbb{Q}^*$ be the equivalent risk-neutral measures for the domestic and foreign economy, respectively. In the absence of arbitrage, the time-t prices of domestic and foreign zero-coupon bonds $P(t,\tau)$ and $P^*(t,\tau)$ that mature at $t+\tau$ are given by (see Egorov et al., 2011):

$$P(t,\tau) = E_t^{\mathbb{Q}} \left[\exp(-\int_t^{t+\tau} r(u)\, du) \right] \text{ and} \tag{1.11}$$

$$P^*(t,\tau) = E_t^{\mathbb{Q}^*} \left[\exp(-\int_t^{t+\tau} r^*(u) du) \right]. \tag{1.12}$$

$E_t^{\mathbb{Q}}$ and $E_t^{\mathbb{Q}^*}$ denote $\mathcal{F}_t$ conditional expectations under $\mathbb{Q}$ and $\mathbb{Q}^*$ at time t.

The international bond study in Chapter 3 offers an econometric analysis of two major government bond markets. A factor analysis segregates the common factors of both yield curves and a principal component analysis reveals their economic intuition. This econometric analysis sets the requirements for specifying a joint ATSM and answering question (3). The model fit and the interaction of yield and model factors are discussed. The μ-σ framework ensures analytical solutions for international bonds. In addition, the expected drift and variance of the common factors are available in closed-form. This is a vital input for the international bond investor. Put differently, the information in the current yield curve can be transferred to an expectation of the future path of economy.

[10] This classification is comparable to that done by Dai and Singleton (2000) for single term structure models.

This topic is discussed in the following.

In Equations (1.11) and (1.12) a joint ATSM is presented for pricing international bonds. The second part of this subsection builds on this model and the investor characterized in Equation (1.7) and asks what this investor's optimal portfolio would look like, thus linking the μ-σ bond investor of Section 1.1 and the portfolio application in Section 3.5.

Institutional investors attempt to diversify their risk by investing in bonds of different economies rather than exposing their entire investment to the interest rate risk in a single asset. That is, the investor sets up a portfolio of bonds. Jones (2011) characterizes portfolio management as identifying and managing risk factors of financial assets. Bond investment is different from equity investment in many aspects. Hence, the application of famous equity portfolio models, such as the portfolio selection model (see Markowitz, 1952), is not straightforward. The ideas, however, are the same and provide a useful starting place. Throughout this Section to Part I of this thesis, the investor is assumed to act according to a μ-σ preference. The present section takes a portfolio approach to this preference.

The main risk factor of a fixed income asset is interest rate risk (also referred to as market risk) (see Fabozzi, 2013, p. 18).[11] Interest rate risk can be characterized as follows. The bond price changes in a direction opposite to that taken by the interest rate. The investor faces a loss if interest rates are going up and he sells prior to maturity. The bond's sensitivity to interest rate changes mainly depends on the coupon payments, the time to maturity, and the interest rate level. A good indicator of this sensitivity is *duration*. Duration is the approximate change in bond price for a given change in yield (see Fabozzi, 2013, p. 75). A serious drawback of the duration is that only small, parallel shifts of the yield curve are appropriately covered. Any hump building or change in slope is forfeited by the linear approximation. A second order approximation can handle these

[11] See Fabozzi et al. (2006) and the references therein for a treatment of other, minor risk factors.

changes, which are referred to as *convexity*. Convexity is a quadratic approximation of the price change due to a changing yield. A simple portfolio model tries to set the net duration and convexity to zero so as to immunize the portfolio against changes of the yield curve (see Hull, 2011, p. 90). This portfolio approach builds on the core of the yield curve characteristics. Yet the investor has no estimate of the risk taken.

Affine term structure models, however, can provide a prediction about the future path of the economy and these expectations as to future drift and variance can be transferred to the portfolio model. In an early study, Wilhelm (1992) proposes an optimal portfolio model where the short rate follows a CIR process. Puhle (2008) extends the model of Wilhelm (1992) by using a Hull-White model and studying an optimal portfolio in continuous time. Korn and Koziol (2006) study German government bonds and find that the model outperforms bond and equity indices in terms of the Sharpe ratio. What all these studies have in common is that they restrict their models to one country. As studies on bond diversification show, however, foreign assets can be beneficial for the investor(see Hunter and Simon, 2004; Hunter and Simon, 2005; Cappiello et al., 2003). To this end, the portfolio application in Chapter 3 contributes to the field by adding foreign bonds to the portfolio. Section 3.5.1 proposes an optimal portfolio model for an international bond investor and answers question (4). In addition, the expected returns and covariances are evaluated in closed-form. The investor's preferences as well as the common factors found in international yields help explain the portfolio weight adjustments.

1.2 Extension to higher order moments

Much of the finance research assumes that the past behavior of asset prices contains information about their future price behavior; that is, history repeats itself. Based on Bachelier (1900), returns are frequently assumed to follow a normal distribution, explicitly or implicitly. Throughout Section 1.1 to Part I of this thesis, the μ-σ bond investor acts accordingly. The model implications were sketched in Section 1.1:

- the existence of closed-form corporate bond prices,
- the existence of closed-form international bond prices, and
- the existence of analytical solutions for expected returns and covariances on portfolio level.

However, nearly since its inception, the normality assumption has been criticized. In early studies, Mandelbrot (1963), Fama (1965), and Press (1967) argue that the normal distribution is not able to fit financial returns accurately. The authors find that the empirical distributions mainly depart from the normal distribution in the tails. The authors thus propose different distributional assumptions that provide more flexibility in the fourth moment.[12] In addition to heavy tails, Peiró (1994) highlights the importance of skewness in financial returns.[13]

In light of this literature, this section introduces a more sophisticated investor who takes more advanced distributions into consideration in his investment decisions. This paves the way for the very important question of the relevance of higher order moments in bond yields. This question is answered in Part II.

In particular, Chapter 4 investigates the statistical distribution of price changes in European government bonds. In the period 1999 to 2012, Euro bonds with one, three, five, and ten years to maturity are tested for normality. Due to the skewness and excess kurtosis found in the data, alternative distributions that can account for these features are proposed.

In what follows, the μ-σ preference, assumed in Section 1.1 is extended to consider this departure from normality. Terminating the Taylor approximation of Equation (1.4)

[12] The normalized fourth moment is defined as $\omega_V^4 = \frac{E[(V-\mu_V)^4]}{\sigma_V^4}$. The fourth moment of the normal distribution is $3\sigma_V^4$ (see Christoffersen, 2012). Higher values of the fourth moment of an empirical distribution indicate a departure from normality and are also referred to as *heavy tails*.

[13] A literature review of studies concerned with alternative distributions for financial returns is provided in Chapter 4.

after the fourth term, the expected utility function accounts for the third and fourth moment:

$$\begin{aligned} E[(U(V)] &= U(\mu_V) + \frac{1}{2} \cdot U''(\mu_V) \cdot \sigma_V^2 + \frac{1}{6} \cdot U'''(\mu_V) \cdot \gamma_V^3 + \frac{1}{24} \cdot U''''(\mu_V) \cdot \omega_V^4 \\ &=: \Theta(\mu_V, \sigma_V, \gamma_V, \omega_V). \end{aligned} \tag{1.13}$$

γ_V^3 and ω_V^4 correspond to skewness and kurtosis, respectively. It is reasonable to assume a positive, decreasing marginal utility (see Breuer et al., 1999, p. 180).[14] An immediate consequence of this assumption is a positive third derivative ($U'''(\mu_V) > 0$) and a negative fourth derivative ($U''''(\mu_V) < 0$) of the utility function.

The investor's preferences result in him favoring greater odd and smaller even moments or, in other words, the investor prefers assets offering high returns on average (positive first moment) and little risk (small second moment). He seeks investments that have a higher probability of positive excess returns than negative excess returns (positive third moment) and he avoids financial instruments that are more likely to realize extreme positive or negative returns (small fourth moment). The derivatives of the preference function support this argument (see Breuer et al., 1999, p. 185):[15]

$$\frac{\partial \Theta}{\partial \mu_V} = U'(\mu_V) + \frac{1}{2} \cdot U'''(\mu_V) \cdot \sigma_V^2 + \frac{1}{6} U''''(\mu_V) \cdot \gamma_V^3 > 0, \tag{1.14}$$

$$\frac{\partial \Theta}{\partial \sigma_V} = U''(\mu_V) \cdot \sigma_V < 0, \tag{1.15}$$

$$\frac{\partial \Theta}{\partial \gamma_V} = \frac{1}{2} U'''(\mu_V) \cdot \gamma_V^2 > 0, and \tag{1.16}$$

$$\frac{\partial \Theta}{\partial \omega_V} = \frac{1}{6} U''''(\mu_V) \cdot \omega_V^3 < 0. \tag{1.17}$$

[14] Raa (2013) defines *marginal utility* as: *The marginal utility of a good is the rate of change in utility, with respect the quantity of goods.*

[15] The Taylor expansion of the expected utility function is terminated after the fourth derivative of $U(\mu_V)$. In other words, the fifth derivative is assumed to be zero ($U'''''(\mu_V) = 0$). To this end, $U''''(\mu_V)$ is the derivative of $\frac{\partial \Theta}{\partial \mu_V}$ exhibiting the highest order.

Higher order moments can be an important feature of bond yields and there are generally two ways to account for them in bond yield models (see Dai and Singleton, 2003, p. 651). [1] *Regime shifts* generate a persistent period of "turbulence" and "quiet" in bond models. [2] *Jumps* add large yield movements at discrete points in time.

A regime shift can be introduced to the short-rate process to model different states of the economy. The best-known example of a regime shift occurred over the period 1979 to 1982, also referred to as the Federal Reserve experiment (see Chapman and Pearson, 2001). The foundation of the European monetary union and the recent Euro crisis constitutes reasons for another regime shift (see Section 4.5). Regime shifts can be introduced to ATSMs by relaxing the restrictions of the market price of risk. The instantaneous short rate $r(t)$ defined in Equation (1.2) can be adjusted for accommodating different regimes $s(t)$ (see Dai and Singleton, 2003, p. 652):

$$r^i(t) = r[s(t) = i; X(t), t] = \delta_0^i + \delta^{i'} X(t). \tag{1.18}$$

The different regimes can be used to account for skewness in bond yields.[16] However, the overall evidence of skewness in (government) bonds is weak (see Section 4.2) and the third moment plays a minor role in modeling bond yields.

Jump extensions of short-rate processes can provide enough flexibility to model excess kurtosis in bond yields. This excess kurtosis in the distribution of financial returns corresponds to rare events that are underestimated by the normal distribution. Large movements in bond yields usually occur around monetary policy news, rating changes, or other exceptional events at discrete points in time. These large movements can be modeled as discontinuous moves or *jumps* in the state vector (see Piazzesi, 2003, p. 19). Allowing the d-dimensional factor process $X(t)$ of Equation (1.3) to follow a

[16] Hamilton (1988), Gray (1996), and Ang and Bekaert (2002) propose different regime-switching models. Dai and Singleton (2003) provide an overview of regime shifts and a classification in the context of general ATSMs.

jump diffusion results in (see Dai and Singleton, 2003, p. 649):

$$dX(t) = \mu(X(t), t)\, dt + \sigma(X(t), t)\, dW(t) + \Delta X(t) dZ(t). \tag{1.19}$$

Z is a Poisson counter, with state dependent intensity $\left\{\lambda^{\mathbb{P}}(X(t)) : t \geq 0\right\}$ that is a positive, affine function of $X(t)$, $\lambda^{\mathbb{P}}(X(t)) = l_0 + l'X(t)$; and $\Delta X(t)$ is the jump amplitude with distribution $\nu^{\mathbb{P}}$ on $\mathbb{R}^N$. Hence, based on the results of Part II of this thesis, jumps might be a starting point for modeling excess kurtosis in bond yields.

1.3 Outline of the thesis

The thesis is organized in two parts. Part I assumes a bond investor with μ-σ preferences. The first chapter of Part I revisits the corporate bond pricing model of Subsection 1.1.1. It presents a convenient multi-rating ATSM with no-arbitrage restrictions, focusing on the application to the term structure of interest rates for corporate bonds. The empirical study covers U.S. treasury and corporate bonds with different levels of credibility (AAA, AA, A, and BBB) in the period 2002 to 2013. In addition to two common treasury factors, the analysis reveals a common credit factor. This leads to the conclusion that credit spreads provide information about the business cycle that is not found in the treasury yield curve. The proposed model supports the finding of a common credit factor by illustrating the link between corporate bond spreads and the state of economy. Seven factors explain 89% of the yield variation and exhibit a clear economic intuition. In summary, the model shows an excellent fit of the treasury and corporate bond yields across all rating classes and provides solid economic intuition for the latent factors.

The second chapter of Part I investigates common factors in U.S. and U.K. treasury yields in the period 1983 to 2012. A factor analysis determines the number of common and local factors that drive both yield curves. A principal component analysis reveals the economic intuition of the latent factors. Based on these, a joint ATSM is proposed that is capable of modeling the variability in treasury yields of both economies. A detailed analysis of yield and model factors illustrates the link between the econometric analysis and the proposed model. In addition, a bond portfolio application demonstrates a possible extension of the proposed model and shows how the investor's choice reflects the common and local factors. In summary, two common factors explain 85% of the yield variation and the yield factors maintain their importance for interpreting the joint ATSM and understanding the investor's portfolio adjustments.

Section 1.2 revealed the shortcomings of the normality assumption and introduced a more sophisticated investor who takes higher order moments of bond yields into consideration. This more sophisticated investor is studied in Part II. Particularly, Chapter 4 assesses the statistical distribution of daily EMU bond returns for the period 1999 to 2012. The normality assumption is tested and clearly rejected for all European countries and maturities. Although skewness plays a minor role, the departure from normality is mainly due to the excess kurtosis of bond returns. Therefore, we (the chapter was co-written with Christian Lau) test the Student's t, skewed Student's t, and stable distributions that exhibit this feature. The financial crisis leads to a structural break in the time series. We account for this and retest the alternative distributions. A value-at-risk application underlines the importance of the findings for investors. In sum, excess kurtosis in bond returns is essential for risk management, and the stable distribution captures this feature best.

Part I

Bond pricing with μ-σ preference

2 Corporate bond pricing: a multi-rating model

2.1 Introduction

Precise modeling of the term structure of corporate debt is crucial to those investing in financial products exposed to corporate default. Therefore, accounting for the interaction of corporate bond spreads of distinct rating classes is an important input to the risk management of corporate loan portfolios.

The chapter presents a convenient multi-rating affine term structure model (multi-rating ATSM) with no-arbitrage restrictions, focusing on the application to the term structure of interest rates for corporate bonds. Default risk is exogenously modeled using a double-stochastic intensity-based process. In this setting, the risk-neutral instantaneous default loss rates are assumed to be affine functions of the state variables. The empirical study covers U.S. treasury and corporate bonds with different levels of credibility (AAA, AA, A, and BBB) in the period 2002 to 2013. A detailed analysis of model and yield factors provides new information on the interplay of corporate bond spreads and the state of economy.[1]

Single ATSMs are a powerful framework for modeling the term structure of interest rates as an affine function of the state variables (see Chapman and Pearson, 2001).

[1] Amato and Luisi (2006) are the first to model multi-rating classes in an intensity-based framework. For a review of intensity-based processes in corporate bond pricing see Duffie (2011).

Beginning with Lando (1998) and Duffie and Singleton (1999), intensity-based models are also used for pricing default risk. In contrast to the structural approach of Merton (1974), intensity-based models assume the default rate to be exogenously given. Duffie (2011) provides an extensive review of models of the defaultable term structure, whereas Giesecke et al. (2011) study corporate bond pricing models with long history data of over 150 years.

There is strong empirical evidence that these corporate bond prices are systematically related to the state of economy. Bernanke et al. (1999) are the first to find a linkage between credit spreads, economic output, and inflation. Pinning it down to U.S. corporate bonds, the monthly correlation between BBB-rated U.S. corporate bonds and real output is -0.52 (see Amato and Luisi, 2006). Default loss rates show a negative correlation to the business cycle (see Altman et al., 2005). Cantor and Mann (2003) find a procyclicality of credit quality changes for Moody's credit ratings data.[2]

Bearing the procyclicality in mind, it is desirable to model this relationship of treasury and corporate bond yields jointly. Amato and Luisi (2006) propose a no-arbitrage term structure model of U.S. treasury bonds and BBB- and B-rated corporate bonds. Mueller (2009) explores credit spreads of different rating classes and their transmission to GDP growth. Speck (2013) uses a joint model of the term structure of U.S. treasury yields and U.S. corporate bond yields to work out whether credit conditions contain information about the business cycle.[3] However, there appears to be "unknown" factors that determine price changes of corporate bond spreads (see Collin-Dufresne et al., 2001). Models with latent factors have already proven to work well for modeling unknown factors in the context of international bond models (see Sarno et al., 2012; Graveline and Joslin, 2011; Egorov et al., 2011).

[2]See Duffie et al. (2007) for an overview of the research concerned with the relationship of macroeconomic factors and corporate default prediction.

[3]Bhar and Handzic (2011) propose a three-factor credit spread model for different rating classes. However, their model does not take the whole term structure of interest rates into account but only ten spreads. Thus, their multi-factor model is in the APT (see Ross, 1976) and not in the term-structure model (see Vasicek, 1977) sense.

The chapter contributes to the literature in modeling these unknown factors of corporate bonds. To the best of my knowledge, the proposed model is the first multi-rating ATSM for corporate bonds exclusively driven by latent factors. A principal component analysis identifies the number of factors that drive treasury and corporate bond yields. Seven factors explain 89% of the variation of five rating classes. The proposed model supports the finding of a common credit factor by illustrating the link between corporate bond spreads and the state of economy. Altogether, the model shows an excellent fit of the treasury and corporate bond yields across all rating classes and provides a solid economic intuition of the latent factors.

The remainder of the chapter is organized as follows. Section 2.2 proposes a multi-rating ATSM to match the common and rating-specific factors. Section 2.3 presents the data and provides a factor analysis of U.S. bond yields in the sample period. Section 2.4 illustrates the results and links the empirical and model factors. The chapter concludes with Section 2.5.

2.2 The corporate bond pricing model

Significant improvements have been made in modeling the single term structure of interest rates for pricing government, sovereign, or corporate bonds and their derivatives.[4] However, investors, who are exposed to corporate default, usually invest in a portfolio of corporate loans rather than in a single corporate borrower. Hereafter, the term multi-rating ATSM is used for those ATSMs that incorporate different rating classes of corporate bonds. Subsection 2.2.1 presents a multi-rating ATSM that is exclusively driven by latent factors and Subsection 2.2.2 introduces the corresponding state space model for applying the corporate bond pricing model to the data presented in Section 2.3.

[4] See Dai and Singleton (2003) for a survey of single term structure models.

2.2.1 The multi-rating model

Dai and Singleton (2000) state that in the absence of arbitrage the time-t price of a zero-coupon bond, maturing at time $t+\tau$, is given by

$$P(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} r(u)du)\right], \tag{2.1}$$

where $E_t^{\mathbb{Q}}$ denotes the expectation at time t under the risk-neutral measure $\mathbb{Q}$. Duffie and Singleton (1999) show that under the risk-neutral probability measure $\mathbb{Q}$, $h^{CB}(t)$ denotes the hazard rate for default at time t and $L^{CB}(t)$ the expected fractional loss due to all defaults up to time t. The corporate bond can then be priced using the default-adjusted short-rate process $R^{CB}(t) = r(t) + h^{CB}(t)L^{CB}(t), t \in [0,T]$. Time-$t$'s price of the corporate bond $P^{CB}(t,\tau)$ with maturity τ is:

$$P^{CB}(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} R^{CB}(u)du)\right]. \tag{2.2}$$

Equation (2.2) specifies the price of a single corporate bond. In what follows, the default-adjusted short rate is defined for multiple rating classes. The proposed model captures the joint variation in the common factors and the individual variability in the rating-specific factors. A multi-rating ATSM is obtained under the assumption that the instantaneous short rates $R^{CR}(t) = r(t) + h^{CR}(t)L^{CR}(t), t \in [0,T]$ with CR soon to be defined. The time-t prices of corporate bonds $P^{CR}(t,\tau)$ with different credibility CR maturing at $t+\tau$ read:

$$P^{CR}(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} R^{CR}(u)du)\right]. \tag{2.3}$$

Equation (2.3) defines a corporate bond pricing model for multiple rating classes. For nesting the different rating classes, $R^{CB}(u)$ changes to $R^{CR}(u)$ in comparison to Equation (2.2).

$R^{CR}(t)$ are affine functions of a vector of latent state variables $X(t) = [X_1(t), X_2(t), \ldots, X_N(t)]'$. $R^{CR}(t)$ are defined for different rating classes $CR = TR$, AAA, AA, A, and BBB. Treasury bonds are commonly assumed to be default-risk free. Therefore, no hazard rate of default determines the treasury bond yields. Here and henceforth, the notation $P^{CR}(t,\tau)$ with $CR = TR$ denotes treasury bonds.[5] Based on Mueller (2009) three common factors and one rating-specific factor for each rating class are used.[6] This corresponds to a $A_0(7)$ multi-rating ATSM in the sense of Dai and Singleton (2000). That is, $N = 7$ and $X(t)$ is a 7×1 vector of latent state variables. The short-rate processes $R^{CR}(t)$ read:

$$\begin{bmatrix} R^{TR}(t) \\ R^{AAA}(t) \\ R^{AA}(t) \\ R^{A}(t) \\ R^{BBB}(t) \end{bmatrix} = \begin{bmatrix} \delta_0^{TR} \\ \delta_0^{AAA} \\ \delta_0^{AA} \\ \delta_0^{A} \\ \delta_0^{BBB} \end{bmatrix} + \begin{bmatrix} \delta_1^{TR} & \delta_2^{TR} & \delta_3^{TR} & 0 & 0 & 0 & 0 \\ \delta_1^{AAA} & \delta_2^{AAA} & \delta_3^{AAA} & \delta_4^{AAA} & 0 & 0 & 0 \\ \delta_1^{AA} & \delta_2^{AA} & \delta_3^{AA} & 0 & \delta_5^{AA} & 0 & 0 \\ \delta_1^{A} & \delta_2^{A} & \delta_3^{A} & 0 & 0 & \delta_6^{A} & 0 \\ \delta_1^{BBB} & \delta_2^{BBB} & \delta_3^{BBB} & 0 & 0 & 0 & \delta_7^{BBB} \end{bmatrix} \begin{bmatrix} X_1^{com}(t) \\ X_2^{com}(t) \\ X_3^{com}(t) \\ X_4^{AAA}(t) \\ X_5^{AA}(t) \\ X_6^{A}(t) \\ X_7^{BBB}(t) \end{bmatrix}. \tag{2.4}$$

δ_i^{CR} (for $i = 0, \ldots, 7$ and $CR = TR$, AAA, AA, A, and BBB) are scalars. Without loss of generality, $X_i^{com}(t)$ for $i = 1, 2, 3$ are assumed to be the common factors. $X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, and $X_7^{BBB}(t)$ are rating-specific factors. Therefore, $X(t)$ nests the common and rating-specific factors that drive the economy. Common factors enter all short rates through non zero δ_i^{CR}'s (for $i = 1, 2, 3$ and $CR = TR$, AAA, AA, A, and BBB). A similar common factor weighting across all rating classes implies the existence

[5] The treasury bond is assumed to be risk-free. That is, $h^{CR}(t)$ is set zero and $R^{TR}(t) = r(t) + 0 \cdot L^{CR}(t) = r(t)$.

[6] Mueller (2009) models three different rating classes with three local factors plus the additional common factors.

of an economy-wide risk factor. If, in contrast, some risk factors only affect specific rating classes the others will have a weighting close to zero. This provides information on whether there are common risk factors in addition to the treasury yield factors. The existence of a common credit factor, indicated by the parameter estimation, might provide insight into the business cycle.

The rating-specific factors are forced to be mutually independent. That is, $\delta_i^{CR} = 0$ for rating-specific factors of other rating classes. Otherwise the rating-specific factors would be common. They may, however, depend on each other through correlated common factors. Postulating the independence, the multi-rating ATSM can be decomposed into five single ATSMs (see Egorov et al., 2011). Mueller (2009), Graveline and Joslin (2011), and Speck (2013) propose using a Gaussian ATSM, since the correlation structure is more flexible. The joint dynamics of $X(t)$ follow an affine diffusion of the form:

$$dX(t) = K[\vartheta - X(t)]dt + \Sigma \cdot dW(t). \tag{2.5}$$

$W(t)$ is a 7-dimensional independent Brownian motion under the physical measure $\mathbb{P}$. K and Σ are 7×7 parameter matrices and ϑ is a 7×1 parameter vector.

The chapter aims to provide an economic intuition of the model factors. To this end, the risk premium is restricted to be non-time-varying. That is, the Gaussian process is *completely* affine (see Duffee, 2002).[7] The risk premium is defined as a constant 7×1 parameter vector λ^{CR}.

As outlined above, the multi-rating ATSM can be decomposed into five single ATSMs. Under the risk-neutral measure $\mathbb{Q}$ the rating-specific affine diffusion

$$dX(t) = K^{\mathbb{Q}}[\vartheta^{\mathbb{Q}} - X(t)]dt + \Sigma \cdot dW^{\mathbb{Q}}(t), \tag{2.6}$$

[7]Feldhütter et al. (2012) argue that investors even prefer simple (completely affine) to more complex (essentially affine) models.

where $dW^{\mathbb{Q}}(t) = dW(t) - \lambda^{CR}dt$, $\vartheta^{\mathbb{Q}}$, and $K^{\mathbb{Q}}$ represent the parameters under the risk-neutral measure. Under the risk-neutral measure $\mathbb{Q}$ the price of a zero-coupon bond reads:

$$P^{CR}(t,\tau) = \exp(-A_\tau^{CR} - B_\tau^{CR'} X(t)). \tag{2.7}$$

A_τ^{CR} and B_τ^{CR} satisfy the ordinary differential equations (ODEs) (see Dai and Singleton, 2000):

$$\frac{dA_\tau^{CR}}{\tau} = \vartheta' K B_\tau^{CR} - \frac{1}{2}\sum_{i=1}^{7}[\Sigma' B_\tau^{CR}]_i^2 - \delta_0^{CR} \text{ and} \tag{2.8}$$

$$\frac{dB_\tau^{CR}}{\tau} = -K B_\tau^{CR} - \frac{1}{2}\sum_{i=1}^{7}[\Sigma' B_\tau^{CR}]_i^2 + \delta^{CR'}. \tag{2.9}$$

Hereby the ODEs are completely specified and their solutions are available in closed-form. Kim and Orphanides (2012) present convenient closed-form solutions in vector notation. The corporate bond yields with maturity τ read:[8]

$$y_\tau(t) = A_\tau^{CR} + B_\tau^{CR} \cdot X(t) \tag{2.10}$$

with

$$\begin{aligned} A_\tau^{CR} =& -\frac{1}{\tau}[(K\vartheta)'(m_{1,\tau} - \tau I)K^{-1'}\delta^{CR} \\ &+ \frac{1}{2}\delta^{CR'}K^{-1}(m_{2,\tau} - \Sigma\Sigma' m_{1,\tau} - m_{1,\tau}\Sigma\Sigma' + \tau\Sigma\Sigma')K^{-1'}\delta^{CR} - \tau\delta_0^{CR}] \text{ and} \\ B_\tau^{CR} =& \frac{1}{\tau} m_{1,\tau}\delta^{CR}, \end{aligned}$$

where

$$\begin{aligned} m_{1,\tau} =& -K^{-1'}(exp(-K'\tau) - I) \quad \text{and} \\ m_{2,\tau} =& -vec^{-1}((K \otimes I) + (I \otimes K))^{-1} vec(exp(-K\tau)\Sigma\Sigma' exp(-K'\tau) - \Sigma\Sigma'). \end{aligned}$$

[8] $vec(C)$ denotes the vectorization of C: $vec(C) = [c_{1,1}, \ldots, c_{i,1}, c_{1,2}, \ldots, c_{i,2}, c_{1,j}, \ldots, c_{i,j}]'$. $\otimes$ denotes the Kronecker product. I is the identity matrix.

A_τ^{CR}, B_τ^{CR} are functions of $K, \vartheta, \Sigma, \delta^{CR}, \delta_0^{CR}$, and τ. The risk-neutral and physical parameters correspond in the following way:

$$K = K^{\mathbb{Q}} \tag{2.11}$$

$$\vartheta = \vartheta^{\mathbb{Q}} - K^{-1}\Sigma\lambda^{CR} \tag{2.12}$$

To avoid over identification, Dai and Singleton (2000) propose parameter restrictions and set K lower triangle, Σ the identity matrix, and $\vartheta^{\mathbb{Q}}$ zero. Since the rating-specific factors are required to be mutually independent, $\kappa_{ij} = 0$ for $4 \leq i, j$ and $i \neq j$. Under the physical measure the diffusion process is given by:

$$d\begin{bmatrix} X_1^{com}(t) \\ X_2^{com}(t) \\ X_3^{com}(t) \\ X_4^{AAA}(t) \\ X_5^{AA}(t) \\ X_6^{A}(t) \\ X_7^{BBB}(t) \end{bmatrix} = \begin{bmatrix} \kappa_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 & 0 & 0 & 0 & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & 0 & 0 & 0 & 0 \\ \kappa_{41} & \kappa_{42} & \kappa_{43} & \kappa_{44} & 0 & 0 & 0 \\ \kappa_{51} & \kappa_{52} & \kappa_{53} & 0 & \kappa_{55} & 0 & 0 \\ \kappa_{61} & \kappa_{62} & \kappa_{63} & 0 & 0 & \kappa_{66} & 0 \\ \kappa_{71} & \kappa_{72} & \kappa_{73} & 0 & 0 & 0 & \kappa_{77} \end{bmatrix} \begin{bmatrix} -X_1^{com}(t) \\ -X_2^{com}(t) \\ -X_3^{com}(t) \\ -X_4^{AAA}(t) \\ -X_5^{AA}(t) \\ -X_6^{A}(t) \\ -X_7^{BBB}(t) \end{bmatrix} dt + d\begin{bmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \\ W_4(t) \\ W_5(t) \\ W_6(t) \\ W_7(t) \end{bmatrix}. \tag{2.13}$$

2.2.2 The state space model

Modeling the term structure of interest rates involves matching the evolution over time (time series) and the different yields depending on time to maturity (cross section). Affine term structure models have the positive feature that they capture the variation in time in the factors and the bond prices are a function of time to maturity. A natural way to approach this panel data is the state space model (see DeJong, 2000).

Modeling yield curves with completely affine term structure models is straightforward because the diffusion processes follow a Gaussian distribution. Thus, the parameters can be estimated via Kalman filtering with direct maximum likelihood estimation (see Babbs and Nowman, 1999). Let there be observations for maturities τ_1 trough τ_3. The coefficients in the vectors $y(t)$ and A and matrix B read (see Dewachter et al., 2006):

$$y(t)=\begin{bmatrix} Y^{TR}_{\tau_1}(t) \\ Y_{\tau_2}TR(t) \\ Y^{TR}_{\tau_3}(t) \\ Y^{AAA}_{\tau_1}(t) \\ Y^{AAA}_{\tau_2}(t) \\ Y^{AAA}_{\tau_3}(t) \\ Y^{AA}_{\tau_1}(t) \\ \vdots \\ Y^{BBB}_{\tau_3}(t) \end{bmatrix}, A=\begin{bmatrix} A^{TR}_{\tau_1} \\ A^{TR}_{\tau_2} \\ A^{TR}_{\tau_3} \\ A^{AAA}_{\tau_1} \\ A^{AAA}_{\tau_2} \\ A^{AAA}_{\tau_3} \\ A^{AA}_{\tau_1} \\ \vdots \\ A^{BBB}_{\tau_3} \end{bmatrix}, B=\begin{bmatrix} B^{TR}_{\tau_1,1} & B^{TR}_{\tau_1,2} & B^{TR}_{\tau_1,3} & 0 & 0 & 0 & 0 \\ B^{TR}_{\tau_2,1} & B^{TR}_{\tau_2,2} & B^{TR}_{\tau_2,3} & 0 & 0 & 0 & 0 \\ B^{TR}_{\tau_3,1} & B^{TR}_{\tau_3,2} & B^{TR}_{\tau_3,3} & 0 & 0 & 0 & 0 \\ B^{TR}_{\tau_1,1} & B^{TR}_{\tau_1,2} & B^{TR}_{\tau_1,3} & B^{AAA}_{\tau_1,4} & 0 & 0 & 0 \\ B^{TR}_{\tau_2,1} & B^{TR}_{\tau_2,2} & B^{TR}_{\tau_2,3} & B^{AAA}_{\tau_2,4} & 0 & 0 & 0 \\ B^{TR}_{\tau_3,1} & B^{TR}_{\tau_3,2} & B^{TR}_{\tau_3,3} & B^{AAA}_{\tau_3,4} & 0 & 0 & 0 \\ B^{TR}_{\tau_1,1} & B^{TR}_{\tau_1,2} & B^{TR}_{\tau_1,3} & 0 & B^{AA}_{\tau_1,5} & 0 & 0 \\ & & & \vdots & & & \\ B^{TR}_{\tau_1,1} & B^{TR}_{\tau_1,2} & B^{TR}_{\tau_1,3} & 0 & 0 & 0 & B^{BBB}_{\tau_3,7} \end{bmatrix}. \tag{2.14}$$

The state space has the form:[9]

$$y(t) = A + BX(t) + \epsilon(t) \quad \text{and} \tag{2.15}$$

$$X(t+h) = exp(-Kh)X(t) + (I - exp(-Kh))\vartheta^{\mathbb{Q}} + \upsilon(t+h). \tag{2.16}$$

h is the time between two observations. The measurement equation (2.15) is a function of the parameters δ_0^{CR}, δ^{CR}, K, λ^{CR}, and an error term $\epsilon(t)$. The dataset consists of more treasury yields than the model has factors for accurately estimating the parameters. According to Duan and Simonato (1999) and Geyer and Pichler (1997), all maturities are observed with a serially and cross-sectionally uncorrelated error $\epsilon(t)$. The transition equation (2.16) corresponds to the conditional mean and variance of the factors. According to $\epsilon(t)$, $\upsilon(t+h)$ is an error term and assumed to be serially and cross-sectionally uncorrelated.

[9] The notation $exp(C)$, where C is a square matrix, denotes the matrix exponential: $exp(C) = I + C + C^2/2 + C^3/6 + \cdots$ (see Kim and Orphanides, 2012).

2.3 Corporate bond data

The chapter aims to provide information on how many common and rating-specific factors drive the variability of bond returns. The subsequent question is: are these common factors covered by treasury yields or do common credit factors exist, that are economically meaningful? To this end, this section illustrates the treasury and corporate bond yield evolution, their correlations across rating classes, and a principal component analysis.

The analysis spans a broad range of rating classes (AAA, AA, A, and BBB) to test whether one multi-rating ATSM is capable of capturing the variability of corporate debt with different levels of credibility. The corporate bond indices are provided by Datastream for the period 12/08/2002 to 19/02/2013. U.S. treasury yields were collected for the same period from the U.S. Federal Reserve (see Gürkaynak et al., 2006). Treasury yields shall reflect economy-wide and credit-independent information on the business cycle. The analysis is based on weekly data. The cross-section of the dataset is two-, five-, and ten-years time to maturity.

Figure 2.1 illustrates the evolution of U.S. treasury and corporate bond yields. The figure shows a normal (upward sloping) yield curve for the majority of the sample period among all rating classes. The term structures exhibit the greatest slope in 2011 and an inverse yield curve in 2007, immediately before the credit crunch. The yield levels continue to remain on a low level from 2011 on.

Table 2.1 reports summary statistics of U.S. treasury and corporate bond yields (see Dewachter and Maes, 2001). The first panel shows the mean and standard deviation of the observed yields. The average yield curve is normal and the yield level is an increasing function of exposure to credit risk. The documented standard deviations indicate that the short end is more volatile than the long end across all rating classes.

The second panel reports the correlations among yields with different maturities within and between rating classes. Correlations are remarkably high within the treasury yield

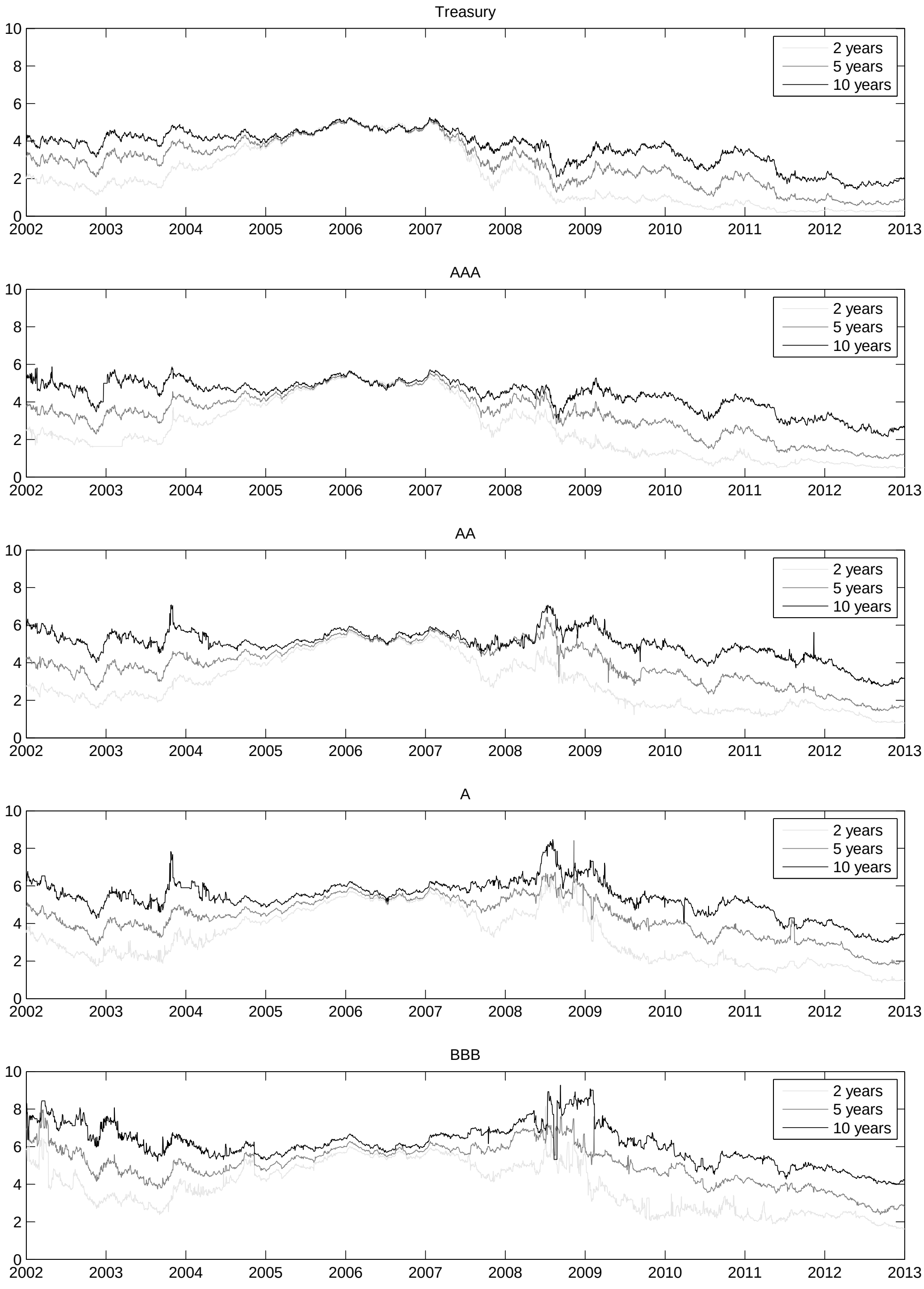

Figure 2.1: Time-series of U.S. treasury and corporate bonds

U.S. treasury and AAA-, AA-, A-, and BBB-rated U.S. corporate bond yields are provided by the U.S. Federal Reserve and Datastream, respectively. The time series of weekly yields covers the period 12/08/2002 to 19/02/2013. The cross-section corresponds to two-, five-, and ten-years time to maturity.

Table 2.1: Summary statistics of U.S. treasury and corporate bond yields

	TR			AAA			AA			A			BBB		
	2y	5y	10y	2y	5y	10y	2y	5y	10y	2y	5y	10y	2y	5y	10y
Mn	2.11	2.86	3.66	2.54	3.38	4.39	2.94	3.95	4.98	3.32	4.33	5.36	3.88	5.02	6.14
Std	1.60	1.28	0.95	1.55	1.23	0.84	1.40	1.14	0.78	1.44	1.09	0.94	1.32	1.07	1.09
							Correlations								
	1.00	0.88	0.71	0.66	0.62	0.55	0.63	0.57	0.18	0.36	0.40	0.24	0.17	0.26	0.13
TR		1.00	0.92	0.62	0.71	0.72	0.59	0.70	0.27	0.35	0.46	0.32	0.17	0.27	0.16
			1.00	0.54	0.67	0.76	0.47	0.68	0.30	0.29	0.44	0.37	0.17	0.26	0.17
				1.00	0.56	0.34	0.54	0.55	0.20	0.32	0.37	0.19	0.22	0.17	0.07
AAA					1.00	0.68	0.51	0.69	0.21	0.33	0.46	0.34	0.20	0.22	0.19
						1.00	0.46	0.67	0.23	0.30	0.47	0.37	0.13	0.28	0.21
							1.00	0.62	0.21	0.39	0.40	0.25	0.26	0.20	0.10
AA								1.00	0.29	0.46	0.54	0.39	0.23	0.23	0.08
									1.00	0.28	0.17	0.16	0.10	0.09	0.05
										1.00	0.60	0.18	0.30	0.22	0.10
A											1.00	0.39	0.33	0.20	0.19
												1.00	0.13	0.13	0.19
													1.00	0.24	0.00
BBB														1.00	0.31
															1.00

Eigenvalue Decomposition

Factor	1	2	3	4	5	6	7
Eigenvalue	0.34	0.18	0.14	0.07	0.06	0.05	0.05
Cumulative value	0.34	0.52	0.66	0.73	0.79	0.84	0.89

The table shows summary statistics of weekly U.S. treasury and corporate bond yields rated AAA, AA ,A, and BBB. Means (Mn) and standard deviations (Std) are reported in p.a. percentage points. The factor analysis is done via eigenvalue decomposition of the yield correlation matrix.

market (up to 0.92% for TR:5y vs TR:10y) and very high between treasury, AAA, and AA markets (up to 0.71 for TR:5y vs AAA:5y). The correlations between markets increase with credibility. However, they are still clearly positive for AA-, A-, and BBB-rated corporated bonds (up to 0.54 for AA five years to A five years). The summary statistics suggest the existence of a common risk factor in addition to the treasury yield factors.

A principal component analysis provides insights into how many factors drive the yield curve variability. To this end, the last panel shows the Eigenvalue decomposition. Seven factors already explain 89% of the yield volatility. This is a remarkably high value, considering that the dataset consists of 15 different bonds among treasury bonds and four different rating classes. On average, the time-varying seven factors, proposed in Section 2.2, are in line with the static principal component analysis reported in Table 2.1.

2.4 Results

This section applies the multi-rating ATSM, presented in Section 2.2, to the data, analyzed in Section 2.3. A presentation of the parameter estimation results in Subsection 2.4.1 precedes a factor analysis in Subsection 2.4.2.

2.4.1 Parameter estimation

Table 2.2 reports the parameters estimated with maximum likelihood. Throughout the table standard errors are given in parenthesis. The first panel shows the factor weighting δ. Factor weightings of other rating-specific factors are zero by definition and have no standard error.

The first common factor ($X_1^{com}(t)$) has high weightings among the different rating classes (ranging from $\delta_1^{AA} = 0.0074$ to $\delta_1^{TR} = 0.0152$). The interpretation of the factors is shifted to Table 2.4. However, the high weightings already indicate that the first common factor captures a large fraction of the yield variation. The importance of the second factor is an increasing function of credibility (ranging from $\delta_2^{A} = 0.0041$ to $\delta_2^{TR} = 0.0093$). In contrast, the third common factor gains influence with probability to default. It seems to be negligible for treasury yields ($\delta_3^{TR} = 0.0025$) but it is of high importance for AA and A rated bonds ($\delta_3^{AA} = 0.0102$ and $\delta_3^{A} = 0.0091$).

The rating-specific factors ($X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, and $X_7^{BBB}(t)$) account for the variation of corporate bond yields that is not captured by the common factors ($X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{com}(t)$). Hence, all rating-specific factors have high weightings ranging from 0.0111 ($= \delta_7^{BBB}$) to 0.0146 ($= \delta_4^{AAA}$). All δ's are estimated with high accuracy (ranging from 0.0011 to 0.0072) and the estimation error is always a small fraction of the parameter estimate itself. The interpretation of the factor loading (see Table 2.4) has to show

Table 2.2: Multi-rating ATSM parameter estimates

	Const	$X_1^{com}(t)$	$X_2^{com}(t)$	$X_3^{com}(t)$	$X_4^{AAA}(t)$	$X_5^{AA}(t)$	$X_6^{A}(t)$	$X_7^{BBB}(t)$
	i=0	i=1	i=2	i=3	i=4	i=5	i=6	i=7
δ_i^{TR}	0.0458	0.0152	0.0093	0.0025	0	0	0	0
	(0.0277)	(0.0019)	(0.0025)	(0.0011)	-	-	-	-
δ_i^{AAA}	0.0031	0.0119	0.0081	0.0064	0.0146	0	0	0
	(0.0255)	(0.0030)	(0.0025)	(0.0027)	(0.0072)	-	-	-
δ_i^{AA}	0.0523	0.0074	0.0058	0.0102	0	0.0120	0	0
	(0.0169)	(0.0054)	(0.0035)	(0.0041)	-	(0.0050)	-	-
δ_i^{A}	0.0278	0.0085	0.0041	0.0091	0	0	0.0129	0
	(0.0280)	(0.0030)	(0.0030)	(0.0026)	-	-	(0.0008)	-
δ_i^{BBB}	0.0628	0.0112	0.0062	0.0053	0	0	0	0.0111
	(0.0132)	(0.0024)	(0.0019)	(0.0013)	-	-	-	(0.0019)
κ_{1i}		0.2486	0	0	0	0	0	0
		(0.0726)	-	-	-	-	-	-
κ_{2i}		-0.3000	0.2493	0	0	0	0	0
		(0.1154)	(0.1646)	-	-	-	-	-
κ_{3i}		-0.1042	0.0675	0.3711	0	0	0	0
		(0.1417)	(0.0652)	(0.0891)	-	-	-	-
κ_{4i}		0.0499	-0.1372	-0.1595	0.1000	0	0	0
		(0.1228)	(0.0914)	(0.1989)	(0.2049)	-	-	-
κ_{5i}		-0.0361	-0.0360	-0.3000	0	0.1928	0	0
		(0.1637)	(0.1931)	(0.2403)	-	(0.2288)	-	-
κ_{6i}		0.1472	-0.1534	-0.1871	0	0	0.1142	0
		(0.1139)	(0.1017)	(0.1261)	-	-	(0.0140)	-
κ_{7i}		0.0162	0.0100	-0.1908	0	0	0	0.1000
		(0.0414)	(0.0770)	(0.0726)	-	-	-	(0.000)
λ_i^{TR}		-0.2011	0.3823	-0.2950	0	0	0	0
		(0.1988)	(0.4514)	(0.4286)	-	-	-	-
λ_i^{AAA}		-0.6725	0.2920	-0.4657	-0.2120	0	0	0
		(0.5882)	(0.5765)	(0.5272)	(0.3224)	-	-	-
λ_i^{AA}		0.7551	0.1309	-1.2082	0	-0.0106	0	0
		(0.8574)	(0.4592)	(1.2907)	-	(1.9622)	-	-
λ_i^{A}		-0.3066	0.2234	-0.5776	0	0	-0.2448	0
		(0.5895)	(0.2141)	(0.3334)	-	-	(0.2934)	-
λ_i^{BBB}		-0.3006	-0.9278	-1.6156	0	0	0	1.4052
		(0.4708)	(0.8339)	(0.7639)	-	-	-	(0.7405)

The table reports the maximum likelihood parameter estimation results of the multi-rating ATSM. Standard errors are given in parentheses. The columns correspond to the constant (Const), common factor ($X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{com}(t)$) and rating-specific factor ($X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, $X_7^{BBB}(t)$) parameters.

whether the different weightings are due to a common credit factor.

The second panel reports the parameter estimates of the correlation matrix K. To avoid over identification K is restricted to be lower triangle (see Dai and Singleton, 2000). Rating-specific factors are mutually independent and their correlation is zero ($\kappa_{ij} = 0$ for $i, j \geq 4$ and $i \neq j$) by definition. The common factors exhibit higher volatility ($\kappa_{11} = 0.2486$ to $\kappa_{33} = 0.3711$) than the local factors ($\kappa_{44} = 0.1000$ to $\kappa_{55} = 0.1928$). Along with high factor weightings δ_i^{CR}, this leads to the conclusion that the common factors capture a considerable proportion of the yield variation. In addition, 10 of 15 parameters that determine the correlation between the factors, are negative. Therefore, allowing the parameter estimates κ_{ij} to become negative is essential for modeling multiple rating classes. This finding is in line with Dai and Singleton (2000). The parameters are estimated with reasonable accuracy. The majority of the estimation errors is smaller than the estimated parameter itself.

The third panel reports the risk premium parameters λ_i^{CR}. All parameters λ_i^{CR} are estimated with large error. This is in line with the results of Dai and Singleton (2000), Duffee (2002), and Feldhütter et al. (2012), who find that accurately estimating the risk premium parameters is a difficult task. Hence, any evidence relying on the risk premium parameter is weak. However, the second factor interpreted as "slope" (see Table 2.4 and the interpretation below) has a positive risk premium. Thus, a risk averse investor wants to be compensated for taking an extra proportion of risk in times of a steep yield curve. The only exception is λ_i^{BBB} where weighting and volatility are rather low.

Table 2.3 documents the model fit. The observational error ϵ_τ^{CR} is the mean of the error term $\epsilon(t)$ (see Equation (2.15)) for the yield with credit-rating $CR = TR$, AAA, AA, A, and BBB and maturity τ. Their standard deviations are given in parentheses. The goodness of fit is excellent for treasury yields (ranging from ϵ_2^{TR} =0.0000 to ϵ_{10}^{TR} =0.0004) and still very good for corporate bond yields (ranging from ϵ_5^{AA} =0.0001 to ϵ_{10}^{AA} =0.0031). The results are slightly better than those reported in the literature. Egorov et al. (2011)

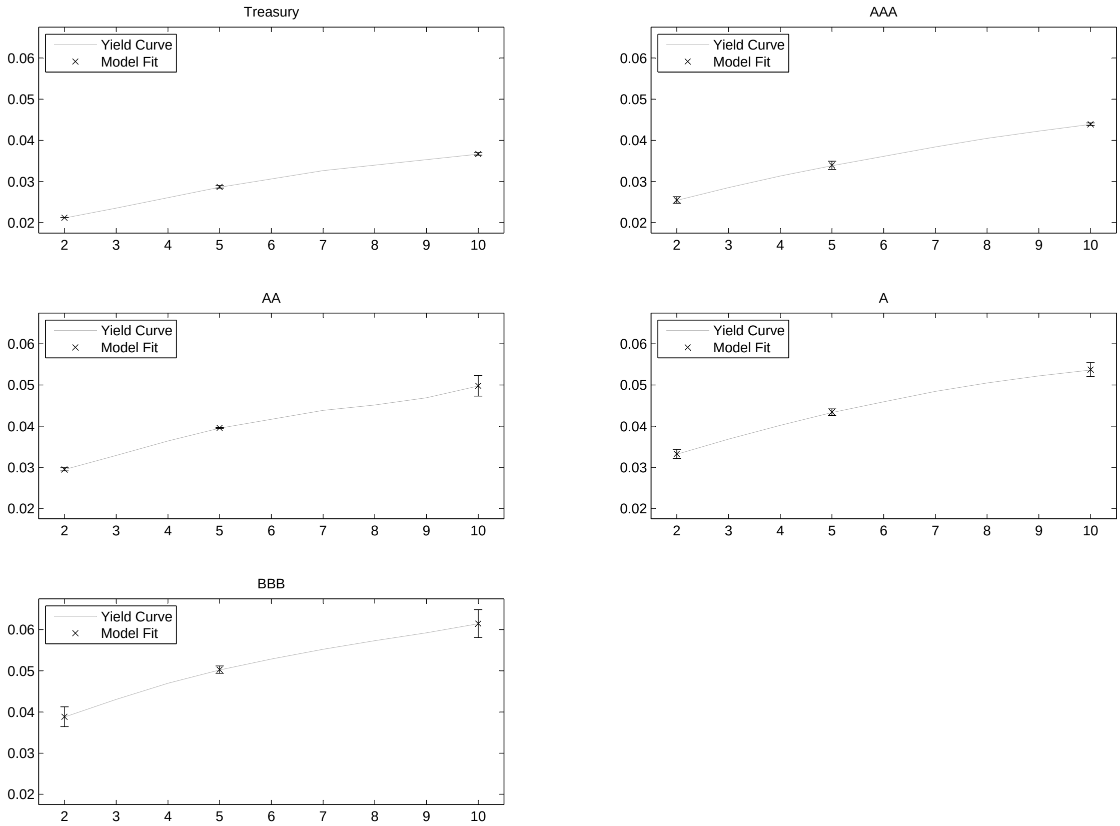

Figure 2.2: Model fit illustration

The figure reports the average fit of the multi-rating ATSM. The solid line corresponds to the average yield curve of U.S. treasury and AAA-, AA-, A-, and BBB-rated U.S. corporate bonds. Crosses represent the average model fit, calculated as the mean. The bars illustrate the standard deviation of the observational error.

Table 2.3: Summary statistics of the model fit

	ϵ_τ^{TR}	ϵ_τ^{AAA}	ϵ_τ^{AA}	ϵ_τ^{A}	ϵ_τ^{BBB}
τ =2 years	0.0000	0.0009	0.0003	0.0011	0.0021
	(0.0000)	(0.0008)	(0.0004)	(0.0011)	(0.0024)
τ =5 years	0.0003	0.0011	0.0001	0.0007	0.0007
	(0.0003)	(0.0010)	(0.0001)	(0.0008)	(0.0009)
τ =10 years	0.0004	0.0002	0.0031	0.0014	0.0026
	(0.0003)	(0.0003)	(0.0027)	(0.0017)	(0.0034)

The table reports the model fit. The observational errors ϵ_τ^{TR}, ϵ_τ^{AAA}, ϵ_τ^{AA}, ϵ_τ^{A}, and ϵ_τ^{BBB} are the mean of the error term $\epsilon(t)$ defined in Equation (2.15). τ denotes the yield's maturity. Their standard deviations are given in parentheses.

document errors up to 0.0011 for U.S. treasury yields and Speck (2013) reports errors up to 0.0007 for U.S. treasury yields and 0.0040 for U.S. corporate bonds yields.

Figure 2.2 illustrates the average fit of the multi-rating ATSM for treasury and corporate bond yields. As documented in Table 2.1, the average yield curve is upward sloping and the interest yield level is an increasing function of probability to default. In line with Table 2.2, Figure 2.2 reports an excellent average fit (crosses) of the empirical yields (solid line) across all rating classes. Nearby bars for treasury yields and most of the corporate bond yields document a remarkably small standard deviation of the observational errors. Widening bars correspond to higher standard deviations for ten-year AA bonds and two- and ten-year BBB bonds.

2.4.2 Factor analysis

Correlations between yield and model factors provide some economic intuition of the evolution of model factors. Litterman and Scheinkman (1991) propose “level”, “slope”, and “curvature” as indicators for yield factor analysis for single term structure models. Driessen et al. (2003) suggest adding the indicator “spread” when studying common factors in international bond returns. They define “spread” as difference between treasury yields of two countries. For corporate bond pricing, the term “spread” is used for describing the difference between treasury and corporate bond yields. Table 2.4 reports the correlations between model factors and the "level", "slope", "curvature", and "spread" of bond yields.

The first common factor $X_1^{com}(t)$ is highly correlated to the treasury yield "level" (TR:0.9960). This correlation is present among all rating classes. It is, however, a decreasing function of probability to default (ranging from AAA:0.9464 to BBB:0.5872). This suggests the existence of a credit factor that is not totally accounted for in the treasury yield "level". The second common factor $X_2^{com}(t)$ is highly correlated to the "slope" of the yields across all rating classes. The correlations range from 0.9972 for treasury bonds to 0.8783 for BBB-rated corporate bonds. This factor seems to correspond to an economy-wide variation without any dependence on the level of credibility. The economic intuition explains the positive risk premiums that are reported in Table 2.2. Table 2.4 supports the conjecture that a risk averse investor wants to be compensated for higher uncertainty in times of a steep yield curve. The third common factor $X_3^{com}(t)$ moves similarly to the "level" of AA-, A-, and BBB-rated corporate bonds (AA:0.7098, A:0.7855, and BBB:0.7151). It seems to have little influence on treasury and AAA-rated bonds (TR:0.2306 and BBB:0.4491). Recalling the low factor weightings $\delta_3^{TR}(= 0.0025)$ and $\delta_3^{AAA}(= 0.0064)$ and the high weightings $\delta_3^{AA}(= 0.0102)$ and $\delta_3^{A}(= 0.0091)$, this factor seems to account for the variability that could not be captured in the "treasury level" factor. Hence, the factor is interpreted as "corporate level".

The local factors model the variation of each rating class that could not be accounted for by the common factors. AAA-rated corporate bonds have by definition high credibility and a very low probability to default. Hence, they behave similarly to treasury bonds. The first two common factors already capture a large fraction of the yield movement. This argument is in line with the low explanatory power of the common credit factor $X_3^{com}(t)$. However, the common credit factor $X_3^{com}(t)$ is almost uncorrelated to AAA-rated corporate bond yields and does not explain any of their volatility. The

Table 2.4: Yield factor correlations

	Level	Slope	Curvature	Spread
		TR		
$X_1^{com}(t)$	**0.9960**	-0.6183	-0.4492	-
$X_2^{com}(t)$	-0.6896	**0.9972**	-0.3066	-
$X_3^{com}(t)$	0.2306	-0.2683	0.0119	-
		AAA		
$X_1^{com}(t)$	0.9464	-0.6074	-0.4584	-0.3246
$X_2^{com}(t)$	-0.6720	0.9638	0.0710	0.1947
$X_3^{com}(t)$	0.4491	-0.3967	-0.5468	0.7865
$X_4^{AAA}(t)$	-0.6969	0.5158	**0.6750**	-0.2730
		AA		
$X_1^{com}(t)$	0.7933	-0.6010	-0.2098	-0.5852
$X_2^{com}(t)$	-0.5707	0.9182	-0.0269	0.3834
$X_3^{com}(t)$	0.7098	-0.3866	-0.5821	0.6839
$X_5^{AA}(t)$	-0.4428	**0.8120**	-0.2476	0.4923
		A		
$X_1^{com}(t)$	0.6877	-0.5291	0.0604	-0.5039
$X_2^{com}(t)$	-0.4769	0.9297	-0.2394	0.3870
$X_3^{com}(t)$	**0.7855**	-0.4367	0.1808	0.7062
$X_6^{A}(t)$	0.4306	0.2509	-0.0386	**0.7860**
		BBB		
$X_1^{com}(t)$	0.5872	-0.5441	0.2735	-0.5643
$X_2^{com}(t)$	-0.3128	0.8783	-0.5302	0.5180
$X_3^{com}(t)$	0.7151	-0.2330	0.1557	0.4738
$X_7^{BBB}(t)$	0.0954	0.7728	-0.1148	**0.7765**

The table reports the correlations of model factors and weekly bond yields. The model factors are organized in common factors ($X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{com}(t)$) and rating-specific factors ($X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, $X_7^{BBB}(t)$). The yields correspond to a common "level" factor (five-years), a "slope" factor (ten-years - two-years), a "curvature" factor ([ten-years + two-years]-two*five-years), and a "spread" factor (two-years rating-specific - two-years U.S. treasury). Figure 2.3 illustrates correlations in **bold** type.

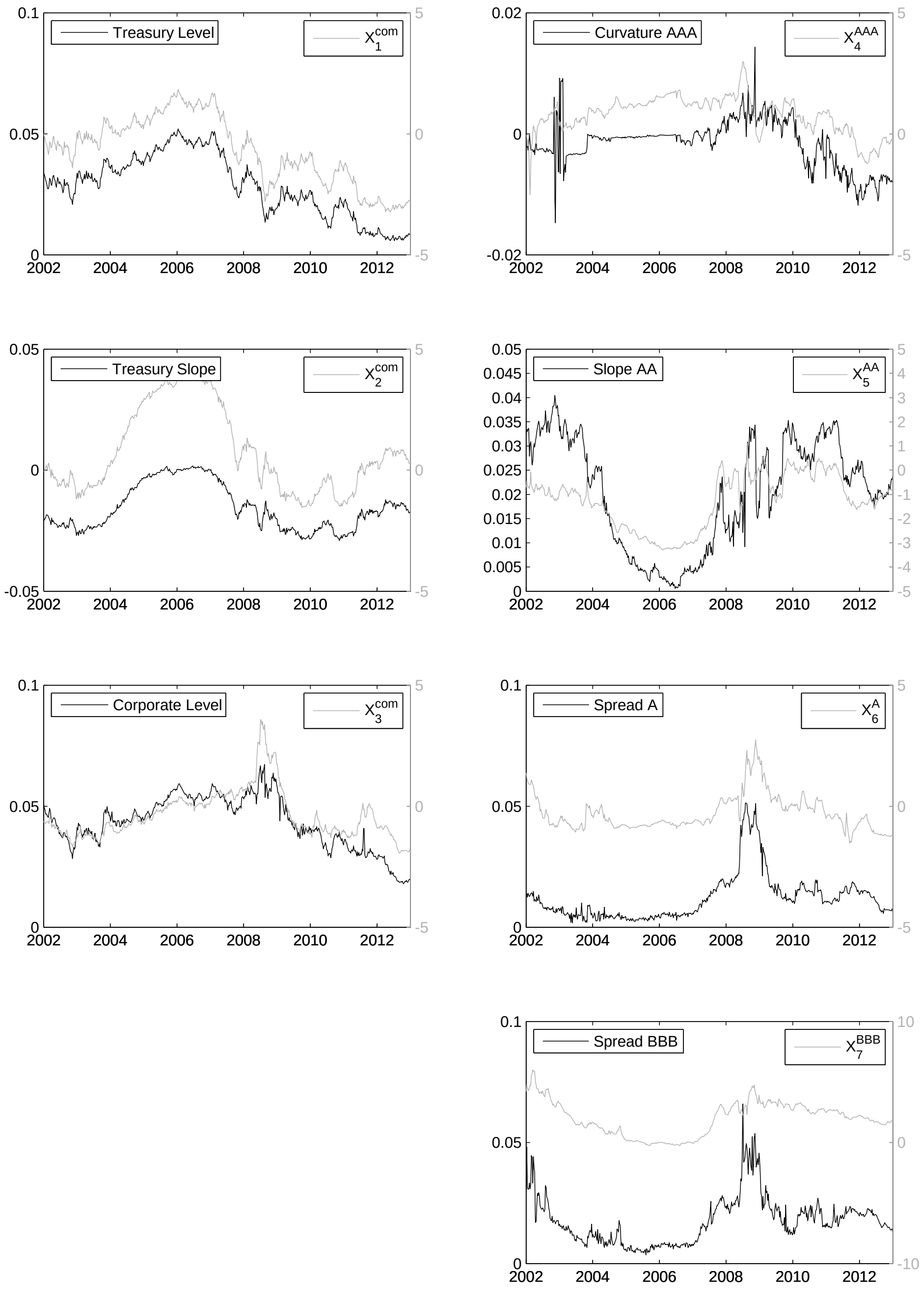

Figure 2.3: Fitted model and yield factors

The figure shows the common (left panel: $X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{com}(t)$) and rating-specific (right panel: $X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, and $X_7^{BBB}(t)$) model factors. Each factor is plotted with its corresponding yield (see correlations in **bold** type in Table 2.4). The yield factors correspond to the following empirical yields: Treasury level= U.S. treasury five-years; Treasury slope= U.S. treasury (ten-years - two-years); Corporate level= Corporate A five-years; Curvature AAA = Corporate AAA ([ten-years + two-years]-two*five-years); Slope AA = Corporate AA (ten-years - two-years); Spread A = (Corporate A two-years - U.S. treasury two-years); Spread BBB = (Corporate BBB two-years - U.S. treasury two-years).

rating-specific factor $X_4^{AAA}(t)$ models the remaining term structure movement that is yet unexplained. The factor is clearly correlated to and models the "curvature" of the AAA yield curve. The rating-specific factors $X_5^{AA}(t)$, $X_6^{A}(t)$, and $X_7^{BBB}(t)$ offer similar results and correspond to "slope", "spread", and "spread", respectively.

Figure 2.3 illustrates the correlations that are reported in Table 2.4 in **bold** type. Each factor is fitted to its corresponding yield. The left and right columns exhibit the common ($X_1^{com}(t)$, $X_2^{com}(t)$, and $X_3^{com}(t)$) and rating-specific ($X_4^{AAA}(t)$, $X_5^{AA}(t)$, $X_6^{A}(t)$, and $X_7^{BBB}(t)$) factors, respectively. Particularly, the common factors have remarkably high correlations to the yields and provide economic intuition behind the model factors.

Table 2.5: Regression of U.S. corporate bond yield factors

	Treasury Level	Treasury Slope	Corporate Level	Curvature AAA	Slope AA	Spread A	Spread BBB
$X_1^{com}(t)$	0.0043	0.0021	-0.0015	0.0013	-0.0023	-0.0051	-0.0034
$X_2^{com}(t)$	0.0025	0.0043	0.0034	-0.0006	-0.0031	-0.0005	0.0007
$X_3^{com}(t)$	-0.0036	0.0026	-0.0008	0.0006	-0.0047	0.0045	0.0035
$X_4^{AAA}(t)$	0.0079	-0.0046	0.0116	0.0018	0.0040	0.0034	0.0009
$X_5^{AA}(t)$	-0.0073	0.0041	-0.0103	0.0003	-0.0040	-0.0037	-0.0032
$X_6^{A}(t)$	0.0010	-0.0004	0.0053	0.0020	0.0005	0.0046	0.0021
$X_7^{BBB}(t)$	0.0063	-0.0037	0.0090	-0.0002	0.0055	0.0022	0.0059
R^2	0.8239	0.8825	0.4684	0.7095	0.7905	0.9367	0.8920

The table reports the results when the $A_0(7)$ model is regressed on US corporate bond yields. The yield factors correspond to the following empirical yields: Treasury level= U.S. treasury five-years; Treasury slope= U.S. treasury (ten-years - two-years); Corporate level= Corporate A five-years; Curvature AAA = Corporate AAA ([ten-years + two-years]-two*five-years); Slope AA = Corporate AA (ten-years - two-years); Spread A = (Corporate A two-years - U.S. treasury two-years); Spread BBB = (Corporate BBB two-years - U.S. treasury two-years). The linear regression model uses least squares. Each column in the upper panel corresponds to a vector of regression coefficients in the linear model. The lower panel reports the R-square statistic for each linear regression of yield factors.

This intuition can be illustrated by way of example of the second common factor. Recall the yield curve evolution plotted in Figure 2.1. The yield curve is flat (up to inverse) in the period 2005 to 2007. That is a rare event in the market and regarded by extreme values of $X_2^{com}(t)$. In the remainder of the sample period (2002 to 2005 and 2007 to 2012) the yield curve is normal. This shape is expressed in values of $X_2^{com}(t)$ that are not significantly different from zero. The greatest slope, reported in Figure 2.1, corresponds to the minimum of $X_2^{com}(t)$ in 2011.

Table 2.5 reports the results of a regression analysis of the proposed model and US corporate bond yields. In sum, the regression analysis supports the proposed yield factors. The first two common factors and every rating-specific factor indicate high explanatory power for the proposed yield factors. Only the common credit factor exhibits a lower R-square statistic when regressed with the linear model.

Overall, Table 2.4 and Figure 2.3 lead to the conclusion that there exists a common credit factor that cannot be captured by treasury yield factors [Table 2.4: A, $X_3^{com}(t)$ vs "level"; Figure 2.3: Corporate Level]. This finding is in line with Amato and Luisi (2006): credit spreads provide information on the business cycle that is not found in treasury yields. Additionally, the parameter estimation and factor analysis shows that the multi-rating ATSM offers an excellent fit of corporate bond yields and, even more, proves there is a clear economic intuition behind the model factors.

2.5 Conclusion

Modeling the term structure of corporate bonds is important to risk managers who are concerned with financial products exposed to corporate default. Significant improvements have already been made in modeling single corporate bond term structures. However, the literature has neglected to provide a multi-rating ATSM with a solid economic intuition of the latent factors.

In this chapter, a multi-rating ATSM with no-arbitrage restrictions has been proposed to analyze corporate bonds. U.S. treasury bonds and U.S. corporate bonds with different levels of credibility have been studied in the period 2002 to 2013. A principal component analysis has shown that seven factors explain 89% of the yield variation. To this end, the model is based on three common factors and one rating-specific factor for each rating class.

The findings can be summarized as follows: the multi-rating ATSM exhibits an excellent fit across all rating classes. The model fit is line with the credit spread literature (see Speck, 2013) and even better than the joint ATSM literature (see Egorov et al., 2011). Secondly, the factor analysis leads to the conclusion that a common credit factor exists that cannot be captured by treasury yield factors. Finally, the latent factors of the multi-rating ATSM provide a clear economic intuition. The common factors can be interpreted as "treasury level", "slope", and "corporate level". The rating-specific factors of AAA-, AA-, A-, and BBB-rated bonds correspond to "curvature", "slope", "spread", and "spread", respectively.

3 Common factors in international bond returns and a joint ATSM to match them

This chapter is based on Gabriel (2014).

3.1 Introduction

Institutional investors do not usually restrict their capital of a fixed income fund to a single country. They rather diversify risk by holding government bonds issued by different countries. If yields across countries depend on each other, investing abroad no longer diversifies the domestic interest rate risk away. Therefore, international investors immediately benefit from identifying and modeling common factors.

This chapter provides an economic analysis of the common factors of two major government bond markets. A factor analysis determines the number of common and local factors that drive both yield curves. A principal component analysis reveals the economic intuition of the latent factors. On this basis, a joint affine term structure model (joint ATSM) is proposed that is capable of modeling the variability of the treasury yields of both economies. A detailed analysis of yield and model factors illustrates the link of the econometric analysis and the proposed model and confirms the factor interpretation. In addition, a bond portfolio application offers a possible extension of the proposed model and shows how the investor's choice reflects the common and local factors. In summary, two common factors explain 85% of the yield variation and the yield factors maintain their importance for interpreting the joint ATSM and understanding the investor's portfolio adjustments.

Litterman and Scheinkman (1991) apply a principal component analysis to U.S. bond returns and document three factors which correspond to the "level", "slope", and "curvature" of the yield curve. Driessen et al. (2003) find that "level", "spread", and "steepness" determine a large part of the variation in bond returns from the U.S., Germany, and Japan. Juneja (2012) studies treasury yields from the U.S., U.K., and Germany and concludes that "level" and "slope" govern most of their variability. No *ex ante* financial theory supports the treasury yield movement analysis in these studies. Joint ATSMs might provide the theoretical background to close that gap in a solid no-arbitrage setting.

Modeling yields of two distinct economies creates the main challenge of modeling two different term structures of interest rates within one term structure model. That is referred to as a joint ATSM. Backus et al. (2001), Bansal (1997), and Hodrick and Vassalou (2002) propose joint ATSMs to match the common factors of international yield curves. Dewachter and Maes (2001) add an additional risk-driving factor to capture the volatility of exchange rate movements. In contrast, Brennan and Xia (2006), Sarno et al. (2012), and Graveline and Joslin (2011) include time variation in the risk premium to cope with the difference in variation of interest rates and exchange rates. Egorov et al. (2011) provide a classification for completely affine ATSMs as proposed by Dai and Singleton (2000). The analysis of the present chapter is based on the classification of Egorov et al. (2011). The proposed joint ATSM forms the basis for the econometric analysis and the subsequent bond portfolio extension.

Bond portfolios have been extensively researched in the fixed income literature (see Black and Litterman, 1992). However, little is known about continuously moving yields (see Sundaresan, 2000). In this field of research most papers also focus on continuously adjusted portfolios. In reality, however, portfolio adjustments are made infrequently rather than continuously. According to Bacchetta and van Wincoop (2010) a discrete time investment horizon is preferable. Korn and Koziol (2006) propose an optimal portfolio model with fixed investment horizon on the basis of an ATSM. However, they restrict the investor to buying domestic bonds. The portfolio application of the present chapter extends their model to international bonds in using a joint ATSM.

The contribution of the present chapter is twofold: to the best of my knowledge, this is the first factor analysis of U.S. and U.K. treasury yields on the basis of a joint ATSM. The solid theoretical framework of the no-arbitrage setting makes it possible to provide an economic intuition of the common and local model factors. In addition, an optimal portfolio model for international bond investors is proposed.

The remainder of the chapter is organized as follows. Section 3.2 provides a factor analysis of the treasury yields. Section 3.3 proposes a joint ATSM to match the common and local factors. Section 3.4 links the empirical and model factors. Section 3.5 offers a possible extension for international bond investors and the chapter concludes with Section 3.6.

3.2 International bond data

The dataset consists of U.S. and U.K. zero-coupon bonds. The data is provided by the U.S. Federal Reserve and the Bank of England (see Gürkaynak et al., 2006). The period 1979 to 1982 is known to be econometrically precarious because of the so-called U.S. Federal Reserve's experiment (see Chapman and Pearson, 2001). Therefore, the analysis starts in January 1983 and runs till July 2012. The six-months, two-, five-, and ten-years treasury yields are collected on a daily basis. Figure 3.1 reports the U.S. and U.K. treasury yields of the sample period. The interest rate level decreases consistently

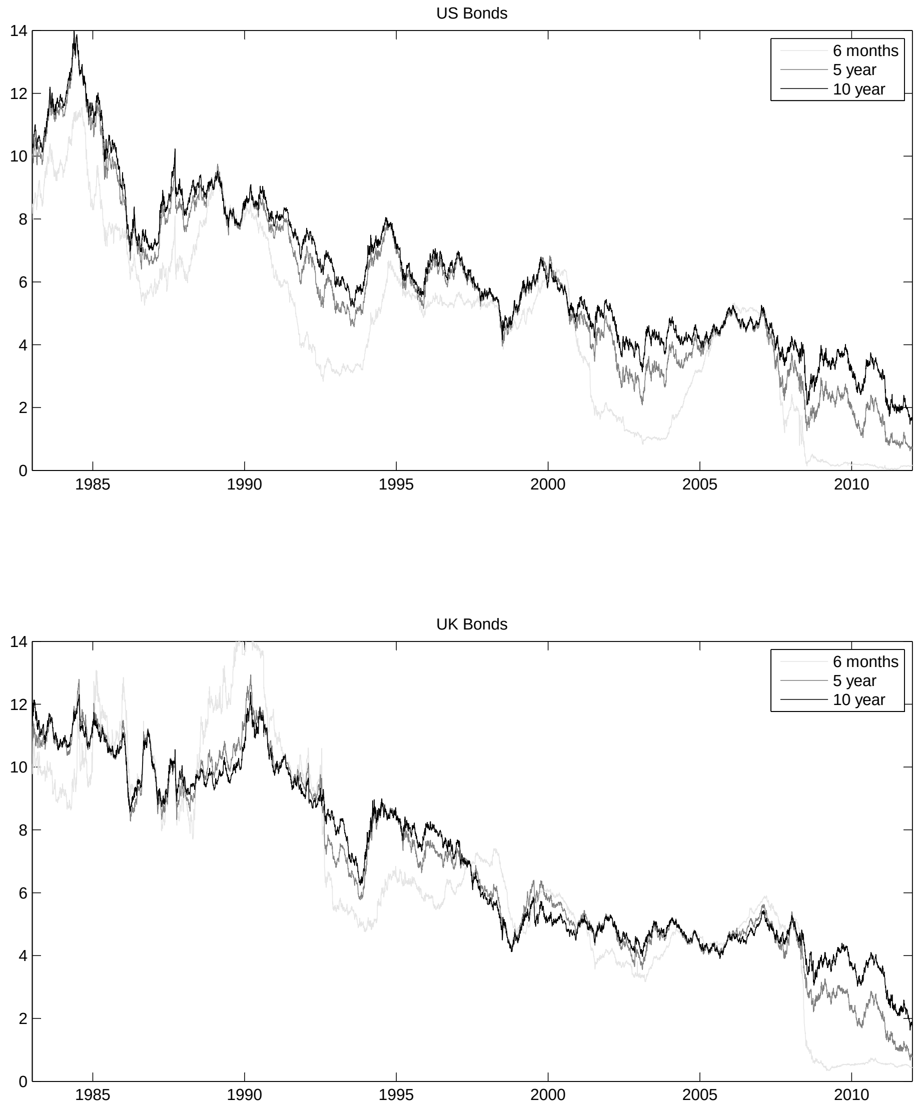

Figure 3.1: Six-months, five-, and ten-years U.S. and U.K. treasury yields

The figure reports daily treasury yields from the 2^{nd} of January 1983 to the 31^{st} of July 2012.

over time for both countries.[1]

Table 3.1: Summary statistics of U.S. and U.K. treasury yields

	U.S.				U.K.			
	6m	2y	5y	10y	6m	2y	5y	10y
Mean	4.88	5.47	6.05	6.49	6.77	6.84	7.06	7.17
Std	2.68	2.77	2.59	2.40	3.31	2.99	2.76	2.61
	correlations							
6m	1.00	0.79	0.69	0.62	0.21	0.31	0.30	0.27
2y		1.00	0.94	0.86	0.18	0.31	0.33	0.33
5y			1.00	0.95	0.15	0.30	0.34	0.36
10y				1.00	0.13	0.28	0.33	0.38
6m					1.00	0.80	0.68	0.52
2y						1.00	0.92	0.73
5y							1.00	0.91
10y								1.00
	Eigenvalue Decomposition							
Factor		1		2		3		4
Eigenvalue		0.56		0.29		0.07		0.04
Cumulative value		0.56		0.85		0.92		0.96

The table reports the summary statistics of daily U.S. and U.K. treasury yields in the period of 1983 to 2012. Means and standard deviations (Std) are reported in p.a. percentage points. Factor analysis is done via eigenvalue decomposition of the yield correlation matrix.

Table 3.1 reports the corresponding descriptive statistics. The average yield curves are normal (upward sloping) and the short ends are more volatile than the long ends. The correlations within national bond markets are high (ranging from 52% to 95%). The cross country correlations are lower but still clearly positive. These high correlations imply that both yield curves depend on a limited number of common risk factors.[2] A principal component analysis indicates how many factors the yield curve is driven by (see Bliss, 1997). The eigenvalue decomposition in Table 3.1 shows that a small number of factors describes a large share of the yield curve variation. In sum, two (four) factors

[1] The interest rates of two-years bonds are very similar to those of five-years bonds throughout the sample period. Hence, the figure does not report them.

[2] For the sake of brevity, the term *factor* substitutes the term *risk factors* in the following.

account for 85% (96%) of the yield curve dynamics.

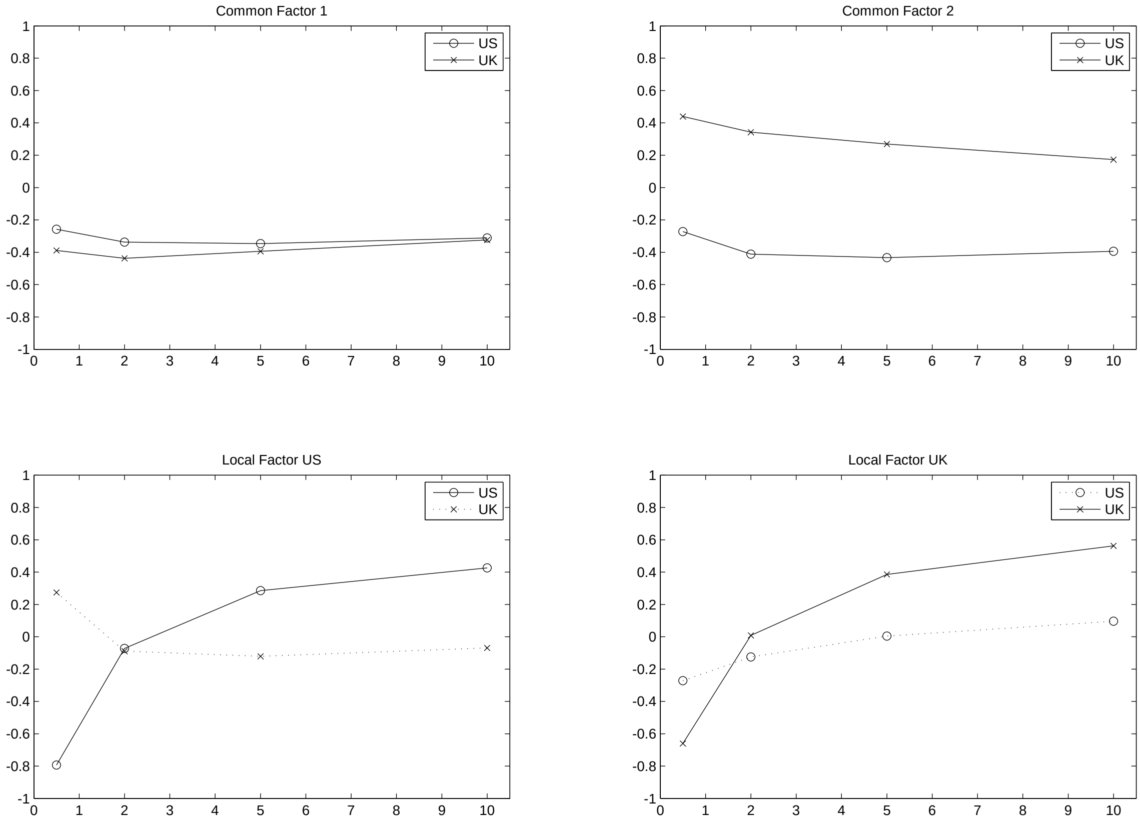

Figure 3.2: Factor analysis

The figure reports factor loadings for U.S. and U.K. yield curves. A principal component analysis is applied to changes in yields of six-months, two-, five-, and ten-years time to maturity. U.S. and U.K. factor loadings are reported as circles ($\circ$) and crosses ($\times$), respectively. Solid lines indicate factor loadings that are economically meaningful. Dashed lines correspond to local factor loadings in the foreign economy.

The principal component analysis, illustrated in Figure 3.2, provide an economic intuition of the yield factors. The first yield principal component in the upper left panel [Common Factor 1] has almost the same loading for both countries and all maturities. It is identified as a common factor and interpreted as “level”. The second yield principal component in the upper right panel [Common Factor 2] loads positively on U.K. bonds and negatively on U.S. bonds. It is interpreted as common “spread” factor. The third yield principal component in the lower left panel [Local Factor US] is negligible for

U.K. bonds. It is, however, highly relevant for the U.S. market. It is the "U.S. slope" factor with negative impact on the short end and positive impact on the long end. The irrelevance of the "U.S. slope" factor for the U.K. economy is highlighted by plotting the U.K. factor loading as dashed line. The fourth yield principal component in the lower right panel [Local Factor UK] draws the precisely opposite picture than the third yield principal component. Whereas U.S. yields play a minor role, the factor loading is a decreasing function of time to maturity of U.K. yields. Hence, it is interpreted as the "U.K. slope" factor. As in the previous graph, the irrelevance of the "U.K. slope" factor for the U.S. economy is indicated by plotting the U.S. factor loading as dashed line. The economic intuition of the international factors as "level", "spread", and "slope" is in line with Juneja (2012). Driessen et al. (2003) find that "level" and "spread" are highly correlated across countries whereas the "slope" factor is country-specific. The factor analysis leads to the conclusion that the yield curve variation corresponds to two common factors and one local factor for each country.

3.3 The international bond pricing model

Single term structure models have proven to work well for pricing bonds, interest rate derivatives and bond portfolios.[3] Two country models are a significant extension of single country models in jointly modeling the dynamics of two yield curves. The previous section suggests that two common factors and one local factor for each country match the variation in U.S. and U.K. treasury yields best.

3.3.1 The joint yield curve model

The zero-coupon bond price is defined in accordance with Dai and Singleton (2000) and Egorov et al. (2011). Let two economies be described by the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where $\mathbb{P}$ denotes the physical measure. $\mathbb{Q}$ and $\mathbb{Q}^*$ shall be the equivalent risk-neutral

[3] See Chapman and Pearson (2001) and Dai and Singleton (2003) for literature reviews.

measures for the U.S. and U.K., respectively.[4] In the absence of arbitrage, the time-t prices of U.S. and U.K. zero-coupon bonds $P(t,\tau)$ and $P^*(t,\tau)$ that mature at $t+\tau$ are given by

$$P(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} r(u)\ du)\right] \text{ and} \tag{3.1}$$

$$P^*(t,\tau) = E_t^{\mathbb{Q}^*}\left[\exp(-\int_t^{t+\tau} r^*(u)du)\right], \tag{3.2}$$

where $E_t^{\mathbb{Q}}$ and $E_t^{\mathbb{Q}^*}$ denote $\mathcal{F}_t$ conditional expectations under $\mathbb{Q}$ and $\mathbb{Q}^*$. A joint ATSM is obtained under the assumption that the instantaneous short rates $r(t)$ and $r^*(t)$ are affine functions of a vector of latent state variables $X(t) = [X_1(t), X_2(t), X_3(t), X_4(t)]'$:

$$r(t) = \delta_0 + \delta'\ X(t) \text{ and} \tag{3.3}$$

$$r^*(t) = \delta_0^* + \delta^{*'} X(t). \tag{3.4}$$

In equations (3.3) and (3.4) δ_0 and δ_0^* are scalars and δ and δ^* are 4×1 vectors. $X(t)$ nests the common and local factors that drive both economies. Common factors enter both expressions of $r(t)$ and $r^*(t)$ through non zero entries of δ and δ^*. The factor weightings are expressed in the value of entires of δ and δ^* for each country. If the common factor weighting tends to zero the dynamics of the short rate are (almost) exclusively driven by the local factor. If, in contrast, the factors are equally weighted for both countries, there is a common factor that drives the dynamics of both economies.

By definition, the local factors are forced to be mutually independent. Without mutually independence they would represent common factors. However, they may depend on each other through correlated common factors. The one joint ATSM can be decomposed into two single ATSMs if the local factors are mutually independent (see Egorov et al.,

[4]In the following, a * shall indicate the British economy.

2011). The joint dynamics of $X(t)$ follow an affine diffusion of the form:

$$dX(t) = K[\vartheta - X(t)]dt + \Sigma \cdot dW(t). \tag{3.5}$$

$W(t)$ is a 4-dimensional independent standard Brownian motion under $\mathbb{P}$ and K and Σ are 4×4 parameter matrices and ϑ is a 4×1 parameter vector.

The chapter aims to provide an economic intuition of the model factors. Therefore, the risk premium is assumed to be non time-varying. That is, the Gaussian setup is defined to be completely affine.[5] The domestic risk premium for U.S. bonds is defined as a constant 4×1 parameter vector λ. The risk premium is country-specific and independent of the foreign risk premium, i.e. the risk premium parameter of the foreign factor is zero. Likewise, the U.K. risk premium is defined as a constant 4×1 parameter vector λ^*.

So far the model describes the joint dynamics of both term structures of interest rates. This joint model can be decomposed into two single ATSMs, if the local factors are mutually independent (see Egorov et al., 2011). Under the risk-neutral measure $\mathbb{Q}$,

$$dX(t) = K^{\mathbb{Q}} [\vartheta^{\mathbb{Q}} - X(t)] + \Sigma \cdot dW^{\mathbb{Q}}(t), \tag{3.6}$$

where $dW^{\mathbb{Q}}(t) = dW(t) - \lambda dt$, $\vartheta^{\mathbb{Q}}$, and $K^{\mathbb{Q}}$ represent the risk-neutral measure $\mathbb{Q}$. Similarly, under the risk-neutral measure $\mathbb{Q}^*$,

$$dX(t) = K^{\mathbb{Q}*}[\vartheta^{\mathbb{Q}*} - X(t)] + \Sigma \cdot dW^{\mathbb{Q}*}(t), \tag{3.7}$$

where $dW^{\mathbb{Q}*}(t) = dW(t) - \lambda^* dt$, $\vartheta^{\mathbb{Q}*}$, and $K^{\mathbb{Q}*}$ represent the risk-neutral measure for the U.K.. Under the risk-neutral measures $\mathbb{Q}$ and $\mathbb{Q}^*$ the prices of U.S. and U.K. zero-coupon

[5] Feldhütter et al. (2012) show that investors prefer simple (completely affine) to more complex (essentially affine) models, if they are aware of parameter risk.

bonds read, respectively:

$$P(t,\tau) = \exp(-A_\tau - B_\tau^{'} X(t)) \text{ and} \tag{3.8}$$

$$P^*(t,\tau) = \exp(-A_\tau^* - B_\tau^{*'} X(t)). \tag{3.9}$$

Duffie and Kan (1996) show that A_τ, B_τ, A_τ^* and B_τ^* satisfy the ordinary differential equations (ODEs):

$$\frac{dA_\tau}{\tau} = \vartheta' K B_\tau - \frac{1}{2}\sum_{i=1}^{4} [\Sigma' B_\tau]_i^2 - \delta_0, \tag{3.10}$$

$$\frac{dB_\tau}{\tau} = -K B_\tau - \frac{1}{2}\sum_{i=1}^{4} [\Sigma' B_\tau]_i^2 + \delta^{'}, \tag{3.11}$$

$$\frac{dA_\tau^*}{\tau} = \vartheta^{*'} K B_\tau^* - \frac{1}{2}\sum_{i=1}^{4} [\Sigma' B_\tau^*]_i^2 - \delta_0^*, \text{ and} \tag{3.12}$$

$$\frac{dB_\tau^*}{\tau} = -K B_\tau^* - \frac{1}{2}\sum_{i=1}^{4} [\Sigma' B_\tau^*]_i^2 + \delta^{*'}. \tag{3.13}$$

Hereby the ODEs are completely specified and their solutions are available in closed-form. Kim and Orphanides (2012) give a very practical closed-form solution for U.S. and U.K. zero-coupon bonds in vector notation:[6]

$$\begin{aligned} A_\tau =& -\frac{1}{\tau}[(K\vartheta)'(m_{1,\tau} - \tau I)K^{-1'}\delta \\ &+ \frac{1}{2}\delta' K^{-1}(m_{2,\tau} - \Sigma\Sigma' m_{1,\tau} - m_{1,\tau}\Sigma\Sigma' + \tau\Sigma\Sigma')K^{-1'}\delta - \tau\delta_0], \end{aligned} \tag{3.14}$$

$$B_\tau = \frac{1}{\tau} m_{1,\tau}\delta, \tag{3.15}$$

$$\begin{aligned} A_\tau^* =& -\frac{1}{\tau}[(K\vartheta^*)'(m_{1,\tau} - \tau I)K^{-1'}\delta^* \\ &+ \frac{1}{2}\delta'^* K^{-1}(m_{2,\tau} - \Sigma\Sigma' m_{1,\tau} - m_{1,\tau}\Sigma\Sigma' + \tau\Sigma\Sigma')K^{-1'}\delta^* - \tau\delta_0^*], \text{ and} \end{aligned} \tag{3.16}$$

$$B_\tau^* = \frac{1}{\tau} m_{1,\tau}\delta^*, \tag{3.17}$$

[6] $vec(C)$ denotes the vectorization of C: $vec(C) = [c_{1,1}, \ldots, c_{i,1}, c_{1,2}, \ldots, c_{i,2}, c_{1,j}, \ldots, c_{i,j}]'$. $\otimes$ denotes the Kronecker product. I is the identity matrix.

where

$$
\begin{aligned}
m_{1,\tau} =& - K^{-1'}(exp(-K'\tau) - I) \quad \text{and} \\
m_{2,\tau} =& - vec^{-1}((K \otimes I) + (I \otimes K))^{-1} vec(exp(-K\tau)\Sigma\Sigma' exp(-K'\tau) - \Sigma\Sigma').
\end{aligned}
$$

A_τ, B_τ, A^*_τ, and B^*_τ are functions of K, ϑ, Σ, δ, δ_0, ϑ^*, δ^*, δ^*_0, and τ. The risk-neutral and physical parameters correspond in the following way:

$$K = K^{\mathbb{Q}}, \tag{3.18}$$

$$\vartheta = \vartheta^{\mathbb{Q}} - K^{-1}\Sigma\lambda, \tag{3.19}$$

$$K = K^{\mathbb{Q}*}, \text{ and} \tag{3.20}$$

$$\vartheta = \vartheta^{\mathbb{Q}*} - K^{-1}\Sigma\lambda^*. \tag{3.21}$$

To avoid over-identification, the restrictions of Dai and Singleton (2000) apply. K is restricted to being lower triangle, Σ to being the identity matrix, and $\vartheta^{\mathbb{Q}}$ to being zero. Under the physical measure the diffusion process is given by:

$$
d \begin{bmatrix} X_1^{com}(t) \\ X_2^{com}(t) \\ X_3^{US}(t) \\ X_4^{UK}(t) \end{bmatrix} = \begin{bmatrix} \kappa_{11} & 0 & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & 0 \\ \kappa_{41} & \kappa_{42} & 0 & \kappa_{44} \end{bmatrix} \begin{bmatrix} -X_1^{com}(t) \\ -X_2^{com}(t) \\ -X_3^{US}(t) \\ -X_4^{UK}(t) \end{bmatrix} dt + d \begin{bmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \\ W_4(t) \end{bmatrix}. \tag{3.22}
$$

Without loss of generality $X_1^{com}(t)$ and $X_2^{com}(t)$ are assumed to be common factors. $X_3^{US}(t)$ is the U.S. local factor and $X_4^{UK}(t)$ is the U.K. local factor. As both local factors are required to be mutually independent κ_{43} is set to zero. This corresponds to an $A_0(4)$ joint ATSM in the Dai and Singleton (2000) sense.

Subsection 1.1.2 raised the question: (3) What are the consequences of common factors in international bond markets for the specification of a joint ATSM? The answer to this question is the $A_0(4)$ joint ATSM defined in Equation (3.22).

3.3.2 The state space model

Term structure models are positive in that they capture the time series dynamics in the factors. The cross section is a resulting function of these factors and the time to maturity (see DeJong, 2000). In dealing with such panel data, the natural approach to take is to define a state space model. The factors can be found in the transition equation and treasury yields of different maturities in the measurement equation. Since the model is completely affine, the parameter estimation can be done using Kalman filtering with straightforward direct maximum likelihood estimation (see Babbs and Nowman, 1999).

At time t let there be observed zero-coupon bond yields with maturity τ_1 through τ_4 in vector $y(t)$. Vector A and matrix B are defined as in Equations (3.14) through (3.17):

$$y(t) = \begin{bmatrix} Y_{\tau_1}^{US}(t) \\ Y_{\tau_2}^{US}(t) \\ Y_{\tau_3}^{US}(t) \\ Y_{\tau_4}^{US}(t) \\ Y_{\tau_1}^{UK}(t) \\ Y_{\tau_2}^{UK}(t) \\ Y_{\tau_3}^{UK}(t) \\ Y_{\tau_4}^{UK}(t) \end{bmatrix}, \qquad A = \begin{bmatrix} A_{\tau_1} \\ A_{\tau_2} \\ A_{\tau_3} \\ A_{\tau_4} \\ A_{\tau_1}^* \\ A_{\tau_2}^* \\ A_{\tau_3}^* \\ A_{\tau_4}^* \end{bmatrix}, \qquad B = \begin{bmatrix} B_{\tau_1,1} & B_{\tau_1,2} & B_{\tau_1,3} & 0 \\ B_{\tau_2,1} & B_{\tau_2,2} & B_{\tau_2,3} & 0 \\ B_{\tau_3,1} & B_{\tau_3,2} & B_{\tau_3,3} & 0 \\ B_{\tau_4,1} & B_{\tau_4,2} & B_{\tau_4,3} & 0 \\ B_{\tau_1,1}^* & B_{\tau_1,2}^* & 0 & B_{\tau_1,4}^* \\ B_{\tau_2,1}^* & B_{\tau_2,2}^* & 0 & B_{\tau_2,4}^* \\ B_{\tau_3,1}^* & B_{\tau_3,2}^* & 0 & B_{\tau_3,4}^* \\ B_{\tau_4,1}^* & B_{\tau_4,2}^* & 0 & B_{\tau_4,4}^* \end{bmatrix}. \tag{3.23}$$

The state space model form is given by:[7]

$$y(t) = A + BX(t) + \epsilon(t) \quad \text{and} \tag{3.24}$$

$$X(t+h) = exp(-Kh)X(t) + (I - exp(-Kh))\vartheta^{\mathbb{Q}} + v(t+h). \tag{3.25}$$

[7] The notation $exp(C)$, where C is a square matrix, denotes the matrix exponential: $exp(C) = I + C + C^2/2 + C^3/6 + \cdots$ (see Kim and Orphanides, 2012).

h is the time between two observations. The measurement equation (Equation (3.24)) is a function of the parameters K, Σ, δ, δ_0, λ, δ^*, δ_0^*, and λ^* and an error term $\epsilon(t)$. DeJong (2000) states that it is important to observe more treasury yields than model factors to accurately estimate the model's parameters. According to Duan and Simonato (1999) and Geyer and Pichler (1997), all maturities are observed with a certain error $\epsilon(t)$. This error is assumed to be serially and cross-sectionally uncorrelated. Equation (3.25) is referred to as the transition equation with the conditional mean and variance of the factors. According to $\epsilon(t)$, $v(t+h)$ is an error term and assumed to be serially and cross-sectionally uncorrelated.

3.4 Results

Section 3.2 showed that two common factors and one local factor for each country best describe the variation in U.S. and U.K. treasury bonds. The corresponding joint ATSM was presented in Section 3.3. This section presents the parameter estimation results in Subsection 3.4.1 and interprets the yield and model factors in Subsection 3.4.2.

3.4.1 Parameter estimation results

Table 3.2 reports the parameter estimation results. Each standard error is given in parenthesis. The subscript $i = 0$ corresponds to the short rate constant parameters δ_0 and δ_0^*. Subscripts $i = 1, 2$ denote common factor parameters. Subscripts $i = 3, 4$ indicate local factor parameters. The local parameters U.S. (U.K.) are set to zero with no standard error for $i = 4$ ($i = 3$). The model estimates rely equally on the common factors ($\delta_{X1}^* = 0.0097$ to $\delta_{X2} = 0.0133$) and local factors ($\delta_{X3} = 0.0103$ to $\delta_{X4}^* = 0.0136$). K determines the factor dependence structure of the joint ATSM and each parameter κ_{ij} for $i, j = 1, \ldots, 4$ is the same for both countries. Yet, the local factors $i = 3, 4$ are mutually independent and the factor dependence is set to zero ($\kappa_{43} = 0$).

Table 3.3 reports the model fit. The observational errors ϵ_τ^{US} and ϵ_τ^{UK} are the mean

Table 3.2: Joint ATSM parameter estimates

	Const	$X_1^{com}(t)$	$X_2^{com}(t)$	$X_3^{US}(t)$	$X_4^{UK}(t)$
	i=0	i=1	i=2	i=3	i=4
δ_i	0.1188	0.0127	0.0133	0.0103	0
	(0.0456)	(0.0069)	(0.0068)	(0.0076)	-
δ_i^*	0.2500	0.0097	0.0131	0	0.0136
	(0.0604)	(0.0060)	(0.0079)	-	(0.0061)
κ_{1i}		0.3314	0	0	0
		(0.5168)	-	-	-
κ_{2i}		-0.1719	0.1000	0	0
		(0.1396)	(0.6555)	-	-
κ_{3i}		-0.1977	0.0841	0.1000	0
		(0.1295)	(0.2149)	(0.6982)	-
κ_{4i}		0.1717	-0.0405	0	0.8161
		(0.1956)	(0.1199)	-	(0.4397)
λ_i		0.0055	-1.5654	0.6850	0
		(1.5463)	(0.5936)	(0.7471)	-
λ_i^*		-0.1609	-0.9873	0	-1.6639
		(1.0456)	(1.8319)	-	(1.2250)

The table reports the maximum likelihood parameter estimation results of the joint ATSM. Standard errors are given in parentheses. The columns correspond to the constant (Const), common factor ($X_1^{com}(t)$ and $X_2^{com}(t)$) and local factor ($X_3^{US}(t)$ and $X_4^{UK}(t)$) parameters. An * denotes parameters corresponding to U.K. treasury yields.

Table 3.3: Summary statistics of the model fit

	τ =6m	τ =2y	τ =5y	τ =10y
ϵ_τ^{US}	0.0000	0.0015	0.0000	0.0017
	(0.0000)	(0.010)	(0.0000)	(0.0011)
ϵ_τ^{UK}	0.0000	0.0015	0.0001	0.0031
	(0.0000)	(0.0011)	(0.0001)	(0.0018)

The table reports the model fit. The observational errors ϵ_τ^{US} and ϵ_τ^{UK} are the mean of the error term $\epsilon(t)$ defined in Equation (3.24). τ denotes the yield's maturity. Their standard deviations are given in parentheses.

of the error term $\epsilon(t)$ defined in Equation (3.24). τ denotes the yield's maturity. Their standard deviations are given in parentheses. The biggest observational error is obtained for ten-years U.K. treasury yields ($\epsilon_{10}^{UK} = 0.0031$). In summary, the model matches the data well.

3.4.2 Yield and model factors

There is consensus in the literature to interpret the first three factors as "level", "slope", and "curvature" for standard three-factor ATSM's (see DeJong, 2000; Litterman and Scheinkman, 1991; Babbs and Nowman, 1999). However, the case is a little more precarious in the present multi-country model. Figure 3.2 indicates that the factors should be interpreted as "level", "spread", "U.S. slope", and "U.K. slope". Figure 3.3 illustrates these economic meanings with their corresponding model factors $X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{US}(t)$, and $X_4^{UK}(t)$ (see Equation (3.22)).

Figure 3.3 shows the fitted model and yield factors. "Common Factor 1" is fitted to the level of U.S. treasury yields. The solid black line illustrates the U.S. treasury yield level. After a short rise at the beginning of the sample period (1983 to 1985) the yield level continues to decrease till 2012. The evolution corresponds to the value of "Common Factor 1" (solid gray line). Beginning with a peak in 1985 it continues to decrease, accordingly. A similarly close relationship can be observed for the other yield and model factors. "Common Factor 2" is fitted to the spread of U.S. and U.K. five-year treasury yields in the second graph. These two common factors explain 85% of the overall variation (see Table 3.1). The variation in yields that cannot be explained by the common factors are captured by the local factors. The "Local Factor US" is fitted to the slope of U.S. yields which corresponds to the difference of the ten-years treasury yield minus the six-months treasury yield. The last graph fits the "Local Factor UK" to the slope of U.K. yields. This is the variation in the U.K. data that cannot be matched by the common factors.

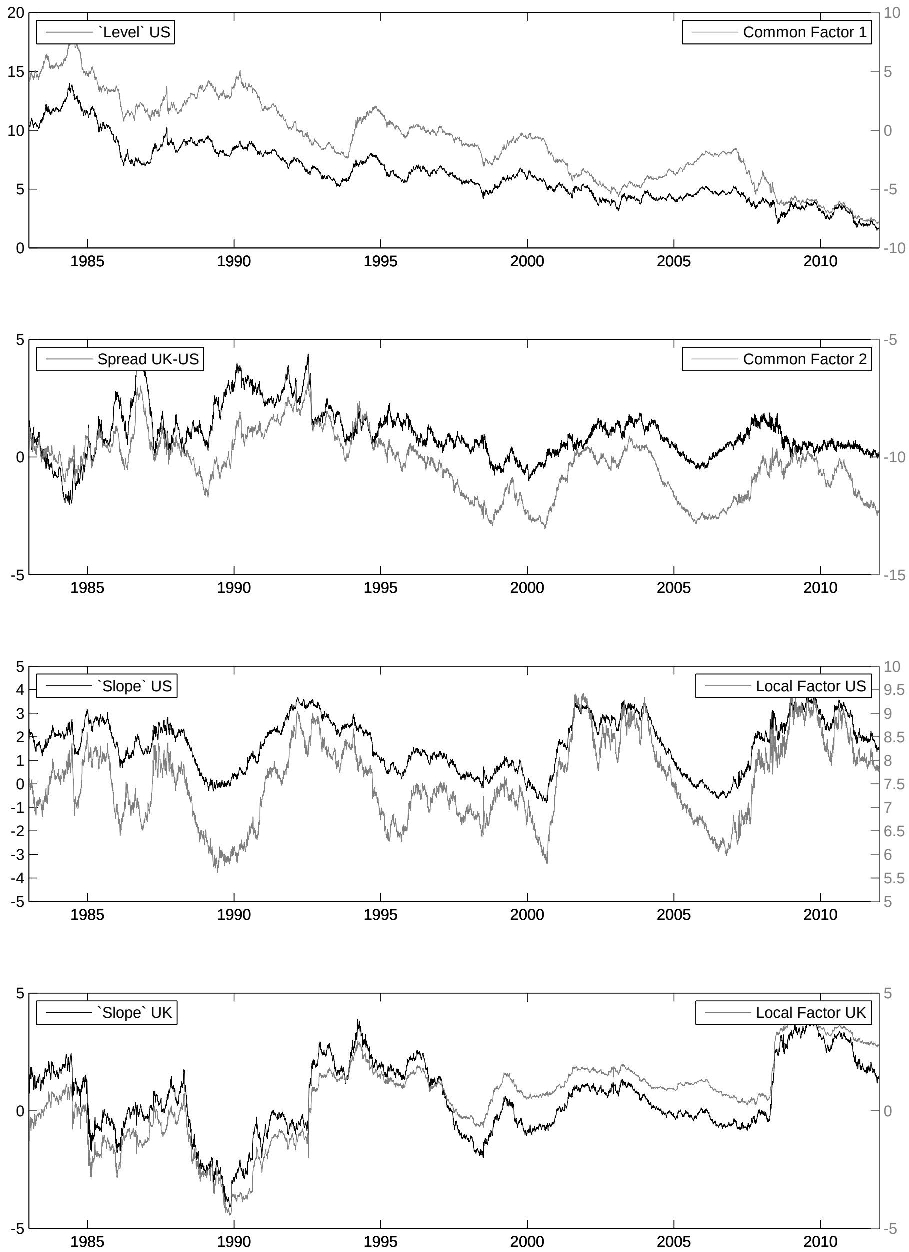

Figure 3.3: Fitted model and yield factors

The figure shows the common and local factors of the estimated joint ATSM. The “Common Factor 1”, “Common Factor 2”, “Local Factor US”, and “Local Factor UK” corresponds to the model factors $X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{US}(t)$, and $X_4^{UK}(t)$, respectively. Each factor is plotted with its corresponding treasury yield. “Common Factor 1” is fitted to the level of U.S. treasury yields (ten years U.S. treasury bond). “Common Factor 2” and the spread between the five years U.S. and U.K. treasury yields are plotted in the second graph. The third graph shows the “Local Factor US” and the slope of the U.S. treasury yields (ten-years - six-months). The last graph shows the “Local Factor UK” and the slope of the U.S. treasury yields. Factors and treasury yields run from the 2^{nd} of January 1983 to the 31^{st} of July 2012.

Table 3.4 the results of a regression analysis of the proposed model and U.S. and U.K. treasury yields. The table supports the previous findings. The model factors $X_1^{com}(t)$, $X_2^{com}(t)$, $X_3^{US}(t)$, and $X_4^{UK}(t)$ have high explanatory power indicated by high R-square statistics. The only exception is the yield spread factor which exhibits less explanatory power.

Table 3.4: Regression of U.S. and U.K. yield factors

	Level US	Spread UK-US	Slope US	Slope UK
$X_1^{com}(t)$	0.7877	-0.1556	-0.0226	0.2013
$X_2^{com}(t)$	-0.1461	0.2433	0.4346	0.2329
$X_3^{US}(t)$	0.7259	0.4853	0.8042	0.3291
$X_4^{UK}(t)$	-0.0402	0.6086	-0.0563	-1.0051
R^2	0.9917	0.5556	0.9500	0.9406

The table reports the results when the $A_0(4)$ model is regressed on US and UK treasury yields. The yield factors correspond to: Level US = ten years U.S. treasury bond; Spread UK-US = five years U.S. - U.K. treasury yields; Slope US = U.S. treasury yields (ten-years - six-months); Slope UK = U.K. treasury yields (ten-years - six-months). The linear regression model uses least squares. Each column in the upper panel corresponds to a vector of regression coefficients in the linear model. The lower panel reports the R-square statistic for each linear regression of yield factors.

The local factors explain additional 11% of the total yield variation. All factors exhibit a very high correlation with their economic intuition (i.e. up to 0.9848 for “Common Factor 1” vs “level”). Figure 3.3 shows that common and local factors are not only an empirical phenomenon. Joint ATSMs perfectly match the variation of international yields. Furthermore, the latent factors of the joint ATSM gain economic intuition. The common factors can be interpreted as “level” and “spread”. The local factors represent the “U.S. slope” and “U.K. slope”.

3.5 Portfolio application

The analyses of the empirical yields (see section 2) showed that international government bond markets share eminent risk factors. If assets exhibit a clear positive correlation a naive diversified portfolio is no longer sufficient. The section draws on this empirical finding. It offers a possible application where the common factors of the joint ATMS are incorporated within the optimal portfolio model in a discrete time framework.

3.5.1 The optimal portfolio model

The proposed joint ATSM already offers expectations of the future short rate drift and volatility. Therefore, all the necessary information is available to calculate the expected returns and covariances of international bonds. Since the model builds on a joint ATSM the optimal portfolio is conditional on the information in the term structures of interest rates. This extends the domestic portfolio model of Korn and Koziol (2006) by adding international bonds.[8]

The investor can choose a combination of zero-coupon bonds from two countries and different maturities $T_1 < \cdots < T_4$ for each country. In sum, the investor is able to invest in $2 \cdot 4$ bonds. $\bar{\mu} \in \mathbb{R}^{2\cdot 4}$ is the vector of expected returns and $\bar{\Sigma} \in \mathbb{R}^{2\cdot 4 \times 2\cdot 4}$ is the matrix of covariances:

$$\bar{\mu} = \begin{bmatrix} \bar{\mu}_{T1} \\ \vdots \\ \bar{\mu}_{T4} \\ \bar{\mu}^*_{T1} \\ \vdots \\ \bar{\mu}^*_{T_4} \end{bmatrix}, \bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{T1} & \cdots & \bar{\Sigma}^{,}_{T1,T4} & \bar{\Sigma}^{,*}_{T1,T1} & \cdots & \bar{\Sigma}^{,*}_{T1,T4} \\ \vdots & \ddots & & & & \vdots \\ \bar{\Sigma}^{,}_{T4,T1} & & & & & \bar{\Sigma}^{,*}_{T4,T4} \\ \bar{\Sigma}^{*,}_{T1,T1} & & & & & \bar{\Sigma}^{*,*}_{T1,T4} \\ \vdots & & & & \ddots & \vdots \\ \bar{\Sigma}^{*,}_{T4,T1} & \cdots & \bar{\Sigma}^{*,}_{T4,T4} & \bar{\Sigma}^{*,*}_{T4,T1} & \cdots & \bar{\Sigma}^{*}_{T4} \end{bmatrix}. \tag{3.26}$$

[8] Korn and Koziol (2006) propose a model for domestic bond portfolio optimization building on a single ATSM with uncorrelated Gaussian factors.

$\bar{\mu}_{Ti}$ and $\bar{\Sigma}_{Ti,Tj}$ can be evaluated as follows:[9]

$$\bar{\mu}_{Ti} = \frac{exp(M_{(1)}(T_i) + \frac{1}{2}S_{(1)}(T_i)^2)}{P\ (0,T_i)} - 1, \tag{3.27}$$

$$\bar{\mu}^*_{Ti} = \frac{exp(M^*_{(1)}(T_i) + \frac{1}{2}S^*_{(1)}(T_i)^2)}{P^*(0,T_i)} - 1, \tag{3.28}$$

$$\bar{\Sigma}_{Ti} = \frac{exp(2 \cdot M_{(1)}(T_i) + S_{(1)}(T_i)^2) \cdot (exp(S_{(1)}(T_i)^2) - 1)}{P\ (0,T_i)}, \tag{3.29}$$

$$\bar{\Sigma}^*_{Ti} = \frac{exp(2 \cdot M^*_{(1)}(T_i) + S^*_{(1)}(T_i)^2) \cdot (exp(S^*_{(1)}(T_i)^2) - 1)}{P^*(0,T_i)}, \text{ and} \tag{3.30}$$

$$\begin{aligned}\bar{\Sigma}^{*,}_{Ti,Tj} =& \frac{exp(M^{*,}_{(2)}(T_i,T_j) + \frac{1}{2}S^{*,}_{(2)}(T_i,T_j)^2)}{P^*(0,T_i) \cdot P\ (0,T_j)} \\ &- \frac{exp(M^*_{(1)}(T_i) + M_{(1)}(T_j) + \frac{1}{2}(S^*_{(1)}(T_i)^2 + S_{(1)}(T_j)^2))}{P^*(0,T_i) \cdot P\ (0,T_j)} \quad \text{for } i \neq j,\end{aligned} \tag{3.31}$$

where

$$\begin{aligned}
M^*_{(1)}(T_i) =& A^*(T_i) + [B(T_i)]^T \cdot E_0^{\mathbb{P}}[X_n(t)], \\
M_{(1)}(T_i) =& A^*(T_i) + [B(T_i)]^T \cdot E_0^{\mathbb{P}}[X_n(t)], \\
S^*_{(1)}(T_i)^2 =& [B(T_i)]^T \cdot [Var_0^{\mathbb{P}}[X_n(t)]] \cdot [B(T_i)], \\
M^{*,}_{(2)}(T_i,T_j) =& A^*(T_i) + A\ (T_j) + [B(T_i) + B(T_j)]^T \cdot E_0^{\mathbb{P}}[X_n(t)], \text{ and} \\
S^{*,}_{(2)}(T_i,T_j)^2 =& [B(T_i) + B(T_j)]^T \cdot [Var_0^{\mathbb{P}}[X_n(t)]] \cdot [B(T_i) + B(T_j)].
\end{aligned}$$

As Equations (3.27) through (3.31) show, the expected returns and covariances are available in closed-form. Subsection 1.1.2 raised the question: (4) How can an optimal portfolio model be set up that accounts for common factors in international bond markets? This subsection has proposed an answer to this question. The model defined in Equation (3.26) constitutes an optimal portfolio model that accounts for common

[9] One uses the fact that the state variable $X(t)$ is normally distributed. Hence, $exp(X(t))$ is log-normally distributed and it is $E[exp(X(t))] = exp(E[X(t)] + \frac{1}{2}Var[X(t)])$ and $Var[exp(X(t))] = E[exp(X(t))]^2(exp(Var[X(t)]) - 1)$.

factors in international bond markets. Having calculated the bond portfolio's $\bar{\mu}$ and $\bar{\Sigma}$, the model is fully specified and it can be applied to the data. This will provide clarity on how the cross section and time series of treasury yields are related to the portfolio weights evolution.

3.5.2 Portfolio strategy

An international government bond portfolio has two main risk-driving factors - interest rate and exchange rate risk. The focus of the present section is the portfolio risk caused by the variation of interest rates. To this end, the analysis is based on hedged international bond portfolio (see Hunter and Simon, 2004).[10] According to Morey and Simpson (2001), three different currency hedging strategies exist. They classify the three strategies as: (a) unhedged, (b) always hedged, and (c) selectively hedged. In line with the former argument, the exchange rate risk is always hedged away (b) in the present portfolio strategy.

The investment horizon is determined to be one year. The investment set consists of U.S. and U.K. zero-coupon bonds with one-, two-, five-, and ten-years time to maturity. The shortest maturity of one year is due to the investment horizon. If the investment horizon is one year it is not straightforward to invest in an asset that exhibits a shorter maturity. In that case, the investor has to reinvest the amount in a cash account after maturity and hold it till the end of the horizon (see Wilhelm, 1992). To avoid this hurdle and to ensure comparability with other studies of the field (see Korn and Koziol, 2006), the one year bond is the shortest to invest in. Since there is no one year bond in the data, the market price of that asset must be interpolated from the available bond prices.

The Federal Reserve uses smoothing splines whereas the Bank of England uses VRP (variable roughness penalty) to interpolate the treasury yield curve (see BIS, 2005). To

[10] Driessen et al. (2003) propose a disregard of exchange rate effects. Since the difference between forward currency rates and current spot exchange rates is usually close to zero, the returns of a hedged and an unhedged portfolio will not differ significantly. Therefore, hedged bond portfolio returns are primarily driven by the variation in the underlying term structure of interest rates.

ascertain consistency, the yield curves are smoothed with the Svensson (1994) model. This model is a type of parametric model, also referred to as function-based models, for fitting the yield curve. The model may also be viewed as a constant term plus a Laguerre function (see Courant and Hilbert, 1953). The yield curve function is specified by the six parameters $\beta_0, \beta_1, \beta_2, \beta_3, \bar{\tau}_1$, and $\bar{\tau}_2$. The spot rate $\bar{y}(T_i)$ for a zero-coupon bond maturing at T_i reads:

$$
\begin{aligned}
\bar{y}(T_i) =& \beta_0 + \beta_1 \frac{1 - exp(-\frac{T_i}{\bar{\tau}_1})}{\frac{T_i}{\bar{\tau}_1}} + \beta_2 (\frac{1 - exp(-\frac{T_i}{\bar{\tau}_1})}{\frac{T_i}{\bar{\tau}_1}} - exp(-\frac{T_i}{\bar{\tau}_1})) \\
&+ \beta_3 (\frac{1 - exp(-\frac{T_i}{\bar{\tau}_2})}{\frac{T_i}{\bar{\tau}_2}} - exp(-\frac{T_i}{\bar{\tau}_2})).
\end{aligned}
\tag{3.32}
$$

The model parameters are determined through minimization of squared deviations of theoretical prices from observed prices (see BIS, 2005). It allows for generalization by adding terms of higher order of the Laguerre function. The Svensson (1994) model is a generalization of the Nelson and Siegel (1987) model by one order. The generalization offers the flexibility of a second 'hump' at the cost of adding two more parameters (β_3 and $\bar{\tau}_2$).

The optimal portfolios are calculated in the following order: (1) The parameters estimated in Subsection 3.4.1 determine the joint ATSM for the entire sample period. The optimal portfolio is set up conditional on the information currently reflected by the term structures of interest rates. The information at the time correspond to the value of the model factors. (2a) The common and local factors are observed on the 2^{nd} of January, 1983 ($[X_1^{com}(t), X_2^{com}(t), X_3^{US}(t), X_4^{UK}(t)]_{02/01/1983}$). Taking the parameters (1) and factors (2a), the optimal portfolio can be calculated using the optimal portfolio model (see Subsection 3.5.1). In the μ-σ sense, the optimal portfolio is calculated for an investment horizon of one year and a fixed volatility of $\sigma = 10\%$. Short selling is permitted. The investment horizon rolls over one day. (2b) The factors observed on the next day $[X_1^{com}(t), X_2^{com}(t), X_3^{US}(t), X_4^{UK}(t)]_{03/01/1983}$ are collected and the procedure is repeated. Since the investment horizon is 260 trading days, the window continues to roll until the 31^{st} of July, 2011.

3.5.3 Portfolio weight evolution

One particularly attractive aspect of term structure models is that the variation in the time series dimension of interest rates is captured in the latent factors. That paves the way for setting up an optimal bond portfolio conditional on the present state of the economy. Strictly speaking, the optimal portfolio is conditional on the present state of the two economies since the common and local factors capture the variation of both countries.

Figure 3.4 reports the portfolio weight evolutions. The first graph fits the U.S. level and U.K. level to the long end portfolio weights. The long end portfolio weights are defined as the amount invested in U.S. and U.K. bonds with ten years to maturity. The correlations of -0.8573 (U.S.) and -0.8617 (U.K.) are highly negative, indicating that an increase of the interest rate level leads to a decrease in the amount invested in long maturity bonds. A reason for that observation is be the decreasing relative attractiveness of long bonds. When the overall level of interest rates rises, the returns of short bonds also increase. In this case, long bonds have relatively lower excess returns with considerably higher risk in comparison to short bonds. Therefore, long bonds become less attractive and the amount invested is reduced. The second graph fits the slope of U.S. and U.K. treasury yield curves to the short end portfolio weights.[11] The short end portfolio weights are the sum of one-year and two-years bonds for the U.S. and U.K.. The correlations of -0.6821(U.S.) and -0.7139(U.K.) indicate that if the slope of the yield curves increases the amount invested in the short end decreases. This follows the same line of argument as applied to the previous graph. The long end bonds are more attractive, the higher the spread between long and short end is. On the other

[11] As in the former section, the slope is defined as the difference of the ten-years minus the six-months maturity bond.

hand, if the yield curve becomes inverse (negatively sloped) the portfolio weights shift to the short end.

The third graph shows the fitted short ends of the U.S. and U.K. yield curves and the portfolio slope. The short end of the yield curve corresponds to the six-months bond. The portfolio slope is defined as the ten-years bond minus the sum of the one-year and two-years bonds of the U.S. and the U.K. portfolio weights. The correlations of -0.7461 (U.S.) and -0.7753 (U.K.) indicate that if the short end yields increase, the money is invested in short maturity bonds and the amount of long maturity bonds is reduced. The last graph fits the short end of the treasury yield curve to the portfolio duration. The portfolio duration is measured as the amount invested in the bonds (w) times the bonds' maturity ($[w_1, \cdots w_{10}, w_1^*, \cdots, w_{10}^*]' \cdot [1, 2, 5, 10, 1, 2, 5, 10]$). The correlations of the short end bonds and the portfolio duration are -0.7686 (U.S.) and -0.8077 (U.K.). Not surprisingly, a risk-averse investor decreases the portfolio duration when the short rate increase and vice versa. The empirical study shows that common factors in international bond returns are not only an empirical phenomenon; the empirical findings can be supported by the proposed model and the model can link the investor's decision conditional on the common factors in international bond returns.

3.6 Conclusion

Investors are aware of the importance of common factors in international bond returns. However, little is known about the linkage and economic intuition of latent factors of joint ATSMs. This chapter has tried to close that gap and has offered an investigation of the common factors of U.S. and U.K. treasury yields in the period 1983 to 2012. Based on these, a joint ATSM has been proposed that is capable of modeling the variability of treasury yields of both economies. In addition, a bond portfolio application has offered a possible extension of the proposed model and shown how the common and local factors are reflected in the investor's portfolio choice.

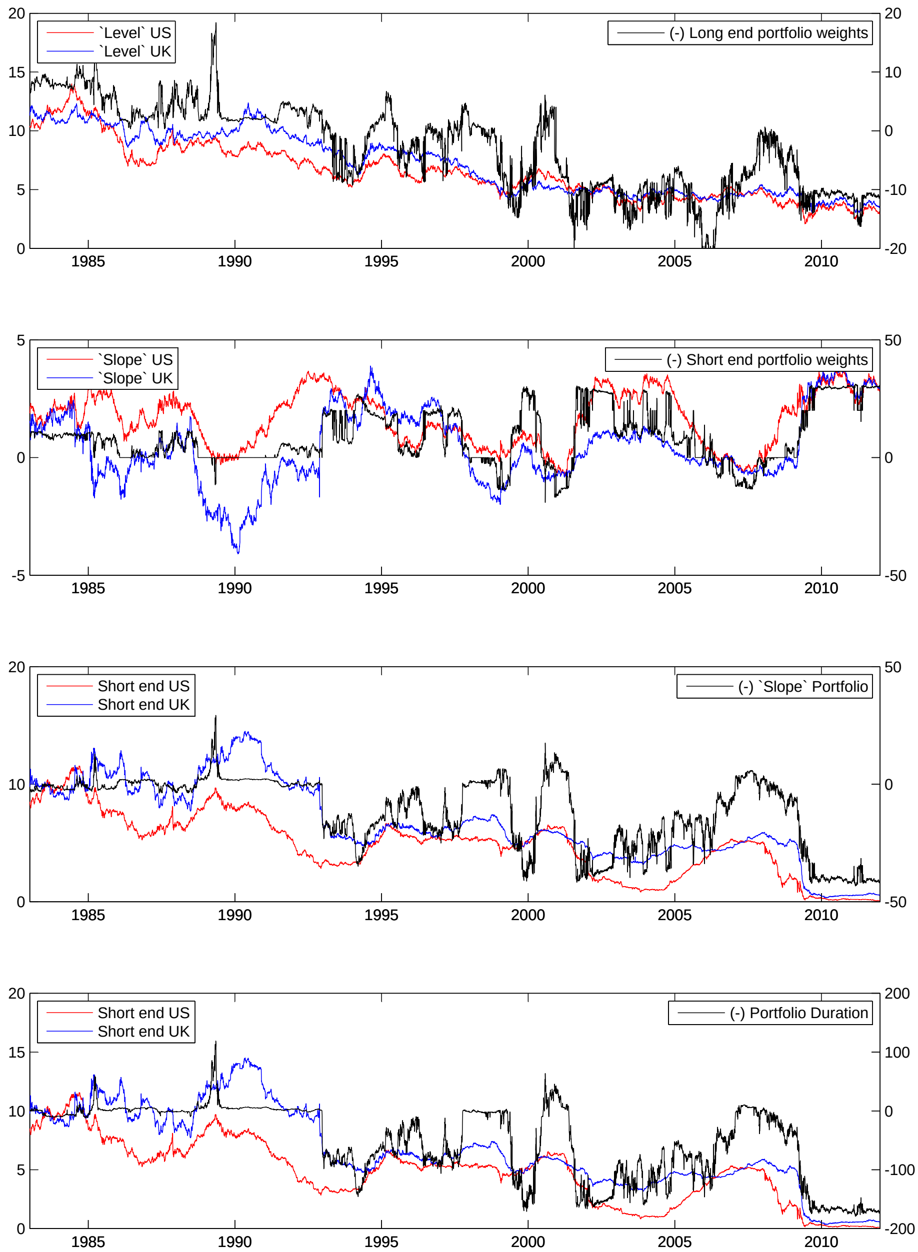

Figure 3.4: Fitted portfolio weights and yield factors

The figure shows the portfolio weights of the conditional bond portfolio. The portfolio weights are fitted to their corresponding U.S. and U.K. treasury yields. The portfolio weights are calculated as the sum of both countries and the following yields: long end = ten-years; short end = six-months + two-years; slope = ten-years - (six-months + two-years); duration = $(w_1, \cdots w_{10}, w_1^*, \cdots, w_{10}^*)' \cdot (1, 2, 5, 10, 1, 2, 5, 10)$. The portfolio weights and treasury yields run from the 2^{nd} of January 1983 to the 29^{th} of July 2011.

The findings of this chapter can be summarized as follows: the factor analysis reveals that two (four) factors already explain 85% (96%) of the overall yield variation. These factors offer a clear economic intuition. A principal component analysis has shown that the first two factors are common whereas the third and fourth factor is local. The yield principal components lead to the conclusion that the latent model factors can be interpreted as “level”, “spread”, “U.S. slope”, and “U.K. slope”. The proposed joint ATSM exhibits an excellent fit and the factor interpretation is also reflected in the model factors. In addition, the chapter offers a possible application of the proposed joint ATSM. The identified common and local factors explain the portfolio adjustments of a fixed income investor. “Level”, “spread”, “U.S. slope” and “U.K. slope” are the predominant decision criteria.

Part II

Higher order moments of bond yields

4 On the distribution of government bond returns: evidence from the EMU

This chapter is based on joint work with Christian Lau (see Gabriel and Lau, 2014).

4.1 Introduction

International bond markets clearly exceed equity markets in terms of capitalization (see Laopodis, 2008). Thus, investigating these markets could have important implications for interest rate modeling, fixed income portfolio management, and monetary policy making. However, equity markets attract considerably more attention in the finance literature than do bond markets. The European Monetary Union (EMU) bond market is particularly unique in that it accommodates economies with different levels of credibility and fiscal discipline in one currency (see Beber et al., 2009).

The objective of this chapter is to investigate the statistical distribution of price changes in European government bonds. For the period 1999 to 2012, we investigate all countries that joined the EMU before 2001. We exclude Luxembourg from our analysis since its public debt market is negligible.[1] The data frequency is daily bond returns with one-, three-, five-, and ten-year maturity. Descriptive statistics and tests of normality lead to a clear rejection of the Gaussian assumption. We therefore propose alternative distributions and fit the Student's t, skewed Student's t, and stable distribution to the

[1] We thank an anonymous referee for highlighting this fact.

data. Since the Euro crisis leads to a shift in the mean and an increase in variance, we test each time series for a structural break and separately study the crisis period.[2] Finally, a value-at-risk (VaR) application contributes to better understanding the implications that can be derived from the distributional assumption.

The type of distribution of financial returns is an essential assumption for mean-variance portfolio theory, pricing of financial derivatives, and many other applications. Mandelbrot (1963) and Fama (1965) reject normality because heavy tails are a key feature of financial returns. They and other authors propose various distributions that account for excess kurtosis (see Press, 1967; Praetz, 1972; Blattberg and Gonedes, 1974; Peiró, 1994). However, investors not only have an aversion to the second and fourth moment, but also a preference for positive first and third moments. Hence, skewness is important for modeling financial returns (see Kon, 1984; Hansen, 1994; Young and Graff, 1995; Peiró, 1999; Rachev et al., 2000; Aparicio and Estrada, 2001). This branch of the literature is mainly concerned with equity returns, whereas the EMU, which is the market of interest in this chapter, is more often discussed in debt capital market research.

Prior to the EMU, we observe converging yields and harmonizing prices of Euro-denominated government bonds (see Baele et al., 2004; Codogno et al., 2003; Hartmann et al., 2003). Decreasing government financing costs are one reason for the significant growth of the European bond market (see Pagano and Thadden, 2004). There is a large body of literature concerned with the European bond market and its interactions with other major bond markets (see Cappiello et al., 2003; Christiansen, 2007; Abad et al., 2010). Laopodis (2008) conducts an extensive empirical study of the link between Euro and non-Euro government bonds for the period 1995 to 2006. However, he draws no conclusions as to which distribution fits the bonds' variation best. Rachev et al. (2003)

[2] It is important to note that our analysis is based on unconditional distributions. GARCH-type models (see Bollerslev, 1986) are beyond the scope of the present chapter.

are the only authors who study the distribution of U.S. corporate bond returns.[3]

The present chapter contributes to the literature by providing a comprehensive study of EMU bond return distributions. To the best of our knowledge, we are the first authors to analyze the daily bond returns of all EMU countries with one-, three-, five-, and ten-year maturity. We test alternative distributions, account for structural breaks in the time series, and offer an application for risk management.

The remainder of the chapter is organized as follows. Section 4.2 reports some descriptive statistics and tests the normality assumption. Section 4.3 presents the theory of the proposed distributions, and Section 4.4 shows the parameter estimation results. Section 4.5 reports the results of a Quandt likelihood ratio test to ascertain if there is a structural break and then takes another look at the Euro crisis period. Section 4.6 presents a VaR application for government bond returns. Section 4.7 concludes.

4.2 Data and test of normality

In terms of capitalization, debt markets clearly exceed equity markets (see Laopodis, 2008). Additionally, the EMU bond market is unique in providing debt for countries with different levels of credibility and fiscal discipline in one currency (see Beber et al., 2009). Therefore, we study government bonds issued by EMU members.

The dataset consists of all countries that joined the EMU before 2001 with the exception of Luxembourg. Countries that joined the Eurozone later are excluded to avoid studying time series of different length. The daily zero bond returns are provided by Datastream. The empirical study starts in 1999, when exchange rates for prospective Euro members were fixed. The sample period is January 1, 1999 to November 30, 2012, resulting in 3,627 data points for each time series.[4] The cross-section of bond returns

[3] Rachev et al. (2003) fit the stable distribution to U.S. corporate bond indices. Interest rate risk, measured with duration, and credit default risk are the main risk-driving factors of bonds. Using indices leads to a clustering of duration and rating, resulting in a less than clear view of the bonds' risk.

[4] The time series for Belgium and Greece start in 2001, resulting in 3,150 data points.

are fixed maturities of one, three, five, and ten years.

Next, we calculate the bonds' daily return. Let $y_\tau(t)$ be the yield of a bond at time t and τ its time to maturity. At each point in time, we take the yield of the previous day $y_{\tau_1}(t-1)$ multiplied by the initial time to maturity τ_1 (= one, three, five, and ten years) and subtract it from today's yield $y_{\tau_2}(t)$ multiplied by the remaining time to maturity τ_2 ($= \tau_1 - 1$ day).[5] The log return $r_t(\tau)$ at time t with maturity τ reads:

$$r_\tau(t) = exp(-\tau_2 y_{\tau_2}(t) + \tau_1 y_{\tau_1}(t-1)) - 1. \tag{4.1}$$

In this manner we calculate the log return for every bond at each point in time.

Table 4.1 summarizes some descriptive statistics of EMU bonds. The left panel of the table shows the first four moments of government bond returns. We apply the Lilliefors and Jarque-Bera goodness-of-fit tests of normality and report the results in the right panel (see Peiró, 1999; Aparicio and Estrada, 2001). In Table 4.1 and henceforth, the order of the countries follows their exposure to sovereign risk.[6]

The table shows that the mean is positive and close to zero for the daily bond returns. The only exception is Greece with slightly higher and negative returns, which we treat as a special case throughout the chapter.[7] It is evident that returns increase with time to maturity. This indicates a normal term structure for most of the time series. Analogously, bond risk is an increasing function of time to maturity. Although yields of the short end are more volatile, the exposure to interest rate risk is much higher for bonds with a longer time to maturity. Values range from a low of $0.405 \cdot 10^{-3}$ (Germany one year) to $17.416 \cdot 10^{-3}$ (Portugal ten years). The order of countries suggests that standard deviation increases with exposure to interest rate and sovereign risk.

[5]We subtract two days for a public holiday and three days for a weekend.

[6]The new phenomenon of sovereign risk in European government bonds is important for interpreting the variation of returns (see Gomez-Puig, 2009; Bernoth et al., 2004; Sgherri and Zoli, 2009). We calculate the average spread of ten-year bonds for each country over ten-year German bonds, which we assume to be the reference.

[7]Due to the imminent default of Greece, results for Greek bonds are more extreme throughout the study. For the sake of brevity, we document the results for Greece only if they provide new insight.

Table 4.1: Descriptive statistics of European government bond returns

	m	Moments: Mn 10^3	Std 10^3	Skew	Kurt	Goodness-of-fit: LF	JB
GER	1	0.111	0.405	0.699	18.461	0.080	36,410
	3	0.144	1.364	-0.109	5.408	0.044	883
	5	0.174	2.370	-0.199	4.708	0.043	464
	10	0.230	4.170	-0.021	5.191	0.043	725
NET	1	0.115	0.643	-0.011	22.512	0.120	57,519
	3	0.147	1.557	0.194	12.273	0.068	13,015
	5	0.175	2.403	-0.150	6.202	0.049	1,562
	10	0.224	6.017	-0.250	11.473	0.068	10,884
FIN	1	0.118	0.586	-0.067	17.188	0.110	30,414
	3	0.148	1.486	-0.297	5.969	0.047	1,384
	5	0.178	2.726	-0.509	19.933	0.064	43,475
	10	0.229	5.807	0.637	17.037	0.070	30,016
FRA	1	0.112	0.534	-0.206	14.684	0.105	20,651
	3	0.147	1.385	-0.054	7.315	0.053	2,814
	5	0.173	2.401	0.087	15.839	0.053	24,910
	10	0.210	3.981	-0.396	11.879	0.044	12,005
AUS	1	0.116	0.656	0.208	18.229	0.161	35,064
	3	0.152	1.336	0.041	12.719	0.148	14,271
	5	0.182	2.345	-0.209	11.000	0.137	9,696
	10	0.229	5.908	-0.165	9.552	0.133	6,503
BEL	1	0.119	0.677	-0.801	30.893	0.141	102,383
	3	0.169	1.667	0.114	15.826	0.069	21,584
	5	0.208	2.640	-0.093	10.664	0.059	7,708
	10	0.268	4.676	-0.151	7.960	0.056	3,238
SPA	1	0.123	1.040	-0.750	29.064	0.170	102,979
	3	0.139	2.105	1.843	45.238	0.171	271,597
	5	0.144	3.122	2.041	38.123	0.145	188,900
	10	0.122	5.599	0.291	12.896	0.106	14,846
ITA	1	0.127	0.744	0.547	34.413	0.166	149,266
	3	0.150	2.151	1.490	38.693	0.125	193,822
	5	0.162	3.410	1.894	41.167	0.107	222,254
	10	0.168	5.614	0.995	28.085	0.097	95,671
IRE	1	0.153	1.647	2.886	69.022	0.209	663,589
	3	0.177	4.180	3.991	93.992	0.200	1,260,538
	5	0.182	4.923	1.060	43.201	0.161	244,843
	10	0.110	11.848	-1.837	53.103	0.186	381,299
POR	1	0.162	2.936	-0.912	46.671	0.226	288,646
	3	0.170	6.339	-2.620	101.927	0.230	1,482,741
	5	0.154	8.073	-1.155	110.194	0.211	1,736,848
	10	0.101	17.416	-0.523	32.179	0.175	128,796
GRE	1	-0.631	29.724	-6.020	268.212	0.409	9,244,928
	3	-0.596	15.794	-3.389	184.364	0.316	4,320,463
	5	-0.583	17.734	-8.747	229.126	0.295	6,747,110
	10	-1.342	62.924	-2.748	182.182	0.341	4,215,222

The table reports the mean (Mn), standard deviation (Std), skewness (Skew), and kurtosis (Kurt) of European government bond returns maturing in m years. For comparison, the normal distribution has zero skewness and a kurtosis of three. The analysis includes 3,627 (3,149) observations for all countries (Belgium and Greece). LF denotes the Lilliefors test statistic defined as $max|S(x) - CDF|$ with $S(x)$ the empirical cdf and CDF the cumulative distribution function of a normal distribution with mean and standard deviation from the empirical data. For all countries and maturities, p-values are well below 0.001 and not provided here. The critical value for a rejection of the null hypothesis that "the data is normally distributed" at the 1% level is 0.0175 (0.0188) for all countries (Belgium and Greece). JB denotes Jarque-Bera test statistic defined as $N \cdot (Skew^2/6 + (Kurt - 3)^2/24)$ with N the number of observations. For all countries and maturities, p-values are well below 0.001 and, again, not provided here. The critical value for a rejection of the null hypothesis that "the data is normally distributed" at the 1% level is 9.4828 (9.5242) for all countries (Belgium and Greece).

The assumption of normality indicates that returns are symmetrically distributed, i.e., exhibit a skewness of zero. The table shows that countries with low sovereign risk tend to have very low skewness (Germany to Belgium), whereas countries with high sovereign risk tend to have higher skewness (Spain to Greece). There seems to be no clear pattern for the sign of skewness. Further tests are needed to discover whether skewness is important for bond returns.

Assuming normality implies a kurtosis of three ($Kurt = 3$). By contrast, the empirical distributions of **all** bond returns exhibit excess kurtosis ($Kurt >> 3$). Returns of German five-year bonds ($Kurt = 4.708$) and Portuguese five-year bonds ($Kurt = 110.194$) are the least and most heavy tailed, respectively. To sum up, the excess kurtosis implies that the returns depart from normality in the tails and indicates that the Gaussian distribution is an inappropriate assumption. Goodness-of-fit tests will provide a more detailed picture.

The second part of Table 4.1 reports test statistics of Lilliefors and Jarque-Bera tests of normality. The Lilliefors test statistic is defined as $max|S(x) - CDF|$ with $S(x)$ the empirical cdf and CDF the cumulative distribution function of the normal distribution. The critical value for a rejection of the null hypothesis that "the data is normally distributed" at the 1% significance level is 0.0175 (0.0188) for all countries (Belgium and Greece). The bonds with the lowest excess kurtosis and, therefore, with the best fit are German five-year bonds. Since the test statistic for these is well above the critical value, normality is nevertheless rejected.

The Jarque-Bera test statistic is defined as $N \cdot (Skew^2/6 + (Kurt - 3)^2/24)$ with N equal to the number of observations. The critical value for a rejection of the null hypothesis that "the data is normally distributed" at the 1% significance level is 9.4828 (9.5242) for all countries (Belgium and Greece). Similar to the Lilliefors test, Germany's five-year bond offers the best fit. However, the test statistic is well above the critical value of 9.4828 and the null hypotheses is rejected. Portuguese five-year bonds exhibit the

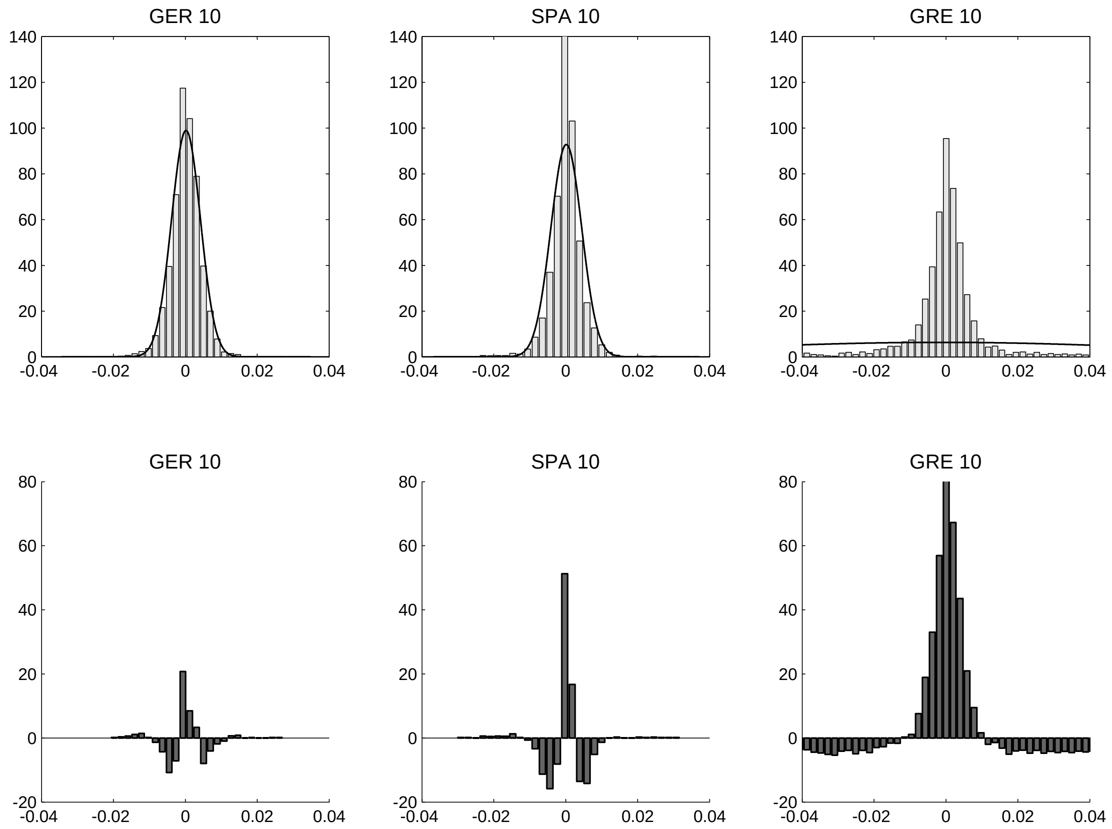

Figure 4.1: Normal distribution and difference in frequency

The figure illustrates the empirical and theoretical distribution of ten year bond returns of Germany, Spain, and Greece in the period 1999 to 2012: 1. Histograms of daily bond returns and fitted probability density functions of the normal distribution. 2. Difference in frequency between the empirical and normal distribution.

highest kurtosis and the worst Jarque-Bera fit. In short, Lilliefors and Jarque-Bera tests overwhelmingly reject the Gaussian distribution for all countries and all maturities at the 1% significance level. The empirical distributions depart from the normal distribution mainly in the tails, which is due to excess kurtosis of bond returns. Figure 4.1 illustrates the difference between the empirical and normal distribution.

As an example, Figure 4.1 shows the fit of the normal distribution for three bonds, selected to be representative of countries with low (Germany), considerable (Spain), and high (Greece) exposure to sovereign risk. The figure illustrates ten-year bonds since this is the most interesting maturity for investors (see Codogno et al., 2003; Bernoth et al., 2004; Gomez-Puig, 2009).

In the first row of Figure 4.1, the histogram of the data and the probability density function of the normal distribution are plotted. The second row shows the difference in frequency between the empirical and normal distributions (see Young and Graff, 1995) and illustrates the goodness-of-fit results reported in Table 4.1. Although the normal distribution fits the German bond somewhat better than the Spanish bond, it still does not exactly match the empirical distribution. The normal density function underestimates empirical bond returns around the mean and in the tails, while it overestimates them in the shoulders of the distribution. The leptokurtic behavior of Greek bonds prohibits the normal density function from fitting the data in any way.

In sum, the Lilliefors and Jarque-Bera tests overwhelmingly reject the Gaussian distribution for all countries. The graphical analysis in Figure 4.1 shows that the misspecification increases with the country's exposure to sovereign risk. Empirical bond distributions depart from the normal distribution mainly in the tails due to excess kurtosis. The clear rejection of normality forces the investor to consider alternative distributions.

4.3 Alternative distributions

The results of the former section lead to the conclusion that the normal distribution is an inappropriate assumption for describing bond returns. However, there is no ex ante financial theory that can aid us in choosing alternative distributions (see Aparicio and Estrada, 2001). Therefore, we focus on the empirical distributions' departure from the normal to identify features that the proposed distributions should have. The results of Table 4.1 show that all bonds have considerable excess kurtosis and some bonds exhibit skewness. Therefore, we consider one distribution that exhibits heavy tails and two that account for skewness and heavy tails.

Praetz (1972) and Blattberg and Gonedes (1974) propose Student's t distribution for modeling financial returns. The density function of the Student's t distribution with unit variance and zero mean is

$$g(X \mid \eta) = \frac{\Gamma\left((\eta+1)/2\right)}{\sqrt{\pi(\eta-2)}\Gamma(\eta/2)}\left(1+\frac{X^2}{\eta-2}\right)^{-(\eta+1)/2} \tag{4.2}$$

with Γ being the Γ-function and $2 < \eta < \infty$ the degrees of freedom. A small value of η implies excess kurtosis. The normal distribution is a special case of the Student's t distribution if η tends to infinity.

Since Kon (1984), it has been standard practice to account for asymmetry when describing financial returns. Extending the density function in Equation (4.2) by a skewness parameter λ results in the skewed Student's t distribution, which is able to capture both skewness via λ and excess kurtosis via η. Following Hansen (1994), the density of the skewed Student's t distribution is given by

$$g(X \mid \eta, \lambda) = \begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bX+a}{1-\lambda}\right)^2\right)^{-(\eta+1)/2}, & X < -a/b, \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bX+a}{1+\lambda}\right)^2\right)^{-(\eta+1)/2}, & X \geq -a/b \end{cases} \tag{4.3}$$

with $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants a, b, and c are given by

$$a = \frac{4\lambda c(\eta-2)}{(\eta-1)},$$

$$b^2 = 1 + 3\lambda^2 - a^2 \text{ and}$$

$$c = \frac{\Gamma\left((\eta+1)/2\right)}{\sqrt{\pi(\eta-2)}\Gamma(\eta/2)}.$$

A positive value of λ implies positive skewness and vice versa. By setting $\lambda = 0$, the skewed Student's t distribution nests the Student's t distribution.[8]

The (α-)stable distribution can also exhibit skewness and excess kurtosis (see Young and Graff, 1995; Rachev et al., 2000). Its four parameters are the index of stability (α), and skewness (β), scale (γ), and location (δ) parameters. In general, there is no closed-form solution, but it is possible to provide the characteristic function. A random variable X is viewed as stable if its characteristic function is (see Nolan, 2001)

$$E\exp(\mathrm{i}tX) = \begin{cases} \exp\left(-\gamma^\alpha|t|^\alpha\left[1+\mathrm{i}\beta\left(\tan\frac{\pi\alpha}{2}\right)(\operatorname{sign}t)\left((\gamma|t|)^{1-\alpha}-1\right)\right]+\mathrm{i}\delta t\right), & \alpha \neq 1, \\ \exp\left(-\gamma|t|\left[1+\mathrm{i}\beta\frac{2}{\pi}(\operatorname{sign}t)(\ln|t|+\ln\gamma)\right]+\mathrm{i}\delta t\right), & \alpha = 1 \end{cases} \tag{4.4}$$

with $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$, $\gamma > 0$, and $\delta \in \mathbb{R}$. The skewness increases with $|\beta|$. As α tends to 2, the distribution becomes Gaussian and β loses its influence. Lower values of α indicate heavy tails. The second moment does not exist for $\alpha < 2$ or, rather, the variance is infinite. For $\alpha < 1$, the stable distribution has no mean either. In contrast to the skewed Student's t distribution, it is not possible to model non-heavy-tailed but skewed returns with this distribution.

The focus of this article is to apply the more popular skewed and fat-tailed distribu-

[8] We decide against a more general formulation of the Student's t distribution that allows more extreme values of the tail parameter and the nonexistence of the first moment as using such a formulation would imply that the parameters are no longer comparable to the parameters of the skewed Student's t distribution.

tions, including the Student's t and the stable model. These distributions have a long history in academia and are prominent in the financial return literature (see Section 1). They provide the foundation for a wide range of commercial applications by leading risk management service providers (see Rachev et al., 2010). To ensure consistency, we use the extension of the Student's t distribution by the third moment and include the skewed Student's t distribution in our analyses.

4.4 Parameter estimation and goodness-of-fit tests

We now present the results of the empirical study. Table 4.2 reports the parameters estimated with maximum likelihood.[9] We assume the returns to be "significantly different from normality" if they:

1. are skewed ($\lambda \neq 0$ for the skewed Student's t or $\beta \neq 0$ for the stable distribution) or
2. exhibit excess kurtosis ($\eta < 30$ for the Student's t and skewed Student's t or $\alpha < 2$ for the stable distribution).

An * (**) indicates significance at the 95% (99%) confidence level.

The results in Table 4.2 show that the location parameter μ is close to zero and positive for all countries with the exception of Greece. The stable location parameter δ is close to zero and positive for all countries. Remember that the first moment does not exist if $\alpha < 1$, which might be responsible for the difference between the first moment (μ) and the stable location parameter (δ).

Not surprisingly, scale parameters increase with time to maturity and sovereign risk, as do the second moments (see Table 4.1). The bond with the lowest standard deviation is the German one-year ($\sigma = 0.41 \cdot 10^{-3}$) and the bond with highest is the Portuguese ten-year ($\sigma = 17.42 \cdot 10^{-3}$). Keeping in mind that the second moment does not exist for

[9] We first standardize the data for the estimation of the Student's t and skewed Student's t distribution.

Table 4.2: Parameter estimates of the alternative distributions

				t	Skewed t		Stable			
	m	$\mu\,10^3$	$\sigma\,10^3$	η	λ	η	$\delta\,10^3$	$\gamma\,10^3$	β	α
GER	1	0.11	0.41	3.22**	0.05**	3.23**	0.10	0.20	0.13*	1.64**
	3	0.14	1.36	5.22**	-0.02	5.22**	0.17	0.84	-0.12	1.80**
	5	0.17	2.37	6.18**	-0.03	6.35**	0.25	1.51	-0.21*	1.84**
	10	0.23	4.17	5.73**	-0.03	5.88**	0.37	2.63	-0.24**	1.82**
NET	1	0.12	0.64	2.60**	0.01	2.60**	0.11	0.25	0.00	1.43**
	3	0.15	1.56	3.45**	-0.01	3.45**	0.17	0.83	-0.05	1.66**
	5	0.18	2.40	4.36**	-0.04*	4.36**	0.24	1.40	-0.13	1.73**
	10	0.22	6.02	3.53**	-0.03	3.53**	0.35	3.21	-0.10	1.66**
FIN	1	0.12	0.59	2.72**	-0.04**	2.72**	0.14	0.24	-0.09*	1.44**
	3	0.15	1.49	4.79**	-0.04	4.79**	0.19	0.90	-0.13	1.76**
	5	0.18	2.73	4.00**	-0.05*	4.01**	0.28	1.55	-0.19*	1.77**
	10	0.23	5.81	3.43**	-0.04*	3.43**	0.41	3.05	-0.16**	1.67**
FRA	1	0.11	0.53	2.77**	0.06**	2.77**	0.09	0.23	0.12**	1.46**
	3	0.15	1.39	4.30**	-0.01	4.30**	0.17	0.81	-0.07	1.75**
	5	0.17	2.40	4.48**	-0.01	4.48**	0.22	1.42	-0.10	1.79**
	10	0.21	3.98	4.88**	0.00	4.88**	0.23	2.41	0.00	1.81**
AUS	1	0.12	0.66	2.40**	-0.10**	2.37**	0.17	0.17	-0.19**	1.02**
	3	0.15	1.34	2.43**	0.00	2.43**	0.15	0.36	0.00	1.02**
	5	0.18	2.35	2.48**	0.01	2.48**	0.16	0.66	0.00	1.01**
	10	0.23	5.91	2.59**	0.01	2.59**	0.18	1.77	0.00	1.01**
BEL	1	0.12	0.68	2.44**	0.02	2.44**	0.11	0.24	0.07	1.42**
	3	0.17	1.67	3.51**	-0.02	3.51**	0.21	0.89	-0.10	1.69**
	5	0.21	2.64	3.88**	-0.03	3.89**	0.28	1.48	-0.13	1.73**
	10	0.27	4.68	4.30**	-0.03	4.34**	0.40	2.72	-0.14	1.75**
SPA	1	0.12	1.04	2.29**	-0.09**	2.28**	0.19	0.27	-0.13**	1.14**
	3	0.14	2.11	2.29**	-0.04**	2.29**	0.19	0.58	-0.10**	1.21**
	5	0.14	3.12	2.42**	-0.03*	2.42**	0.21	1.02	-0.10**	1.29**
	10	0.12	5.60	2.78**	-0.03*	2.78**	0.27	2.37	-0.08*	1.42**
ITA	1	0.13	0.74	2.32**	-0.01	2.32**	0.13	0.22	0.00	1.33**
	3	0.15	2.15	2.53**	-0.03	2.53**	0.19	0.82	-0.09*	1.49**
	5	0.16	3.41	2.71**	-0.02	2.71**	0.23	1.44	-0.11*	1.56**
	10	0.17	5.61	2.84**	-0.04*	2.84**	0.29	2.50	-0.11*	1.57**
IRE	1	0.15	1.65	2.18**	0.01	2.18**	0.15	0.36	0.00	1.16**
	3	0.18	4.18	2.18**	-0.01	2.18**	0.18	0.93	0.00	1.22**
	5	0.18	4.92	2.32**	-0.02	2.32**	0.26	1.49	-0.08*	1.38**
	10	0.11	11.85	2.24**	-0.04**	2.24**	0.38	3.04	-0.10**	1.26**
POR	1	0.16	2.94	2.10**	0.00	2.10**	0.17	0.38	-0.07**	0.85**
	3	0.17	6.34	2.11**	0.02	2.11**	0.14	0.97	0.00	0.99**
	5	0.15	8.07	2.16**	-0.01	2.16**	0.15	1.60	0.00	1.12**
	10	0.10	17.42	2.24**	-0.01	2.24**	0.12	4.11	0.00	1.10**
GRE	1	-0.63	29.72	2.00**	-0.45**	2.00**	0.12	0.25	0.00	0.77**
	3	-0.60	15.79	2.03**	-0.22**	2.03**	0.21	1.07	-0.11**	0.94**
	5	-0.58	17.73	2.04**	-0.17**	2.04**	0.30	1.67	-0.11**	1.00**
	10	-1.34	62.92	2.01**	-0.16**	2.01**	0.37	3.31	-0.10**	0.99**

The table reports the estimated parameters of the Student's t (t), skewed Student's t (Skewed t), and stable (Stable) distribution. The analysis includes European government bond returns maturing in m years. For the estimation of the Student's t and skewed Student's t distribution, the data is standardized first ((X-μ)/σ). $2 < \eta < \infty$ are the degrees of freedom of the Student's t and skewed Student's t distribution. As η tends to infinity the Student's t distribution becomes Gaussian. $-1 < \lambda < 1$ is the skewness parameter of the skewed Student's t distribution. $\delta \in \mathbb{R}$ and $\gamma > 0$ are the location and scale parameters of the stable distribution, respectively. $-1 \leq \beta \leq 1$ and $0 < \alpha \leq 2$ denote its skewness parameter and index of stability. Lower numbers of α indicate heavy tails. As α tends to 2, the distribution becomes Gaussian and β loses its influence. An * (**) implies statistical significance of non-normality ($\eta < 30$, $\lambda \neq 0$, $\alpha < 2$, and $\beta \neq 0$) at a 95% (99%) confidence level.

the stable distribution, scale parameters still imply the same interactions. They range from $\gamma = 0.20 \cdot 10^{-3}$ for the German one-year bond to $\gamma = 4.11 \cdot 10^{-3}$ for the Portuguese ten-year one.

Skewness parameters of the skewed Student's t distribution are close to zero for all countries with the exception of Greece. Austrian one-year bonds ($\lambda = -0.10^{**}$) have the lowest estimate and French one-year bonds ($\lambda = 0.06^{**}$) the highest. However, the departure from normality ($\lambda \neq 0$) is mainly insignificant. Only 17 (11) of 44 skewness parameters are significantly different from normality at a 95% (99%) confidence level. Estimates of the skewness parameter β of the stable distribution imply similar results. Parameters reflect almost symmetric returns, ranging from $\beta = -0.19^{**}$ (Austrian one-year) to $\beta = 0.12^{**}$ (French one-year). Altogether, only 21 (12) of 44 skewness parameters are significantly different from normality at a 95% (99%) confidence level. In sum, skewness appears to play a minor role in European government bond returns.

All alternative distributions have a kurtosis parameter for providing a better fit in the tails. According to expectations, estimates of the parameter η are almost identical for the Student's t and skewed Student's t distribution (see Section 4.3). Parameter estimates range from $\eta = 6.35^{**}$ (German five-year) to $\eta = 2.10^{**}$ (Portuguese five-year), implying considerably heavy tails. For all countries and maturities, the tail parameter is significantly different from normality at the 99% confidence level. The tail parameters of the stable distribution show similar characteristics. Parameter estimates for α indicate heavy tails among all bond returns and vary between $\alpha = 1.84^{**}$ (German five-year) and $\alpha = 0.85^{**}$ (Portuguese one-year). Analogous to the other distributions, tail parameters are significantly different from normality at a 99% confidence level for **all** countries and maturities, leading to the conclusion that a tail parameter is necessary for matching the characteristics of government bond returns. Goodness-of-fit tests will provide insight into the reliability of the parameter estimation.

Table 4.3 reports statistics and p-values of a χ^2 goodness-of-fit test. The test follows a χ^2 distribution with degrees of freedom depending on the number of parameters and with the null hypothesis that "the empirical distribution equals the distributional assumption." Note that the p-value of the French ten-year bond for the Student's t distribution ($p = 0.355$) is higher than the p-value for the skewed Student's t distribution ($p = 0.336$), although the test statistic is identical ($Stat = 29.64$). Following the parsimonious argument, the difference in p-values is due to the various degrees of freedom. The Student's t distribution offers the best fit for the Belgium three-year bond. With a low test statistic of 22.40, the null hypothesis cannot be rejected at a 5% level. The table reports the worst fit for high sovereign risk countries. With a test statistic of 858.42, the null hypothesis has to be rejected for Portuguese one-year bonds. In summary, the Student's t distribution provides a poor fit for European government bond returns. For 24 (17) of a total of 44 bonds, the null hypothesis cannot be rejected at the 1% (5%) significance level.

The skewed Student's t distribution offers similar results. It fits the Belgium three-year bond best and the Portuguese one-year bond worst. Overall, for 22 (17) bonds, the null hypothesis cannot be rejected at the 1% (5%) significance level. This is slightly worse than the fit of the Student's t distribution and somewhat surprising at first sight. Since the skewed Student's t distribution has an additional parameter, one would expect it to provide a better fit. There are two reasons for this underperformance. First, skewness plays only a minor role in European government bonds and the additional parameter does not result in a better fit of the distribution. Second, the poorer p-values might be due to simulated test statistics.

The results of the stable distribution are considerably different. This distribution fits the Italian three-year bond best ($Stat = 23.69$); however, even for the Austria ten-year bond ($Stat = 394.94$), the stable distribution cannot be rejected at the 1% level. In sum, the assumption of stable distributed returns cannot be rejected for any (37) bond(s) at

Table 4.3: χ^2 goodness-of-fit test

	m	t		Skewed t		Stable	
		Stat	p	Stat	p	Stat	p
GER	1	33.11	0.321	26.31	0.554	35.68	0.228
	3	33.51	0.204	32.52	0.198	58.80	0.075
	5	45.44	0.061	41.99	0.076	67.28	0.067
	10	36.79	0.153	32.54	0.235	57.33	0.088
NET	1	47.78	0.147	47.04	0.132	35.31	0.250
	3	33.93	0.295	32.48	0.272	52.13	0.109
	5	24.99	0.614	22.55	0.684	51.32	0.094
	10	35.57	0.191	31.62	0.257	59.95	0.096
FIN	1	89.72	0.002	78.94	0.004	65.43	0.077
	3	51.19	0.035	47.10	0.031	80.13	0.053
	5	72.74	0.004	67.47	0.006	56.03	0.065
	10	42.47	0.101	42.93	0.066	50.08	0.110
FRA	1	71.27	0.011	62.35	0.021	62.47	0.087
	3	23.75	0.701	24.02	0.610	54.42	0.096
	5	31.04	0.326	30.33	0.299	27.01	0.387
	10	29.64	0.355	29.64	0.336	31.71	0.297
AUS	1	221.87	0.001	243.09	0.000	209.07	0.033
	3	385.67	0.000	386.60	0.000	274.61	0.013
	5	444.19	0.000	438.74	0.000	283.37	0.016
	10	422.50	0.000	418.84	0.000	394.94	0.013
BEL	1	68.28	0.044	67.95	0.029	33.11	0.317
	3	22.40	0.760	23.25	0.668	29.34	0.361
	5	31.46	0.349	32.01	0.286	39.35	0.193
	10	34.28	0.210	33.62	0.235	49.19	0.115
SPA	1	242.41	0.002	242.98	0.002	95.35	0.049
	3	157.98	0.007	162.80	0.006	57.98	0.092
	5	109.88	0.007	120.26	0.009	79.56	0.064
	10	129.59	0.001	115.66	0.000	114.19	0.028
ITA	1	108.25	0.020	108.37	0.011	35.58	0.272
	3	48.13	0.156	48.26	0.118	23.69	0.542
	5	51.35	0.066	49.18	0.079	29.64	0.363
	10	61.64	0.014	64.12	0.018	37.25	0.228
IRE	1	241.65	0.011	241.82	0.005	57.06	0.108
	3	178.33	0.006	176.81	0.005	40.27	0.235
	5	115.46	0.014	113.37	0.005	42.71	0.207
	10	206.47	0.004	208.11	0.003	51.12	0.131
POR	1	858.42	0.001	858.40	0.000	147.55	0.030
	3	423.80	0.002	420.14	0.005	70.70	0.096
	5	204.65	0.007	205.04	0.008	42.78	0.201
	10	285.68	0.004	287.42	0.002	74.48	0.090
GRE	1	7,370.08	0.000	38,248.24	0.000	39.30	0.251
	3	1,360.60	0.001	1,571.63	0.000	54.06	0.138
	5	765.19	0.006	913.97	0.002	44.10	0.172
	10	1,772.95	0.001	1,890.03	0.000	31.23	0.265

The table shows the test statistics (Stat) and p-values (p) of a χ^2 goodness-of-fit test of European government bond returns maturing in m years. We test the null hypothesis that "the empirical distribution equals the distributional assumption." The test follows a χ^2 distribution with $p - k - 1$ degrees of freedom, where $p = 30$ is the number of intervals and k is the number of parameters estimated for each distribution. Degrees of freedom are 26 for the Student's t (t), 25 the for skewed Student's t (Skewed t) and 25 for the stable (Stable) distribution. For the sake of consistency, all p-values are calculated with simulation techniques for all distributions based on 1,000 repetitions.

the 1% (5%) level.

Table 4.3 reveals the following conclusions. (1) High exposure to sovereign risk yields a worse fit of the Student's t and skewed Student's t distribution; on the other hand, however, the stable distribution's fit does not seem to depend on sovereign risk. (2) The skewed Student's t provides an even worse fit than the Student's t distribution, meaning that skewness plays a minor role in European government bond returns. (3) The stable distribution outperforms both alternative distributions. At the 1% significance level, it cannot be rejected for any bond.

Figure 4.2 illustrates parameter estimation results and goodness-of-fit tests. Following the design of Figure 4.1, Figure 4.2 presents plots of the goodness of fit of the German, Spanish, and Greek ten-year bonds. Since the stable distribution offers the best fit of the alternative distributions, the figure shows its probability density function and difference in frequency.

In comparison to Figure 4.1, we see a slight improvement in the fitting of the German bond's empirical distribution. The stable distribution captures the peaked returns around the mean better and no longer overestimates the shoulders of the distribution. The plot shows a similarly clear improvement for Spain. A closer look reveals that the stable distribution can capture the departure from normality in the tails. Even for the special case of Greece, the alternative distribution offers a reasonable fit. Considering the difference between the empirical and normal distribution of Greek returns, the improvement is remarkable. In particular, the stable distribution does a good job of describing the peaked returns around the mean. Figure 4.2 supports the conclusion drawn from Table 4.3: the stable distribution clearly improves the fit for European bond returns.

4.5 Euro crisis

From an investor's perspective, the Euro crisis reveals the existence of sovereign risk in Euro bonds. Due to a change of market circumstances, these bond returns begin to

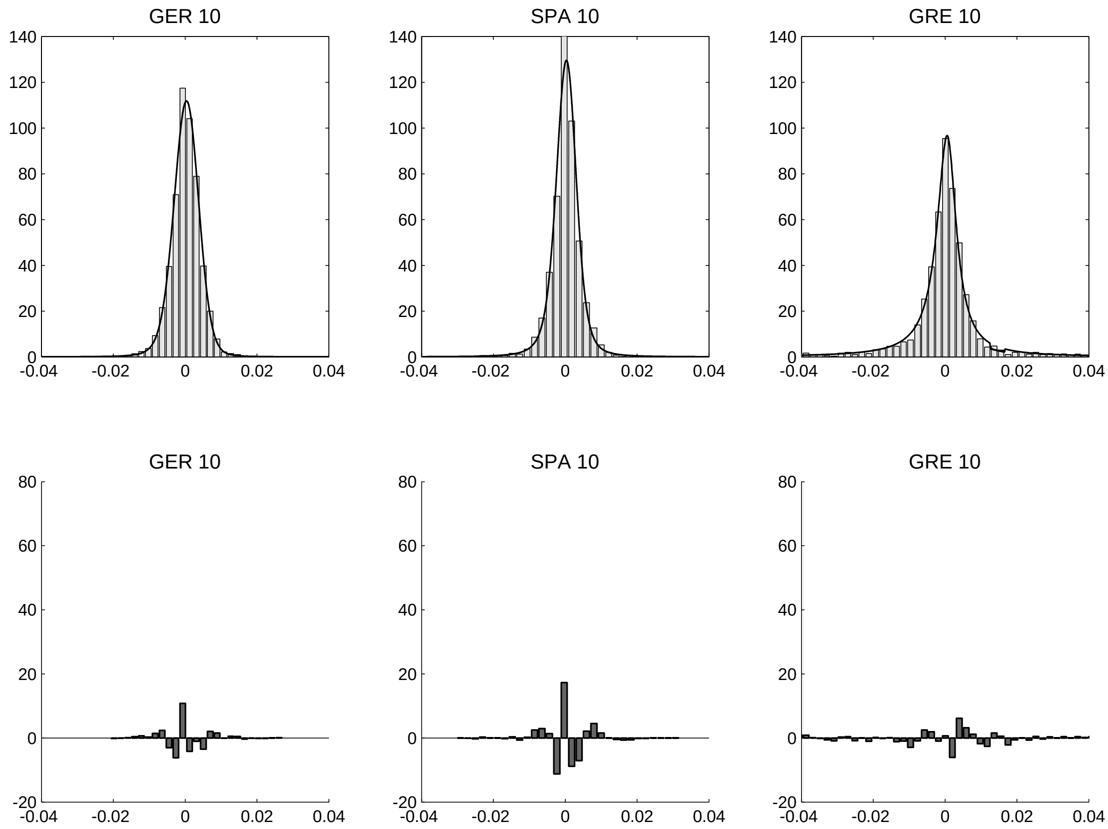

Figure 4.2: Stable distribution and difference in frequency

The figure illustrates the empirical and theoretical distribution of ten year bond returns of Germany, Spain, and Greece in the period 1999 to 2012: 1. Histograms of daily bond returns and fitted probability density functions of the stable distribution. 2. Difference in frequency between the empirical and stable distribution.

behave differently, i.e., the spreads of sovereign risky government bonds increase and yields diverge significantly, leading to the conclusion that structural breaks occur in the time series. The issue to address is whether the significant parameters of skewness and kurtosis are in fact caused by a structural break in the time series. It might well be that (the weak evidence of) skewness is due to a shift in the mean, and that excess kurtosis is the consequence of an increase of standard deviation.

Table 4.4: QLR test results and corresponding headlines

	m	Date	News
GER NET FIN FRA AUS	1	17/05/2007	Risk and reward: Worried about credit risk? (Flight to quality draws yields to historic low level.)
GER NET FIN FRA	3	02/06/2008	Just bury it: It is time to accept that the Lisbon treaty is dead. The European Union can get along well enough without it. (Lehman bankruptcy rumors)
AUS BEL ITA	3,5 all all	10/11/2011	That's all, folks: For the Euro to survive, Italy must not fail. That will require leadership and courage. (Berlusconi resigned.)
SPA	all	09/07/2012	The flight from Spain: Spain can be shored up for a while; but its woes contain an alarming lesson for the entire Euro zone.
IRE	all	01/07/2011	Can Europe's recovery last? Only if its governments take advantage of sunnier times to make deeper reforms. (Rescue aid for Bank of Ireland approved.)
POR	all	16/01/2012	A false dawn: The recession has been mild so far. But things are likely to get much worse. (S&P downgrade of Portugal)
GRE	1,5	13/10/2011	Nowhere to hide: Investors have had a dreadful time in the recent past. The immediate future looks pretty rotten, too. (Rumors about Greece' debt cut)
	3	04/08/2011	Central bankers to the rescue? They can buy a little time, but the real remedy must come from Western politicians. (Once again the ECB bought government bonds on the secondary market.)
	10	01/12/2011	Is this really the end? Unless Germany and the ECB move quickly, the single currency's collapse is looming. (Greek credit tranche of eight billion Euro)

The Table reports the date resulting from the QLR test. "all" indicates maturities of one, three, five, and ten years. The news reported are title page headlines of "The Economist". Notes of the authors are given in parenthesis.

Since the existence and, if found, exact date of the structural breaks are unknown, a Quandt likelihood ratio (QLR) statistic is calculated for each time series. For this aim, we exclude the first and last three months of the data. For each remaining data point, we perform a Chow (1960) test with null hypothesis being "no structural break in the time series." The maximum of all Chow test statistics corresponds to the QLR test statistic. If the maximum is greater than a critical value at a significance level of 1%, we reject the null hypothesis and report the corresponding date in Table 4.4.[10] Bonds with the same date are grouped in the table. In addition, we report news headlines that might help explain the considerable rise or fall of yields in the right column of the table. In sum, 31 of 44 time series show evidence of a structural break in the data.

For instance, the table reveals a straightforward link between yield movements and a news headline for Portugal on 14/01/2012. This structural break (Table 4.4; 16/01/2012) is due to a considerable increase in yields in the subsequent period. The increased yields, in turn, might well be caused by the S&P downgrade of Portugal that dominated the newspaper headlines that week.

Subsequently, we reestimate the parameters for the period after the structural break (if one is found). Those periods correspond to the financial crisis in the Eurozone. Like Table 4.2, Table 4.5 reports the maximum likelihood parameter estimates of the Student's t, skewed Student's t, and stable distribution. An * (**) indicates that the parameter is significantly different from normality at a 95% (99%) confidence level.

Depending on the exposure to sovereign risk, Table 4.5 documents a considerable positive shift in location parameters. The mean parameter of Portuguese five-year bonds varies from $\mu = 0.15 \cdot 10^{-3}$ (whole time series) to $\mu = 2.07 \cdot 10^{-3}$ (after structural break). Greece is the only country that experiences a drastic negative shift in the location parameter. Not surprisingly, the standard deviation increases considerably during the period of the Euro crisis and depends on the exposure to sovereign risk. Table 4.5 reports

[10]In this case, we refrain from giving critical values and test statistics.

Table 4.5: Parameter estimates of the alternative distributions after structural break

				t	Skewed t		Stable			
	m	$\mu\,10^3$	$\sigma\,10^3$	η	λ	η	$\delta\,10^3$	$\gamma\,10^3$	β	α
GER	1	0.09	0.48	2.73**	0.11**	2.73**	0.05	0.20	0.23**	1.48**
	3	0.17	1.42	4.68**	0.01	4.69**	0.15	0.85	0.08	1.76**
NET	1	0.10	0.69	2.60**	0.07**	2.60**	0.06	0.27	0.16*	1.44**
	3	0.17	2.04	2.98**	0.01	2.98**	0.14	0.95	0.07	1.54**
FIN	1	0.10	0.72	2.93**	-0.03	2.92**	0.12	0.33	-0.08	1.48**
	3	0.17	1.67	3.83**	0.01	3.83**	0.16	0.93	0.09	1.67**
FRA	1	0.10	0.51	2.86**	0.08**	2.86**	0.06	0.23	0.17*	1.49**
	3	0.18	1.58	3.48**	0.02	3.49**	0.14	0.83	0.10	1.65**
AUS	1	0.10	0.80	2.40**	-0.02	2.40**	0.11	0.23	-0.09*	1.09**
	3	0.17	1.69	2.49**	0.04	2.49**	0.13	0.60	0.11	1.37**
	5	0.32	3.08	2.67**	0.07	2.66**	0.20	1.22	0.20	1.39**
BEL	1	0.11	0.86	2.18**	0.13**	2.18**	0.05	0.19	0.19	1.20**
	3	0.39	2.79	2.32**	0.02	2.33**	0.30	0.81	0.09	1.25**
	5	0.58	4.32	2.53**	0.04	2.53**	0.47	1.57	0.09	1.34**
	10	0.87	7.48	2.70**	-0.02	2.69**	0.88	3.00	0.00	1.37**
ITA	1	0.29	1.83	3.16**	0.06	3.16**	0.22	0.88	0.00	1.48**
	3	0.59	5.40	3.10**	-0.00	3.10**	0.60	2.56	-0.11	1.50**
	5	0.77	8.28	3.27**	0.02	3.28**	0.67	4.13	-0.09	1.56**
	10	1.23	12.46	3.79**	0.02	3.81**	0.77	6.86	0.08	1.65**
IRE	1	0.57	3.41	2.30**	0.11**	2.28**	0.32	0.90	0.19*	1.19**
	3	1.30	9.66	2.25**	0.15**	2.24**	0.44	2.36	0.22*	1.21**
	5	1.52	9.92	2.27**	0.14**	2.25**	0.61	2.30	0.23**	1.07**
	10	1.09	30.00	2.87**	0.02	2.87**	0.95	13.21	0.13	1.45**
POR	1	0.76	5.05	2.54**	0.08	2.52**	0.56	1.82	0.27*	1.35**
	3	1.68	15.03	2.62**	0.11	2.60**	0.59	5.74	0.32*	1.38**
	5	2.07	18.98	3.35**	0.05	3.33**	1.68	9.72	0.12	1.58**
	10	1.36	39.77	3.75**	0.04	3.77**	-0.41	21.64	0.15	1.61**
GRE	1	-6.56	95.60	2.03**	-0.28**	2.02**	-0.26	2.90	-0.09	0.63**
	3	-5.34	45.55	2.36**	-0.13**	2.36**	-1.45	13.70	-0.16	1.29**
	5	-5.31	52.84	2.54**	-0.14**	2.56**	0.48	19.11	-0.15	1.38**
	10	-8.89	193.85	2.11**	-0.15**	2.10**	5.03	26.76	-0.14	0.93**

The table reports the estimated parameters of the Student's t (t), skewed Student's t (Skewed t), and stable (Stable) distribution for the period after the structural break. The analysis includes countries and maturities (m) where the QLR test (see Table 4.4) indicates a structural break, namely Germany, Netherlands, Finland, and France (one and three years), Austria (one, three, and five year), and Belgium, Italy, Ireland, Portugal, and Greece (all maturities). For the estimation of the Student's t and skewed Student's t distribution, the data is standardized first ($(X\text{-}\mu)/\sigma$). $2 < \eta < \infty$ are the degrees of freedom of the Student's t and skewed Student's t distribution. As η tends to infinity the Student's t distribution becomes Gaussian. $-1 < \lambda < 1$ is the skewness parameter of the skewed Student's t distribution. $\delta \in \mathbb{R}$ and $\gamma > 0$ are the location and scale parameters of the stable distribution, respectively. $-1 \leq \beta \leq 1$ and $0 < \alpha \leq 2$ denote its skewness parameter and index of stability. Lower numbers of α indicate heavy tails. As α tends to 2, the distribution becomes Gaussian and β loses its influence. An * (**) implies statistical significance of non-normality ($\eta < 30$, $\lambda \neq 0$, $\alpha < 2$, and $\beta \neq 0$) at a 95% (99%) confidence level.

a slight increase of $\sigma = 0.41 \cdot 10^{-3}$ to $\sigma = 0.48 \cdot 10^{-3}$ for German one-year bonds and a clear increase of $\sigma = 17.42 \cdot 10^{-3}$ to $\sigma = 39.77 \cdot 10^{-3}$ for Portuguese ten-year bonds. The same results hold for scale parameters of the stable distribution.

By studying the skewness parameters λ and β, we can discover if a shift of the location parameter in the time series causes the skewness in daily returns. For the slight negative shift of Finnish one-year bonds (from $\mu = 0.12 \cdot 10^{-3}$ to $\mu = 0.10 \cdot 10^{-3}$), the former significant negative skewness parameter ($\lambda = -0.04^{**}$) does indeed become insignificant ($\lambda = -0.03$). In contrast, the skewness parameters of Irish one-, three-, and five-year bonds become significant after the structural break. Altogether, we find weak evidence for skewness in the bonds, which add support to the findings of Section 4.4.

Are the significant excess kurtosis parameters of the whole sample period due to an increase of standard deviation during the Euro crisis period, similar to the findings of the skewness parameters? We find overwhelmingly clear evidence that the answer to this question is no. The kurtosis parameters (λ and α) are still significantly different from normality at a 99% confidence level for **all** time series and **all** alternative distributions. In sum, we find strong evidence for heavy tails even after correcting for structural breaks in the data.

In Table 4.6 we report the test statistics and p-values of χ^2 goodness-of-fit tests with null hypothesis being that "the empirical distribution equals the distributional assumption." Since the parameters are estimated for the period after each structural break, the lengths of the time series differ. Hence, the p-values gain credibility because they are invariant to time series length.

Compared to the whole sample period, the goodness-of-fit statistics for the Euro crisis are even more pronounced. The stable distribution, for instance, offers the best fit for Belgian three-year bonds ($p = 0.820$). In comparison to the best fit of the whole series ($p = 0.542$, Italian three-year bonds), this is a clear improvement. The worst fit after the structural break is for Austrian one-year bonds ($p = 0.062$). However, the null

Table 4.6: χ^2 goodness-of-fit test after structural break

		t		Skewed t		Stable	
	m	Stat	p	Stat	p	Stat	p
GER	1	52.05	0.070	33.54	0.314	23.61	0.542
	3	28.20	0.406	28.32	0.372	36.68	0.208
NET	1	58.86	0.046	47.23	0.079	25.76	0.463
	3	33.73	0.257	33.80	0.249	31.93	0.289
FIN	1	79.09	0.006	77.35	0.006	66.95	0.063
	3	58.59	0.009	58.64	0.000	67.33	0.061
FRA	1	61.90	0.021	55.85	0.016	47.42	0.134
	3	23.78	0.674	23.03	0.684	26.60	0.439
AUS	1	127.47	0.005	133.92	0.002	88.61	0.062
	3	48.03	0.131	49.08	0.055	30.07	0.340
	5	43.93	0.131	48.80	0.037	29.50	0.316
BEL	1	73.36	0.058	74.97	0.031	21.69	0.557
	3	41.03	0.268	40.18	0.222	13.63	0.820
	5	42.56	0.196	42.11	0.128	23.24	0.502
	10	34.89	0.317	34.66	0.248	25.69	0.405
ITA	1	47.39	0.047	46.10	0.033	39.84	0.163
	3	35.82	0.192	35.85	0.153	32.58	0.263
	5	42.52	0.084	41.93	0.051	42.09	0.121
	10	32.66	0.248	31.75	0.194	33.47	0.205
IRE	1	64.99	0.072	56.27	0.078	23.09	0.499
	3	64.52	0.082	57.68	0.077	16.16	0.749
	5	129.67	0.009	130.29	0.002	46.94	0.140
	10	47.14	0.073	47.56	0.042	41.05	0.157
POR	1	36.88	0.295	39.28	0.164	17.62	0.693
	3	35.49	0.273	37.55	0.147	20.92	0.609
	5	27.62	0.491	28.73	0.381	27.14	0.328
	10	17.66	0.925	17.39	0.885	18.69	0.692
GRE	1	511.87	0.001	1,303.68	0.000	36.62	0.225
	3	90.00	0.018	66.94	0.020	30.34	0.313
	5	38.65	0.258	26.78	0.582	19.40	0.684
	10	154.40	0.013	172.60	0.004	27.98	0.383

The table shows the test statistics (Stat) and p-values (p) of a χ^2 goodness-of-fit test for countries and maturities (m) where the QLR test (see Table 4.4) indicates a structural break, namely Germany, Netherlands, Finland, and France (one and three year), Austria (one, three, and five year), and Belgium, Italy, Ireland, Portugal, and Greece (all maturities). We test the null hypothesis that "the empirical distribution equals the distributional assumption." The test follows a χ^2 distribution with $p - k - 1$ degrees of freedom, where $p = 30$ is the number of intervals and k is the number of parameters estimated for each distribution. Degrees of freedom are 26 for the Student's t (t), 25 the for skewed Student's t (Skewed t) and 25 for the stable (Stable) distribution. For the sake of consistency, all p-values are calculated with simulation techniques for all distributions based on 1,000 repetitions.

hypothesis can no longer be rejected at the 5% level. In contrast, the worst fit of the whole series ($p = 0.013$, Austrian ten-year bonds) is rejected at the 5% level.

Overall, the fit of the alternative distributions clearly improves after correcting for the structural break. The Student's t distribution cannot be rejected at the 1% (5%) significance level for 26 (21) of a total of 31 bonds. We obtain similar results for the skewed Student's t distribution; it cannot be rejected at the 1% (5%) significance level for 25 (19) bonds. The stable distribution cannot be rejected at the 5% significance level for **any** bond. We further conclude that the goodness of fit diminishes with exposure to sovereign risk. After correcting the time series for structural breaks, the mean shifts and some skewness parameters become insignificant. The overall evidence for skewness is weak. The empirical distribution's deviation mainly occurs in the tails. Therefore, excess kurtosis is highly relevant for an alternative distribution: all proposed alternative distributions exhibit this feature. However, the stable distribution clearly offers the best fit.

4.6 Risk management implications

The previous sections underline the importance of considering higher-order moments when describing European government bond returns. We now analyze the consequence for downside risk when assuming different distributions. Since the VaR is the most widely used tool in risk management (see Ammann and Reich, 2001), we apply VaR calculations when investigating whether the alternative distributions are able to adequately capture the bond risk. Note that the 99% confidence level is crucial for VaR calculations (see e.g. Berkowitz and O'Brien, 2002).

In light of their importance for fixed income management, bonds maturing after ten years are predominantly discussed in finance literature (see Codogno et al., 2003; Bernoth et al., 2004; Gomez-Puig, 2009). As we did in Figures 4.1 and 4.2, we again examine a country with low (Germany), considerable (Spain), and high (Greece) exposure to sovereign risk. We use a rolling window approach for all three time series to account for the shift in parameters (see Section 4.5). We take the first 100 returns, fit the distribution, and calculate the 0.01 quantile (that is, the VaR at a 99% confidence level for the 101^{st} day). If the return on the 101^{st} day does (not) exceed the VaR, we assign a one (zero) to the date. We continue rolling this window until the end of the time series. Thus, a hitting sequence is generated consisting of ones and zeros.

Three standard techniques dominate analyses of hitting sequences (see Christoffersen, 2012). First, an unconditional coverage test determines if the expected fraction of VaR violations π_{exp} differs significantly from the realized fraction π_{real}. In the limit, the resulting test-statistic

$$LR_{uc} = -2\ln[(1-\pi_{exp})^{T_0}\pi_{exp}^{T_1}/((1-T_1/T)^{T_0}(T_1/T)^{T_1})] \tag{4.5}$$

follows a χ^2 distribution with one degree of freedom. T_1 (T_0) is the number of times the VaR is (not) violated and $T = T_1 + T_0$ is the total number of VaR observations.

The unconditional converge test provides no information on whether VaR violations are clustered. Therefore, we formulate the independence test statistic

$$LR_{ind} = -2\ln[L(\hat{\pi})/((1-\hat{\pi}_{01})^{T_{00}}\hat{\pi}_{01}^{T_{01}})]. \tag{4.6}$$

$L(\hat{\pi})$ is the likelihood under the alternative hypothesis of Equation (4.5), T_{00} (T_{01}) is the number of observations where no (a) VaR violation follows no violation, and $\hat{\pi}_{01} = T_{01}/(T_{00} + T_{01})$. As for the unconditional coverage test, the test statistic is χ^2 distributed with one degree of freedom.

The conditional coverage test combines the insights of the unconditional and independence test in one test statistic:

$$LR_{cc} = LR_{uc} + LR_{ind}. \tag{4.7}$$

The test statistic is χ^2 distributed with two degrees of freedom.

Table 4.7: VaR calculation

	Distribution	π_{real}	π_{exp}	p_{unc}	p_{ind}	p_{cc}
GER	t	0.0118	0.01	0.2935	0.3273	0.3564
	Skewed t	0.0118	0.01	0.2935	0.4999	0.4587
	Stable	0.0121	0.01	0.2250	0.3153	0.2893
SPA	t	0.0077	0.01	0.1624	0.1967	0.1638
	Skewed t	0.0062	0.01	0.0177	0.6081	0.0528
	Stable	0.0101	0.01	0.9655	0.3555	0.6519
GRE	t	0.0135	0.01	0.0753	0.5538	0.1725
	Skewed t	0.0121	0.01	0.2756	0.0733	0.1110
	Stable	0.0135	0.01	0.0753	0.3022	0.1207

The table reports a comparison of VaR calculations at the 99% confidence level for a maturity of ten years for German, Spanish, and Greek government bond returns assuming the Student's t (t), skewed Student's t (Skewed t), and stable (Stable) distribution. π_{real} (π_{exp}) gives the realized (expected) ratio of VaR violations, p_{unc}, p_{ind}, and p_{cc} represent the p-values of the unconditional, independence, and conditional coverage test (see Christoffersen, 2012) with null hypothesis being that "the VaR model is correct."

Table 4.7 sets out the VaR calculations at the 99% confidence level for ten-year bonds of Germany, Spain, and Greece. π_{real} (π_{exp}) is the realized (expected) ratio of VaR violations, p_{unc}, p_{ind}, and p_{cc} represent the p-values of the unconditional, independence, and conditional coverage test with the null hypothesis being that "the VaR model is correct." At the 5% significance level, we reject the skewed Student's t model only for the Spanish bond in case of the unconditional test. This confirms our finding that skewness is not an important issue in European government bond returns. The strong result of the stable distribution, which almost realizes the expected violation rate ($\pi_{real} = 0.0101$ and $\pi_{exp} = 0.01$), is remarkable. The Student's t and stable distribution cannot be rejected at the 5% significance level for any country and therefore both provide a reliable framework for the VaR calculations. Since the stable distribution offers the best fit and provides the greatest p-values in VaR calculations, we propose using the stable distribution for risk management purposes.

4.7 Conclusion

The assumption that financial returns follow a Gaussian distribution, implicitly or explicitly, is frequently made in the finance literature. However, this assumption of normality has important consequences for portfolio theory, derivative pricing, and other financial applications. Whereas a large body of literature is concerned with the empirical distribution of equity returns, little is known about the distribution of bond returns. This is surprising, as international bond markets exceed international equity markets in terms of capitalization.

In the present chapter, we remedy this situation by studying the distribution of daily European government bond returns in the period 1999–2012. We find that the Lilliefors and Jarque-Bera tests overwhelmingly reject the Gaussian distribution for all countries. The empirical distribution departs from the normal distribution mainly in the tails due to the excess kurtosis of bond returns. Therefore, the kurtosis parameters of the Student's t, skewed Student's t, and stable distribution are highly significant, whereas we find only weak evidence for the significance of skewness parameters.

The goodness-of-fit tests show that sovereign risk is a crucial factor in bond returns. Hence, the importance of flexibility in the tails increases with exposure to sovereign risk. The stable distribution clearly offers the best fit of the tested alternatives. We find a shift in location parameters and a drastic increase of scale parameters caused by the Euro crisis. Due to this shift, some skewness parameters become insignificant and the overall evidence for their influence remains weak. However, even after correcting the time series for structural breaks, excess kurtosis parameters are highly significant. Indeed, the goodness of fit of the stable distribution becomes even better.

Taking excess kurtosis of bond price variations into account has immediate consequences for risk management. We show in a VaR application that risk management clearly improves when assuming the stable distribution for EMU bonds.

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Erklärung gemäß §7 Abs. 4 Nr. 2. der Promotionsordnung

Hiermit erkläre ich, dass es sich bei der eingereichten Dissertation um meine selbstständig verfasste Leistung handelt. Ich habe nur die angegebenen Quellen und Hilfsmittel benutzt und mich keiner unzulässigen Hilfe Dritter bedient. Insbesondere habe ich wörtlich oder sinngemäß aus den Schriften anderer Autoren entnommene Stellen als solche kenntlich gemacht.

Düsseldorf, den 14.05.2015

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