# Method for Determining the Number of States of the Markov Model of Damage Accumulation in Predicting the Technical Condition of a Fiber-Optic Cable

Elena Ionikova[1], Kirill Karpov[1,2] and Viacheslav Shuvalov[1]
[1] *Department of Transmission of Discrete Data and Metrology, Siberian State University of Telecommunications and Information Sciences, Kirova Str. 86, 630102 Novosibirsk, Russian Federation*
[2] *Future Internet Lab Anhalt, Anhalt University of Applied Sciences, Bernburger Str. 57, 06366 Köthen, Germany*
*ionikova.lena@mail.ru, kirill.karpov@hs-anhalt.de*

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Abstract: Estimation of the residual service life of operating fiber-optic cables is an urgent t ask. Usually this problem is solved based on the use of the Markov chain model. However, due to the nonlinear dependence of the probability of rejection on the rate of gradual failures, the task of selecting the number of states of a Markov chain becomes difficult. The article discusses a technique for determining the required number of states of the Markov model of damage accumulation based on a given value of the modeling accuracy. The characteristic values of the time and the probability of failure are found for the model of the destruction of optical fibers made of silica glass. The determination of the required number of states of a Markov chain in the article is carried out using the Python programming language.

## 1 INTRODUCTION

Since 1993, more than 2.2 billion km of optical fiber has been laid in the world, which is used to transmit more than 20% of global information traffic. With the proliferation of cloud technologies, distributed computing and databases, the role of fiber-optic communication systems is growing steadily [1].

The service life of fiber-optic communication lines is about a quarter of a century [2, 3] (not to be confused with the warranty service life, which, according to most manufacturers, does not exceed two years). Depending on the design of the cable and its field of application, the value of the service life can vary from 2 to 40 years. Often the requirements for tendering indicate that the service life of an optical cable must be at least 25 years.

Lifetime of the cable is defined [4, 5] as the average service life - the mathematical expectation of the service life. Gamma percent service life is the calendar duration of operation during which it will not reach the limit state with a given probability γ, expressed as a percentage.

A. Yu. Tsym in his work [6] proposed to supplement the list of lifetime criteria with an indicator of disproportionate risk of loss of network connectivity.

This indicator is relevant for the Russian information infrastructure due to the limited possibilities of network redundancy and the need for an additional assessment of the fact of loss of network connectivity when the optical cable goes to the limit state. The criteria for the limiting state is a set of features established in the technical documentation [7].

During its service life, a fiber-optic cable belongs to the class of recoverable objects, after passing to the limit state, it is a non-recoverable object. The transition to the limiting state occurs gradually as the static fatigue of the fiber accumulates (aging or deterioration).

In addition to damage to the cable sheath, the aging of optical fibers is influenced by such internal factors as fiber stretching, moisture, and hydrogen [8, 9]. The lifetime of optical cables is mainly determined by the amount of tension on the fibers. Since under tension, optical fibers gradually decrease their strength due to the growth of cracks on their surface [10, 11], the number of failures caused by cable breaks increases. For example, the ITU-T recommendation [11] provides test data for cable sections that have been buried in the ground since 1979, 1986 and 1991. The probability of failure values for the 1979 cable are significantly higher than those for the 1991 cable. In the work of I. V. Bogachkov and N.I. Gorlov [12] it is shown that the established service life of 25 years is ensured in the presence of an elongation of less than 0.26%, which determines the permissible value of the local mechanical tensile load within 3 N.

## 2 RELATED WORK

Gradual failure models are designed to analyze changes in the physical parameters of technical systems under stress. The parameter $Y$, called *defining*, changes during wear, reaching a limit value, after which the system becomes inoperative. The mean time to failure of the system is determined by the formula (1).

$$T = \frac{Y_{lim} - Y_0}{\alpha} \quad (1)$$

where $Y_{lim}$ — limiting value of the defining parameter; $Y_0$ — its initial value; $\alpha$ — the rate of change of the defining parameter $\frac{dY}{dt}$.

Many works have been devoted to assessing the lifetime of various objects, including fiber-optic lines, in which various models of the transition of an object to the limiting state are described.

It was shown in [13] that the processes of damage accumulation (regardless of their nature) can be described by Markov models, on the basis of which it is possible to construct fairly accurate models of cumulative damage accumulation. In [14], this model was used to describe the accumulation of damage in polymer high-voltage insulation. In [15], this probabilistic approach was used to model the life cycle of road bridge elements based on the Markov stochastic degradation model. The paper presents a graph of the degradation process for a model of five discrete states and a method for determining the degradation parameter, which is considered as the failure rate $\lambda$.

In [16], the Markov model with discrete states and continuous time is used to predict the parametric reliability of the Monitoring System. Application of this model makes it possible to determine several operational states of the Monitoring System with different levels of operational efficiency, determined by the probability of no-failure operation.

In [17], a Markov branching process was used to build a model for predicting changes in the parameters of an electronic system during operation. The model is recommended to be used to predict the parametric reliability and technical condition of radio-electronic systems depending on the time of operation.

In [18], Markov models were developed for predicting the parameters of computer networks, taking into account the nonstationarity of the operation modes. The calculation of the parameters is carried out on the basis of the results of the wavelet - analysis of the dynamics of changes in operating parameters.

In [19], using the theory of semi-Markov processes, models of operation of communication systems equipment are considered, taking into account the physical aging of the elements included in it.

A similar approach to assessing the time to reach the limit state can be used for fiber-optic communication lines.

Griffiths is considered the founder of the mechanical concept of optical fiber destruction [20, 21]. According to Griffiths, a solid contains microcracks, which begin to expand under the action of tensile stress. Crack growth occurs when the tensile force reaches a certain threshold value. When this value is reached, the crack begins to grow at a limiting rate.

Today, optical fiber fracture models are actively used, built on the basis of the empirical concept of the power-law dependence of the rate of development of microcracks $V$ on the tensile stress intensity factor $K_e$, which characterizes the overstress at the crack tip.

$$V = A \cdot K_e^n \quad (2)$$

where $n$ is the parameter of resistance to fatigue (corrosion coefficient); $A$ is a constant depending on the parameters of the material and the environment.

In [22], it is noted that the use of a simple power law to describe the statistical fatigue of an optical fiber leads to the neglect of the possible existence of regions where crack growth follows other mechanisms and patterns (regions with a limited rate of moisture diffusion to the crack tip, as well as regions of thermal fluctuation crack growth in the absence of moisture).

A similar approach to estimating the time to reach the limit state can be used for fiber-optic communication lines.

This article discusses a technique for determining the required number of states of a Markov damage accumulation model based on a given value of modeling accuracy. The characteristic values of the time and the probability of failure are found for the model of the destruction of optical fibers made of quartz glass according to the Weibull parameters determined in the article [23].

## 3 GRADUAL FAILURE MODELS

The theoretical aspects of mechanical reliability are described in sufficient detail in the document [24] and articles [25, 26], where the following formula for calculating the time to failure for static fatigue is presented:

$$t = \frac{2}{A \cdot Y^2(n-2) \cdot K_{IC}^{n-2}} \cdot \frac{\sigma_c^{n-2}}{\sigma_{exp}^n} \tag{3}$$

Where $\sigma_c$ — fiber strength in an inert environment; $\sigma_{exp}$ — applied tension; $A$ — a constant depending on the material and the environment; $Y$ — coefficient depending on the geometry of the crack; $n$ — fatigue parameter; $K_{IC}$ — the stress intensity factor corresponding to the inert environment.

For a statistical assessment of the mechanical strength of an optical fiber, the most suitable type of distribution is the Weibull law, written in the form [27]:

$$P(\sigma, L) = 1 - \exp\left(-\frac{L}{L_0}\left(\frac{\sigma}{\sigma_0}\right)^n\right) \tag{4}$$

where $L$ is the length of the optical fiber; $L_0$ — the length of the optical fiber sample during testing; $\sigma$ — tensile strength of the fiber; $\sigma_0, m$ — the parameters of the Weibull distribution are determined experimentally.

At present, usually, a two-stage optical fiber destruction model is used: the first mode is valid for the probabilities of optical fiber destruction $P_{crit} \leq P(\sigma, L) \leq 1$, the second is for probabilities $0 \leq P(\sigma, L) \leq P_{crit}$. $P_{crit}$ corresponds to the probability of destruction at the boundary of two modes.

$$P(\sigma, L) = \begin{cases} 1 - \exp\left(\frac{L\sigma^{m_1}}{L_0\sigma_{01}^{m_1}}\right) & if\ P_{crit} \leq P(\sigma, L) \\ 1 - \exp\left(\frac{L\sigma^{m_2}}{L_0\sigma_{02}^{m_2}}\right) & if\ P(\sigma, L) \leq P_{crit} \end{cases} \tag{5}$$

However, for gradual failures, it is more reasonable to consider only the time interval of the two-stage fiber failure model, characterized by a slow decrease in the availability factor (a slow increase in the probability of failure).There is no need to take into account the time interval corresponding to the transition to the limit state, since the second stage proceeds in fractions of a second [28] and the value of the unavailability coefficient tends to 1.

From equations (3) and (4) it follows that the probability of failure of an optical fiber during its aging is determined as:

$$P(t) = 1 - e^{-\frac{L}{L_0}\cdot\left[\frac{\sigma_c}{\sigma_0} - \left(\left[\frac{\sigma_c^{n-2}}{\sigma_0^{n-2}}\right] - \frac{t\cdot\sigma_{exp}^n}{B\cdot\sigma_0^{n-2}}\right)^{\frac{1}{n-2}}\right]^m} \tag{6}$$

where

$$B = \frac{2}{A \cdot Y^2(n-2) \cdot K_{IC}^{n-2}} \tag{7}$$

The Markov model of damage accumulation with discrete states and continuous time can be represented as Figure 1.

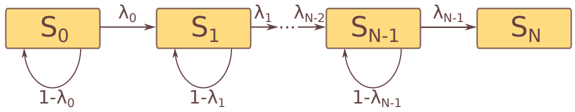


Figure 1: Markov Damage Accumulation Model.

The probability of the transition of the system to the state $S_j$ during the time interval $\Delta t$, counted from the moment $t$, will be denoted by $P_{ij}(t + \Delta t)$.

$$P_{ij}(t + \Delta t) = P(S(t + \Delta t) = S_j | S(t) = S_i) \tag{8}$$

In this case, the events of the Markov chain are mutually exclusive and create a complete group:

$$\sum_{k=1}^{N} P_k(t) = 1 \tag{9}$$

The probability density of the transition (or the intensity of the transition) of the system from state $S_i$ to state $S_j$ is:

$$\lambda_{ij} = \lim_{\Delta t \to 0} \frac{P_{ij}(t + \Delta t) - P_{ij}(t)}{\Delta t} = P'_{ij}(t) \tag{10}$$

Note that for $\Delta t \to 0$ the intensity $\lambda_{ij}(t) = \tan(\alpha)$, where $\tan(\alpha)$ is the tangent to the function $P_{ij}(t)$.

$$P_{ij}(t + \Delta t) \approx \lambda_{ij}\Delta t \tag{11}$$

Thus, the intensities of state-to-state transitions can be found by means of a piecewise linear approximation of the failure probability function determined by formula (6). The accuracy of the correspondence of the piecewise linear approximation of the original function will depend on the number of linear sections of the approximating function, the number of which will correspond to the number of states of the Markov model. The choice of the optimal number of states in this case is determined by the required simulation accuracy.

## 4 DATA FOR MODELING

The following characteristics are taken as the initial data for finding the required number of states of the Markov damage accumulation model for an optical fiber 100 km long:

1) $L_0 = 0.012$ m, $\sigma_0 = 5.222$ GPa, $m = 5.187$, $n = 23.287$, $\ln B = -24.7711$ [23]

2) Inert strength for category B singlemode fibers $\sigma_c = 20$ APa according to [29];
3) Applied Aechanical Atress $_{exp}$ =2 APa Taking into account the adopted characteristics, expression (6) takes the form:

$$P(t) = 1 - e^{-\left[53.0476-(5.164\cdot 10^{36}-6.1\cdot 10^{26}\cdot t)^{\frac{1}{21.287}}\right]^{5.187}} \quad (12)$$

## 5 FINDING THE REQUIRED NUMBER OF STATES OF THE MARKOV MODEL

Piecewise-linear approximation of the function of the probability of failure of an optical fiber at any moment of time is obtained in the following form:

$$P(t) = \begin{cases} k_1 t + C_1 & if\, 0 \le t \le t_1 \\ k_2 t + C_2 & if\, t_1 \le t \le t_2 \\ \dots & \dots \\ k_v t + C_v & if\, t_{v-1} \le t \le t_v \end{cases} \quad (13)$$

where $v$ is the number of sections in the piecewise linear approximating function, the number of states of the Markov model of damage accumulation.

To find the number of states of the Markov model of damage accumulation, it is necessary to set the permissible simulation error $\varepsilon_{mod}$, which will correspond to the approximation error $\varepsilon_{approx}$.

$$\varepsilon_{mod} \ge \varepsilon_{approx} \quad (14)$$

The solution of the problem of approximation with the required error within the framework of the article is implemented by numerical methods.

Finding the optimal linear equations for piecewise linear approximation for a function is carried out using the least squares method [30]. The software functionality is implemented in the pwlf library for the Python programming language. The approximating function is found for a given number of linear equations. To determine the required number of linear equations, the criterion of the maximum approximation error $\varepsilon_{approx}$ is used for a time interval from $N_1$ to $N_2$ days. The number of approximating linear equations increases until condition (14) is satisfied. The maximum error between the approximating and approximating functions in percent is found as:

$$dmax(f, \pi f) = \max(f - \pi f) \cdot 100 \quad (15)$$

where $f$ is the approximated function, $\pi f$ - approximating function.

As an example, an algorithm of operation is presented at $\varepsilon_{mod} = 0.005$, for a time interval from 1 year to 60 years.

```
import pwlf
import numpy as np

P(t) # equation 11

e_max_threshold = 0.005
nLine = 1
t0, t1 = 3.1536e7, 1.89216e9
t = np.linspace(t0, t2, 10000)

maxError = lambda (f, pf) :
          max(abs(f - pf)) * 100

pf = pwlf.PiecewiseLinFit(t, P(t))

while (e_max > e_max_threshold):
    nLine += 1
    pf.fit(nLine)
    e_max = maxError(P(t),
              pf.predict(t))
```

Based on the results of the algorithm, an approximating function is determined, consisting of $v$ linear equations with the required approximation accuracy.

## 6 RESULTS

The article discusses the dependence of the number of states of the Markov model of damage accumulation on the required modeling error. Considered modeling errors: 0.0001, 0.0005, 0.001, 0.005, 0.01 and 0.05. The obtained values are presented in Table 1. The dependence of the approximation accuracy $\varepsilon_{approx}$ on the number of states of the Markov model is shown in Figure 6.

The plots of the approximating functions *vs* the absolute approximation error, are shown in Figures 2-5.

Table 1: Approximation results with different modeling errors.

| $\varepsilon_{mod}$ | $\varepsilon_{approx}$ | # of states | Equation |
|---|---|---|---|
| 0.0005 | 0.00048 | 9 | (16) |
| 0.001 | 0.00095 | 6 | (17) |
| 0.005 | 0.0038 | 3 | (18) |
| 0.01 | 0.009 | 2 | (19) |
| 0.05 | 0.009 | 2 | (19) |

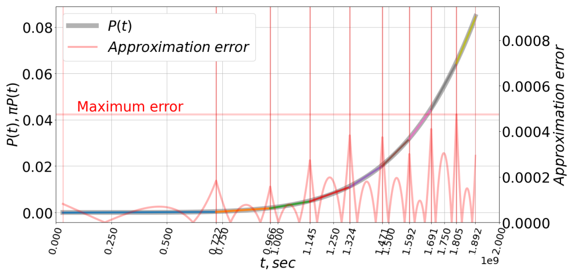


Figure 2: The plot of the approximating function combined with the absolute error of approximation with the accuracy of modeling $\varepsilon_{mod} = 0.0005$.

$$\pi P(t) = \begin{cases} 4.3 \cdot 10^{-13} x & if\ 3.2e7 > t \geq 7.2e8 \\ 6.5 \cdot 10^{-12} x - 0.004 & if\ 7.2e8 > t \geq 9.7e8 \\ 1.6 \cdot 10^{-11} x - 0.014 & if\ 9.7e8 > t \geq 1.1e9 \\ 3.4 \cdot 10^{-11} x - 0.034 & if\ 1.1e9 > t \geq 1.3e9 \\ 6.2 \cdot 10^{-11} x - 0.07 & if\ 1.3e9 > t \geq 1.5e9 \\ 9.5 \cdot 10^{-11} x - 0.12 & if\ 1.5e9 > t \geq 1.6e9 \\ 1.3 \cdot 10^{-10} x - 0.17 & if\ 1.6e9 > t \geq 1.7e9 \\ 1.7 \cdot 10^{-10} x - 0.25 & if\ 1.7e9 > t \geq 1.8e9 \\ 2.2 \cdot 10^{-10} x - 0.34 & if\ 1.8e9 > t \geq 1.9e9 \end{cases} \quad (16)$$

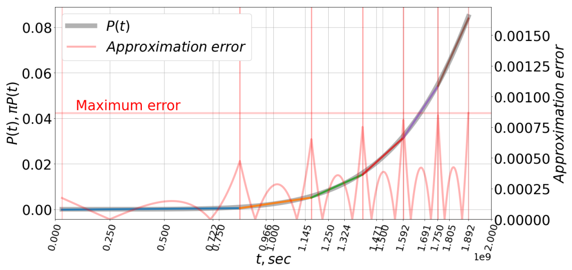


Figure 3: The plot of the approximating function combined with the absolute error of approximation with the accuracy of modeling $\varepsilon_{mod} = 0.001$.

$$\pi P(t) = \begin{cases} 8.0 \cdot 10^{-13} x & if\ 3.2e7 > t \geq 8.5e8 \\ 1.4 \cdot 10^{-11} x - 0.01 & if\ 8.5e8 > t \geq 1.2e9 \\ 4.3 \cdot 10^{-11} x - 0.04 & if\ 1.2e9 > t \geq 1.4e9 \\ 8.6 \cdot 10^{-11} x - 0.1 & if\ 1.4e9 > t \geq 1.6e9 \\ 1.4 \cdot 10^{-10} x - 0.2 & if\ 1.6e9 > t \geq 1.8e9 \\ 2.1 \cdot 10^{-10} x - 0.3 & if\ 1.8e9 > t \geq 1.9e9 \end{cases} \quad (17)$$

$$\pi P(t) = \begin{cases} 3.2 \cdot 10^{-12} x - 0.001 & if\ 3.2e7 > t \geq 1.2e9 \\ 5.8 \cdot 10^{-11} x - 0.06 & if\ 1.2e9 > t \geq 1.6e9 \\ 1.7 \cdot 10^{-10} x - 0.2 & if\ 1.6e9 > t \geq 1.9e9 \end{cases} \quad (18)$$

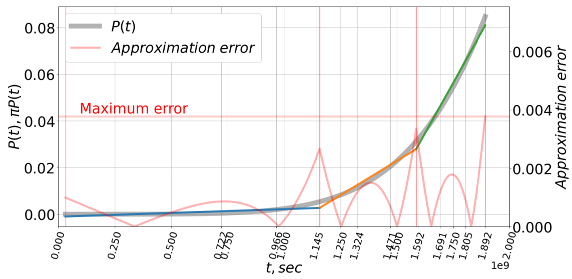


Figure 4: The plot of the approximating function combined with the absolute error of approximation with the accuracy of modeling $\varepsilon_{mod} = 0.005$.

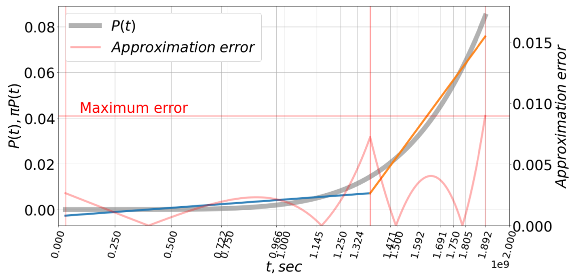


Figure 5: The plot of the approximating function combined with the absolute error of approximation with the accuracy of modeling $\varepsilon_{mod} = 0.01$.

$$\pi P(t) = \begin{cases} 7.2 \cdot 10^{-12} x - 0.003 & if\ 3.2e7 > t \geq 1.4e9 \\ 1.3 \cdot 10^{-10} x - 0.17 & if\ 1.4e9 > t \geq 1.9e9 \end{cases} \quad (19)$$

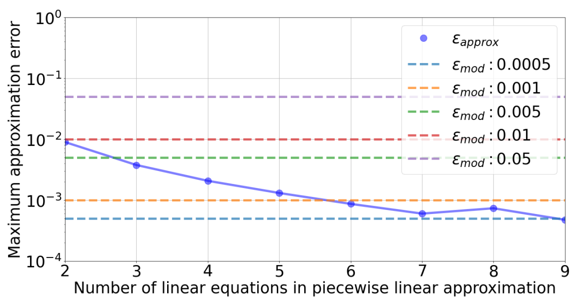


Figure 6: The plot of the dependence of the number of states of the Markov model on the required modeling error $\varepsilon_{mod}$.

Thus, using this algorithm, it is possible to determine the number of states of the Markov damage accumulation model for a given value of the simulation accuracy. In this case, the intensities of transitions between the states of the Markov model can be found from the obtained system of equations of approximating functions.

For a modeling error of 0.01, only two equations are enough to describe a given function, for a modeling error of 0.005, three equations are enough. However, such accuracy is often unacceptable with the required reliability indices of 0.982 for the backbone primary network, 0.998 for the intra-zone primary network [31]. At the same time, with an increase in the number of equations (when there are more than 8), the modeling error decreases slightly. The optimal value in this case is modeling with an error of 0.001.

However, the accuracy of the modeling should be determined by the infrastructure owner based on many aspects: the category of users, the economic costs of downtime, etc.

# 7 CONCLUSIONS

The article proposes a method for solving the problem of determining the number of states of a Markov chain associated with a nonlinear dependence of the probability of rejection on the rate of gradual failures. From the system of equations of approximating functions, the values of time and probability of failure can be found for the model of failure of optical fibers made of silica glass.

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