Supply chain coordination through contracts in supply chains with random production yield: Analytical insights and behavioral aspects

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1. Introduction

The underlying dissertation addresses specific aspects of supply uncertainties in supply chains. Risks in supply gain more practical relevance as their occurrence increases steadily due to the increasing complexity of production processes and procurement strategies. The reasons for uncertainties from the supply side are manifold, ranging from unreliable production processes over perishability and transportation risks to disruptions through major events such as natural or man-made disasters or smaller ones such as strikes. The outcome, however, is basically the same: the quantity gained, production output or delivery, cannot be predicted with certainty, given a fixed input, and thus, affects the performance of the business. Subsequently, it is illustrated how supply uncertainties affect different industries and different processes.

A major supply disruption for several industries worldwide was caused by the 2011 earthquake and tsunami in Japan. The global automobile industry, for instance, lost production of approximately 4.2 billion units in the aftermath of the natural disaster due to destroyed or affected plants of manufacturers and parts suppliers. Toyota reported to have closed Japan-located plants for several weeks as well as shortages of up to 150 parts at their North American plants, forcing them to operate at 30 percent of their capacity (Canis (2011)). Another example is a consumer packaged goods company which incurred highly increased costs due to customs strikes in a South America country they were procuring raw material from. The two-week strike lead to a plant closure for one week as the company only carried one week worth of raw material inventory (Schmitt/Singh (2012)).

Increasing off-shore procurement strategies can lead to supply uncertainty due to long and dangerous shipments which bear the risk of partial or total loss of the delivery (see Ray et al. (2005)). Danger to goods or raw material arriving safely arises from accidents on water, rail, or road. According to the Association of American Railroads, rail shipments of crude oil in the United States has increased by 850% between 2006 and 2013. However, derailments and broken tanks have caused a spill-rate of 1.54 gallons per million ton-miles (McDonald (2014)). Only recently, an Iranian oil freighter collided with an US grain tanker in the East China Sea, sinking the oil tanker with cargo worth $60 million (Martina/Qiao (2018)). Another threat originates from robberies along the way to a product’s destination, affecting ships, trains, and trucks likewise. In Mexico, cargo theft has increased by 20% per year from 2006 to 2010 while rail theft amplified in 2011 by 120% (Segura (2013)). Data from Brazil highlights similar problems. The number of robberies dropped from 2010 to 2011, however, the value of the products stolen increased by approximately 6%. Among the most vulnerable to theft are pharmaceuticals with a total loss of $14.4 million in 2011 (Queiroz (2012)).

In the area of unreliable production processes, the most common and present example is agriculture. Depending on the geography, crops can be highly vulnerable to external conditions such as weather or pests infestation which can cause partial or total losses in harvest.

Apple yields in Germany, for instance, show variations between 24 and 37 tons per hectare in the years from 2002 to 2016. Up- or downward leaps between years were as high as 45% from 2013 to 2014 and as low as -22% between 2009 and 2010. Recently, bad weather conditions such as heavy rain, hail, and storms caused the 2017's apple harvest in the State of Saxony-Anhalt to drop to 70 % of a 5-year average (see dpa (2017)).

Olives are produce where yield losses cause even larger damage as olives can only be harvested every two years. In Turkey, one of the large olive and olive oil producing countries, production of olives averaged at 527,000 tons per crop year between 2011 and 2015. Exemplary data from the region of Çanakkale in north-west Turkey shows an oscillation of over 200% in yield between 2013 and 2014 when crops leaped from 12,5 kg to 26,5 kg per tree (see Kazaz (2004) and Kaleci, Gündogdu (2016)).

The production of vaccinations and other pharmaceuticals is another example for processes which underlie random yield (Arifoglu et al. (2012) and Chick et al. (2008)). A current BCG-report on the vaccine industry in Germany states that yields are highly uncertain with losses up to 100 percent per lot due to complex production processes and perishability of components. Production runs of 6 to 25 months combined with seasonality of vaccine demand eliminate the chance of producing additional lots in order to compensate for yield losses (Lücke et al. (2016)).

Highly influential to yield losses is the semiconductor industry which produces highly sophisticated products at constantly shortening lifecycles. In 2004, Apple postponed the release of its Power Mac G5 due to limited supply from one of its main suppliers for chips, namely IBM. Chip production at IBM struggled at the time due to quality issues of input material as well as the complexity of the production process resulting in low yields (Fried (2004) and McMillan (2017)). Two years later, Sony had to manage losses in sales after releasing its Playstation 3 due to shortages from its parts supplier IBM. At the time, yield rates for IBM's cell processors which were used in Sony's Playstation 3, ranged between 10 and 20 percent (Nguyen (2006)).

Obviously, the field of supply uncertainty is wide and the previous examples sheds light on the subject that production and delivery risks not only harm the supplier but also affect decisions and performance of subsequent actors in the supply chain. This topic highlights the interdependences of individual firms and decision makers in complex chains and generates news challenges for coordinating the decisions of all participants.

This thesis narrowly focuses on the specific area of coordinating supply chains under random production yield, such as in the IBM case.

Literature is extensive for coordination issues in newsvendor-type settings where demand is stochastic. Rather complex contract types are shown to facilitate supply chain coordination under demand uncertainty, especially if they enable appropriate risk sharing between the actors. However, for settings with production yield uncertainty, few research exists on coordination through appropriate contracts. The analytical part of this thesis investigates the coordination properties of various contract types, applied to different types of yield uncertainty, namely stochastically proportional yield and binomial yield. In the empirical part of this thesis, human decision making behavior in random yield supply chains is observed and analyzed with respect to theoretical predictions.

The remainder of this thesis is organized as follows. Section 1.1 introduces various forms of modeling yield randomness while the supply chain model underlying this thesis is presented in § 1.2. A review on existing literature in this field of study is given in § 1.3. Finally, the single papers' contributions are summarized in § 1.4. The three papers constituting this thesis follow after that.

1.1 Random yield in supply chains

Modelling random yield in supply chains can take various forms. Depending on the underlying nature of the risks, common types are stochastically proportional yield, binomial yield, but also an all-or-nothing approach is common.

Stochastically proportional yield is defined such that the usable output from production is a random fraction of the production input. The so-called yield rate is denoted by z (with pdf $\varphi(z)$, cdf $\Phi(z)$, and mean μ_z) and is arbitrarily distributed between 0 and 1. Accordingly, the production output $Y(Q)$ from producing Q units amounts to $z \cdot Q$ with $0 \leq z \cdot Q \leq Q$. This model applies to situations where an entire production lot is exposed to a set of unpredictably or uncontrollable conditions which affect the outcome of the lot. Hence, the distribution of the yield rate represents the likelihood of the different conditions to occur (see Yano/Lee (1995) and Inderfurth/Vogelgesang (2013)). A suitable example is agriculture whereweather conditions or pest infestations can affect a whole crop and influence the amount harvested given a fixed acreage.

Another rather simple way of modeling random yield is to assume that the production of a good unit follows a Bernoulli process and the random production yield $Y(Q)$ follows a binomial distribution with the probability of a unit to turn out 'good' (or usable) being θ $(0 \leq \theta \leq 1)$. A unit is unusable with the counter probability $1-\theta$. The probabilities for certain yields from a production batch are given by:

$$Pr\{Y(Q)=k\}=\binom{Q}{k}\cdot\theta^{k}(1-\theta)^{Q-k} \qquad \forall\, k=0,1,...,Q$$

The mean production yield and its standard deviation are given by $\mu_{Y(Q)}=\theta\cdot Q$ and $\sigma_{Y(Q)}=\sqrt{\theta\cdot(1-\theta)\cdot Q}$, respectively. This yield type gains practical relevance when randomly material or processes fail. The aforementioned IBM chips are an example when they are used as input for other products.

Contrary to stochastically proportional yield there is no autocorrelation in binomial yield, i.e. producing one unit (usable or unusable) is independent from producing another one. Furthermore, the coefficient of variation ($\sigma_{Y(Q)}/\mu_{Y(Q)}$) decreases with increasing input quantities, i.e. the risk of production losses diminishes with larger lots (see Yano/Lee (1995) and Inderfurth/Vogelgesang (2013)).

Yield uncertainty can also be modeled as a result of a disruption. Depending on the intensity, a disruption can cause, but is not limited to, the loss of a whole lot. Failure in handling, accidents, or destruction can destroy entire batches of production or input material and can affect basically any industry or process. However, the probabilities of occurrence can differ widely. A simple way of modeling "all-or-nothing"-type disruption risks is by introducing a disruption probability α which is independent from the lot size. The yield quantity is then given by:

$$Y\ Q=\begin{cases}0 & \text{with prob.} \quad \alpha\\ Q & \text{with prob.} \quad 1-\alpha\end{cases}$$

However, the estimation of appropriate parameter values is challenging especially when considering disruptions through disasters of any kind (compare Yano/Lee (1995) and Gümüs et. al (2012)).

In this thesis, only the first two types of modeling yield randomness, namely stochastically proportional and binomial yield are addressed. The reasoning behind this choice is threefold: First, their modeling is rather simple and their implementation is easy. Second, both types have a large dissemination in existing literature on yield randomness. Most common is the use of stochastically proportional yield. However, practice shows that yield types depend strongly on the industry and thus, binomial yield is equally applicable. This leads to: Third, both types have a high practical relevance. As mentioned above, yield randomness in agriculture or highly sophisticated manufacturing is among the most common and those can be illustrated using stochastically proportional and binomial yield, respectively.

1.2 Model description

The basic model underlying all three papers in this thesis is a single-period interaction within a serial supply chain with one buyer and one supplier. All information on cost, price, and yield is common knowledge. However, deterministic end customer demand may not be common knowledge but only known to the buyer. As the supplier decision is totally independent from end customer demand, this is a reasonable assumption. The setting connects to the field of contracting in a principal-agent context with information asymmetry (compare Corbett and Tang (1999) or Burnetas et al. (2007)) where the principal (buyer) is better informed than the agent (supplier).

The figure below illustrates the supply chain as well as the course of action with direction of decisions and flow of goods. In order to fulfill deterministic end customer demand D that generates a revenue p per unit, the buyer orders an amount X from the supplier at a per unit wholesale price w. The supplier can be considered a manufacturer who, due to production lead times, can realize only a single regular production run and, after receiving the buyer's order, has to decide upon the respective production input quantity Q. The cost for processing one unit of input is c. However, the supplier's production process is not reliable which leads to random output. Depending on the underlying type of yield uncertainty, production output $Y\ Q$ lies between 0 and Q and thus, can fall below order quantity. As mentioned above, an additional production run is not possible. Furthermore, it needs to be noted that business is always profitable, i.e. expected production cost per non-defective unit is lower than the wholesale price per unit which in turn is lower than the retail price.

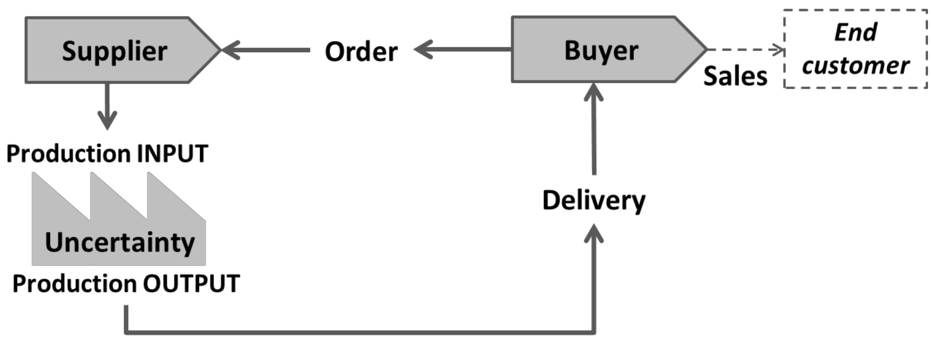

Figure: Serial two-tier supply chain and course of interaction

Unlike in deterministic yield, random demand settings both actors face specific risks of overage and underage while their decisions impact not only the own but also the counterpart's profit. In order to reveal coordination deficits in this basic model, profits are derived from the perspective of the central decision maker as well as from the perspectives of the decentralized supply chain members. Total supply chain profit and decisions then serve as a benchmark for the profits and decisions generated by the individual actors. Profits for the total supply chain (1) and the actors (supplier (2) and buyer (3a) and (3b), respectively) are as follows:

$$\Pi_{SC}(Q_{SC}) = p \cdot E\left[\min(Y(Q_{SC}), D)\right] - c \cdot Q_{SC} \tag{1}$$

$$\Pi_{S}(Q_{S} \mid X) = w \cdot E\left[\min(Y(Q_{S}), X)\right] - c \cdot Q_{S} \tag{2}$$

If the order quantity is above demand, the buyer's profit amounts to

$$\Pi_{B}(X \mid Q_{S}) = p \cdot E\left[\min(Y(Q_{S}), X, D)\right] - w \cdot E\left[\min(Y(Q_{S}), X)\right] \tag{3a}$$

In case it is lower than demand, the profit generated by the buyer is given by

$$\Pi_{B}(X \mid Q_{S}) = p \cdot E\left[\min(Y(Q_{S}), X)\right] - w \cdot E\left[\min(Y(Q_{S}), X)\right] \tag{3b}$$

The analysis of the above profit functions gives insight into the actors' decisions on their respective quantities in the supply chain. Depending on the choice of parameters, analyses reveal how buyers, if so, inflate their demand in order to safeguard against lost sales due to unreliable deliveries from the supplier. Furthermore, is can be analyzed to what extent the supplier raises her production quantities above incoming orders in order to fill them to the best of profitability. In the remainder of this thesis, light will be shed on how different contract types, simple or sophisticated, can be used to drive the actors' behavior and if the contracts' properties depend on the underlying type of yield uncertainty. Finally, subject behavior is studied in order to detect to what extent decision making in theory and reality coincide.

1.3 Literature review

Various streams of literature lead the path to the area of research in this thesis. As supply risks increase steadily because production processes and procurement strategies becoming more and more complex, their origin and management is of great importance. Tang (2006) devotes his work to these general aspects of supply risks.

More specific is the random yield problem in production systems which is fundamental to this research. An overview of articles in this field is given by Yano and Lee (1995). Studies by Gerchak et al. (1988) and Henig and Gerchak (1990) analyze optimal production policies in periodic review systems while Gerchak et al. (1994), Gurnani et al. (2000), and Pan and So (2010) extend these approaches by considering random yields in assembly systems.

Most deeply embedded in this thesis is the field of random yields in a supply chain interaction context as well as coordination through contracts in supply chains under uncertainty. A major survey article in the area of coordination which considers several types of risk sharing contracts was published by Cachon (2003). More recently, Arshinder et al. (2011) provide a review on managing uncertainty and sharing risks in a supply chain through appropriate coordination mechanisms, among others contract design. However, both articles do not address the case of random yield in the supply chain. Several articles exist in the area of yield uncertainties in a supply chain interaction context; however, some contributions just consider different types of risk sharing contracts without analyzing their ability to coordinate a supply chain. Here, He and Zhang (2008, 2010), Keren (2009), Wang (2009), and Xu (2010) can be named. In a study by Güler and Bilgiç (2009) forced compliance of the supplier is assumed so that the typical coordination problem under random yield is not addressed. Another group of contributions considers supply chain coordination under voluntary compliance. Gurnani and Gerchak (2007) and Yan et al. (2010) address contracting issues in a random-yield assembly system with two suppliers and one buyer. They propose various types of coordinating contracts which all leave the suppliers with zero profits. However, they do not consider ordering decisions of the buyer at demand level which is totally reasonable as Chapter 2 of this thesis will prevail. Another stream of articles considers supply chain coordination under demand as well as yield uncertainty. Yan and Liu (2009), He and Zhao (2012), and Ma et al. (2012) analyze contracts for supply chain coordination in such cases. However, this area of research is outside of this thesis` scope. Considering other than stochastically proportional yield is not addressed in any of the above literature.

The last adjacent field of literature concerns behavioral economics in supply chains under uncertainty. While studies on supply risks in supply chains are scarce, various literature exists which investigate behavioral aspects in a random demand setting (newsvendor problem). The fundamental study by Schweitzer and Cachon (2000) who found subjects to order suboptimally but were able to explain their behavior with simple heuristics, has been extended notably. Relevant to the underlying research are experimental studies which consider learning effects, information sharing between supply chain member, social preferences as well as probabilistic choices in decision making. Benzion et al. (2008), Bolton and Katok (2008) as well as Bostian et al. (2008) investigate the role of learning over time in newsvendor type experiments. The impact of information sharing on decision making under demand uncertainty has been investigated by Bolton et al. (2012). Probabilistic choices as a form of bounded rational behavior (Luce (1959) and McKelvey and Palfrey (1995)) in a supply chain context were subject matter to the works of Lim and Ho (2007), Ho and Zhang (2008), Su (2008), Kremer et al. (2010), Chen et al. (2012), Wu and Chen (2014), and Pavlov et al. (2016). Relevant studies concerning social preferences (Fehr and Schmidt (1999)) in a supply chain setting were conducted by Loch and Wu (2008), Katok and Pavlov (2013), Katok et al. (2014), and Hartwig et al. (2015).

Closest to this thesis' scope come studies which consider behavioral aspects in supply chains under supply uncertainties. Only recently, first approaches were made by Gurnani et al. (2014) who study converging supply chains with two suppliers and one buyer where one supply source underlies two types of risk, namely disruption risk and yield uncertainty. However, they model a supply chain without interaction, i.e. decisions are only made by the buyer. Goldschmidt (2014) as well as Goldschmidt et al. (2014) come close to the above approach but focus solely on an all-or-nothing type of supply risk. Finally, Craig et al. (2016) conduct field experiments in the apparel industry to analyze how buyers behave when uncertainty on the supply side is reduced (in terms of increased supplier fill rate).

1.4 Contribution

The underlying thesis contributes in three ways to the existing research on random yield in supply chain management.

First, analytical insight is given into the subject of coordination by contracts in supply chains under stochastically proportional yields. Applied is the above model of a two-level supply chain under deterministic demand. This section discusses the coordination ability of basic and advanced contract types for supply chain settings with and without an emergency procurement option which is, if available, fully reliable and uncapacitated.

In the basic case of one unreliable supplier as the only source of material, both parties are exposed to risks of overage and underage, respectively, if a simple wholesale price contract is applied. As coordination in a supply chain is not achieved under this simple form of contract, more sophisticated contract types are introduced and analyzed. The concepts of penalizing and rewarding are implemented which changes the risk distribution between the actors. While the buyer orders at demand level, the supplier is incentivized to increase her production quantity efficiently in order to safeguard against the yield risk. Resulting from that is a coordinated supply chain.

Another way of coordinating the supply chain other than using more complex contract types is to add further sources. In this context, two cases are distinguished where either the supplier or the buyer has access to an emergency source. It can be proven that allowing the buyer to source from a second (emergency) supplier which is more costly, but reliable, can coordinate the supply chain under the simple wholesale price contract. In case the supplier sources from an additional reliable source, coordination is not obtained.

Second, insight is given into the above analyses of contract types and procurement sources in case of binomially distributed yield. For reasons of simplicity, existing research is mostly limited to modeling stochastically proportional yield. As mentioned earlier, other yield types may be of greater practical relevance as it is not reasonable to assume that yield losses are always caused by external effects which have a joint impact on the complete production lot. The special case of binomial yield is characterized by a zero yield correlation of units within a production lot. Since this property is contrary to the perfect yield correlation under stochastically proportional yield, the coordination properties of the contracts and scenarios may not be transferrable to supply chains under binomial yield.

The analyses show that, indeed, all previously considered advanced contract types retain their ability to coordinate the supply chain with conditions for coordinating parameters and profit split being identical to stochastically proportional yield. Also, the coordinating property of an additional (emergency) procurement option for the buyer holds under binomial yield while the simple wholesale price contract fails to coordinate the chain.

However, the yield type crucially alters the supplier’s decision making. While she inflates buyer demand by a constant factor under stochastically proportional yield, the inflation under binomial yield is not constant. Rather, it increases or decreases with increasing demand – depending on the set of price and yield parameters – and approaches the reciprocal of the expected yield rate the larger demand gets. Another interesting difference is the simple wholesale price contract’s ability to achieve coordination for large values of demand. Both aforementioned properties result from the specific characteristic of binomial yield that the risk diminishes with increasing demand and production lots, respectively.

For each analytical section, numerical examples are provided in order to illustrate the results.

Third, behavioral analyses are provided on the subject of interaction in random yield supply chains. Unlike for newsvendor-type settings of random demand, literature barely investigated decision making under yield randomness from a behavioral perspective. Even further, interaction between members in a random yield supply chain is not addressed at all.

The allocation of risks in this supply chain forms a more complex decision making space for both actors as the risk is not only directly present at the supplier stage but also impacts the performance of the buyer. Laboratory experiment with human subjects shed light on how decision makers perceive and handle this risk allocation. In order to gain insight on each actor's behavior, experiments were conducted with automated counterparts as well as with human subjects as the other supply chain member. Furthermore, the availability of key information within the supply chain is investigated as a driver for performance.

Interestingly, results show that buyers have a good understanding of the situation and mostly follow a probabilistic choice rule. Additionally, they try to hedge against delivery risks from the supplier. Suppliers on the other hand show learning effects as their performance improves over time. However, sharing of crucial information seems to be no cure for inefficient behavior in the supply chain.

In the above order, the next three sections provide two articles which are published in refereed journals (OR Spectrum and Business Research) as well as one paper which is available in the FEMM working paper series of the Faculty of Economics and Management of the Otto-von-Guericke University, Magdeburg.

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REGULAR ARTICLE

Supply chain coordination by risk sharing contracts under random production yield and deterministic demand

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Abstract From numerous contributions to literature we know that properly designed contracts can facilitate coordinated decision making of multiple actors in a supply chain (SC) so that efficiency losses for the whole SC can be avoided. In a newsvendor-type SC with stochastic demand it is well-known that the double marginalization effect hampers the simple wholesale price contract to achieve coordination. More complex contracts however can bring about coordination, especially those which enable appropriate sharing of risks between the actors. While the effectiveness of risk sharing contracts is well understood for SC situations with random demand and reliable supply, less is known about respective problems if demand is deterministic but supply is unreliable due to random production yield. This paper shows how in a buyer-supplier SC the distribution of risks affects the coordination of buyer's ordering and supplier's production decision in a basic random yield, deterministic demand setting. Both parties are exposed to risks of over-production or under-delivery, respectively, if a simple wholesale price contract is applied. The resulting risk distribution always impedes SC coordination. However, more sophisticated contract types which penalize or reward the supplier can change risk distribution so that SC coordination is possible under random yield. Additionally, it is proven that the wholesale price contract will guarantee SC coordination if the supplier has a second (emergency) procurement source that is more costly, but reliable. Moreover, it is shown that under wholesale price contracts it can be beneficial to utilize this emergency source even if it is unprofitable from a SC perspective. However, if such an emergency option is available to the buyer as opposed to the supplier, a wholesale price contract will not be able to coordinate the SC.

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1 Introduction

Uncertainty in SCs can occur in various forms with the most prominent types being demand and supply uncertainties. Regarding the supply side, business risks primarily result from yield uncertainty. While random demand can be found in almost all industries, random yield is not as wide-spread. However, it frequently occurs in the agricultural sector or in the chemical, electronic and mechanical manufacturing industries (see Gurnani et al. 2000, Jones et al. 2001, Kazaz 2004, Nahmias 2009). Here, random supply can appear due to different reasons such as weather conditions, production process risks or imperfect input material. As a consequence of yield uncertainty, the same production input might result in different production output quantities. Another source of supply unreliability stems from the recent trend towards off-shore production or sourcing (from developing countries) mainly due to cost considerations. Such off-shore strategies can lead to supply uncertainty in two ways. On the one hand, appropriate choice of input material and production processes are hard to monitor from a distance. As a consequence, the quality of delivered products can vary from predetermined requirements and thus, be unacceptable for further processing. On the other hand, the transportation of products is a considerable source of uncertainty as long shipments bear the risk of partial or total loss of the delivery (see Ray et al. 2005). In a SC context, yield or supply randomness obviously will affect the risk position of the actors and, therefore, will have an effect on the buyer-supplier relationship in a SC. The question that arises is to what extent random yields affect the decisions of the single SC actors and the performance of the whole SC. In this study we limit ourselves to a problem setting with deterministic demand. This is to focus the risk analysis of contracting on the random yield aspect. As stated in Bassok et al. (2002), this setting is of practical relevance for production planning in some industries.

The main purpose of this paper is to explore how contracts can be used in this context in order to overcome inefficiencies arising from uncoordinated behavior. Therefore, in addition to the simple wholesale price contract, various contract types with risk sharing characteristics containing penalty or reward elements for the supplier are introduced and analyzed with respect to their coordination ability. Comparable to the newsvendor setting with stochastic demand but reliable supply, the double marginalization effect of the wholesale price contract is found in our setting. Therefore, two more contracts with alternative risk distribution are studied. One contract type which rewards the supplier and thus shifts risk to the buyer is the over-production risk sharing contract. This contract, first proposed in He and Zhang (2008), ensures that excess units from production are subsidized by the buyer. The analysis also includes a penalty contract as introduced in Gurnani and Gerchak (2007) under which the supplier is penalized for every unit of under-delivery so that risk is transferred to the supplier. Both advanced contract types can be shown to facilitate SC coordination if contract parameters are chosen appropriately. In case a reliable but more costly second procurement option exists, it can be shown that the wholesale price contract is already entirely sufficient to enable SC coordination as long as this option is in the hand of the supplier.

In literature there exist three major streams which are related to our research. The first one considers the context of ordering and producing under random yield. An overview of articles in this field is given by Yano and Lee (1995). Among others, the reader is referred to Gerchak et al. (1998) and Henig and Gerchak (1990) for analyzing optimal production policies in periodic review systems.

An extension to the approaches above is provided by Gerchak et al. (1994), Gurnani et al. (2000) and Pan and So (2010) who consider random yields in assembly systems.

The second body of literature concerns coordination through contracts in supply chains where risks stem from uncertain demand. A major survey article in this area which considers the coordinating properties of several types of risk sharing contracts was published by Cachon (2003). More recently, Arshinder et al. (2011) provide a review on managing uncertainty and sharing risks in a SC through appropriate coordination mechanisms including contract design.

Most relevant to our research is a third stream of articles which covers stochastic production yields in the SC interaction context, a topic which has only recently received attention in the literature. Partly, these contributions just consider different types of risk sharing contracts without analyzing their ability to facilitate SC coordination. He and Zhang (2008, 2010), Keren (2009), Wang (2009) and Xu (2010) belong to this group. In a study by Güler and Bilgic (2009) forced compliance on the supplier's side is assumed so that the typical coordination problem under random yield is not addressed.

A remaining group of contributions considers SC coordination under voluntary compliance and partly covers a broader type of random yield problems than we do. For instance, Gurnani and Gerchak (2007) and Yan et al. (2010) address contracting issues for SC coordination in a random-yield assembly system with two suppliers and a single buyer. Gurnani and Gerchak propose two types of penalty contracts, but are only able to show that they coordinate if the suppliers are left with zero profits. Insights into contract parameter determination and interaction are not given. Yan et al. extend the work of Gurnani and Gerchak by considering a salvage value for over-produced items and additionally investigate the properties of a specific type of over-production risk sharing contract where the buyer accepts the supplier's total production output, even if he has ordered less. In difference to some of the direct statements in the paper, they also observe that this contract will not coordinate unless the supplier ends up with zero profit. Their analysis does not consider the possibility that the buyer will not order more than what is externally demanded. Our analysis will show that it is essential for constructing SC coordinating contracts in a random yield environment that this buyer-policy of ordering at demand level (i.e. the buyer will not purchase any more than what is externally demanded) is taken into consideration.

Articles by Yan and Liu (2009), He and Zhao (2012) and Ma et al. (2012) analyze contracts for SC coordination in serial systems where they in addition to random yield also include stochastic demand, which is outside of our scope. While He and Zhao as well as Ma et al. limit their analysis to the special case where the supplier has a second fully reliable procurement source at her disposal, Yan and Liu come closest to the scope of our contribution as they address a situation without emergency procurement and consider both advanced contract types that we also examine. Besides considering a SC interaction where a powerful buyer can dictate all contract conditions, they also investigate the standard situation as we do where the power within the SC is more evenly distributed. For this setting they analyze the same type of over-production (where more than the current demand is produced) risk sharing contract as in Yan et al. (2010) where the buyer accepts the supplier's total production output. They show that under such a contract SC coordination which allows for the participation of the SC actors is not possible, even if this contract type is combined with a penalty contract. Coordination can only be achieved if a subsidy for over-production is combined with a buy-back arrangement for the buyer's excess stock. The contract terms, however, are very unrealistic as they form a situation where the supplier's shipment is completely independent from the buyer's order.

Our work fills research gaps from above literature and presents a comprehensive and complete analysis of the coordination properties of different contract types under voluntary compliance with different risk sharing elements in a serial SC with random yield and deterministic demand. It discusses basic and advanced contract types for SC settings with and without an emergency procurement option which is, if available, fully reliable and uncapacitated. In this matter, the analysis distinguishes the two cases where either the supplier or the buyer has access to such an emergency source. From a modeling perspective, our research addresses a special case of literature contributions that refer to assembly systems or to serial systems with additional demand uncertainty. However, as described above, we extend results from literature in multiple ways. This is mainly done by explicitly considering participation constraints and their impact on necessary properties of contract parameters in order to guarantee SC coordination. This is missing in, e.g. Gurnani and Gerchak (2007) and Yan et al. (2010) and makes their analysis incomplete. Our approach also reveals that in case of over-production risk sharing it makes a crucial difference for the coordination property of a contract if over-production quantities are pushed to the buyers or not. This falls outside of the scope of the analyses in Yan and Liu (2009), He and Zhao (2012) and also Yan et al. (2010). The case of an emergency source under the control of the buyer is not discussed in the literature.

The rest of this paper is organized as follows. In Sect. 2 the SC scenarios and risk aspects considered in this research are introduced. In Sect. 3 the above mentioned contract designs are analyzed with respect to their SC coordination potential and general insights are presented for the case without emergency procurement. The impact of the emergency option is analyzed in Sect. 4. Section 5 summarizes main results and suggests aspects of further research.

2 Supply chain scenarios and risk aspects

The basic scenario considered in this paper refers to a single-period interaction within a serial SC with one buyer (indicated by subscript B) and one supplier (indicated by subscript S). All cost, price, and yield information is assumed to be common knowledge. In contrast to that, deterministic end customer demand is not common knowledge but only known to the buyer. This is a reasonable assumption because the supplier decision is, as will be revealed in the analysis, totally independent from

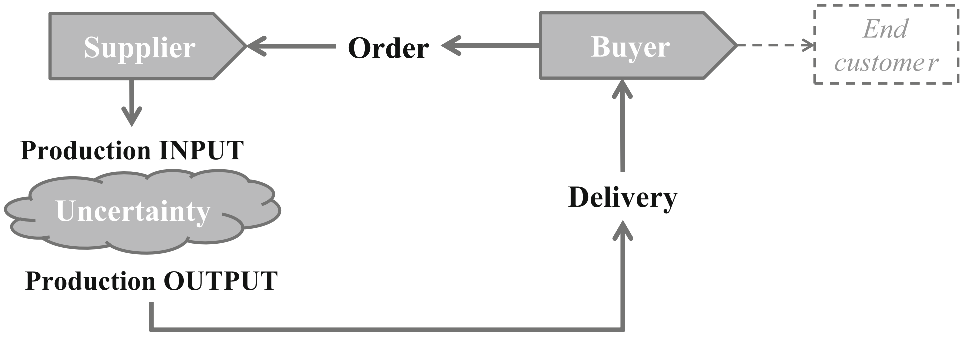

Fig. 1 Serial supply chain and course of interaction

end customer demand. This setting connects to the field of contracting in a principal-agent context with information asymmetry (see Corbett and Tang 1999 or Burnetas et al. 2007) where the principal (buyer) is better informed than the agent (supplier). Nevertheless, this property has no effect on the agent's payoff as it is not a direct function of the principal's information on demand (compare Maskin and Tirole 1990).

The SC and the course of interaction (explained below) are depicted in Fig. 1.

In order to fulfill a deterministic end customer demand D that generates a revenue p per unit, the buyer orders an amount X from the supplier at a per unit wholesale price w. The supplier can be considered a manufacturer who, due to production lead times, can realize only a single regular production run and, after receiving the buyer's order, has to decide upon the respective production input quantity Q associated with a per unit production cost c. However, the supplier's production process is subject to risks which lead to random output. It is assumed that the production yield is stochastically proportional, i.e. the usable output is a random fraction of the production input Q. The production output rate is denoted by z(with $pdf\ \varphi(z)$, $cdf\ \Phi(z)$ and mean μ_z) and is arbitrarily distributed between 0 and 1. Accordingly, the production output amounts to $z \cdot Q$ with $0 \leq z \cdot Q \leq Q$. So, the quantity delivered to the buyer can fall below the order size and is uncertain. Production output that exceeds the buyer's order quantity is assumed to be worthless and does not generate revenue for the supplier.

From the described scenario that implicitly refers to a business relationship under a wholesale price contract, it is clear than unlike in the deterministic yield, random demand setting both actors make self-interested decisions that mutually affect the respective profits. It is also evident that both parties face specific risks. The supplier, on the one hand, is exposed to the risk of over-production if the production output exceeds the buyer's order size as any production overshoot is worthless. On the other hand, she faces the risk of under-delivery if the production output is so low that the buyer's order cannot be satisfied completely and part of her potential revenue gets lost. It is a reasonable assumption that, under a simple wholesale price contract, the supplier is not further penalized (in addition to losing potential revenue) if end customer demand cannot be satisfied due to under-delivery. In typical business transactions the supplying side is usually measured in terms of its ability to deliver to the buyer and not to the end customer. As the mechanism to satisfy end customer demand is not in the control of the supplier, she cannot be held responsible for potential sales losses. Nevertheless, the terms of ‘penalty’ for the supplier in case of under-delivery highly depend on the contract in use.

This under-delivery risk does also affect the buyer who additionally can be exposed to an over-ordering risk. This can occur if he orders more than externally demanded to compensate for a potential under-delivery by the supplier, but receives a quantity that exceeds his demand.

In a second scenario we examine the case where an additional opportunity to procure extra units from a reliable source exists that can be used to compensate for yield losses after output from standard production has materialized. Under reasonable conditions this source has a per unit cost c_E which is larger than the expected cost of regular production ($c_E > c/\mu_z$). This emergency source can be interpreted as an (expensive) short-term external procurement option that might be available to the supplier as well as to the buyer. At the supplier's side this option might also stand for the possibility to rework units from the first production run, provided that such rework is perfectly reliable. The quantity procured from this uncapacitated source is denoted by $\hat{Q}^E$. If under these conditions the supplier has the opportunity of emergency procurement she is always in a position to fulfill the buyer's order completely so that no risk of under-delivery exists. Accordingly, the buyer has no incentive to order above the demand level so that the over-ordering risk vanishes and a SC situation occurs where only one party, viz. the supplier, is bearing any risk. If, on the other hand, the buyer has the emergency option at his disposal he will equally be able to fulfill the customer demand completely, albeit with some cost risk.

The general underlying assumption in this analysis is that profitability of the business for both parties is assured, i.e. the retail price exceeds the wholesale price which in turn exceeds the expected costs of regular production, i.e. $p > w > c/\mu_z$.

3 Contract analyses without emergency procurement

We start by assuming that once production output from regular production has realized there exists no further option in the SC to procure extra products from another source. Thus, that the available quantity for filling end customer demand is random. Later, this assumption will be relaxed. Starting with the development of a benchmark, different contract types are studied afterwards. Finally, the results are compared to the benchmark case which will provide insight into the coordination potential of the specific contract types.

3.1 Centralized decision making

Under centralized or global decision making, i.e. where all actions are conducted by one company (indicated by subscript SC), the only decision is on the production input quantity Q_{SC}. The profit Π_{SC} can then be formulated in the following way:

$$\begin{aligned}\Pi_{\mathrm{SC}}(Q_{\mathrm{SC}}) &= p \cdot E\,[\min(z \cdot Q_{\mathrm{SC}}, D)] - c \cdot Q_{\mathrm{SC}} \\ &= p \cdot \left[\int_{0}^{D/Q_{\mathrm{SC}}} z \cdot Q_{\mathrm{SC}} \cdot \varphi(z)dz + \int_{D/Q_{\mathrm{SC}}}^{1} D \cdot \varphi(z)dz \right] - c \cdot Q_{\mathrm{SC}}. \qquad (1)\end{aligned}$$

Due to concavity of the profit function (for proof see Appendix 1), the optimal production input quantity is obtained by taking the first-order derivative of the profit function and setting it equal to zero. From the first-order condition (FOC) the optimal decision can be derived as (see also He and Zhang 2008)

$$Q^*_{SC} \quad \text{from} \quad \int_0^{D/Q_{SC}} z \cdot \varphi(z)dz = \frac{c}{p}. \tag{2}$$

The integral term in (2) stands for the expected marginal sales quantity induced by a marginal increase of production input. So equation (2) reflects that the input quantity should be fixed such that marginal sales revenue equals marginal production cost.

Obviously, the SC optimal production input quantity depends on all problem data in a specific way. In detail, the structure of the optimal policy is such that the production quantity equals demand D inflated by a factor $K_{SC} > 1$ which depends on the relation of production cost and retail price as well as on the yield rate distribution, or more specifically

$$Q^*_{SC} = K^*_{SC} \cdot D \quad \text{with} \quad K^*_{SC} > 1 \quad \text{and} \tag{3}$$

$$K^*_{SC} \quad \text{from} \quad \int_0^{1/K_{SC}} z \cdot \varphi(z)dz = \frac{c}{p}. \tag{4}$$

$K^*_{SC} > 1$ follows due to $0 < c/p < \mu_Z$. As mentioned before, the condition $p > c/\mu_Z$ always holds as profitability of the business is required. The multiplier K^*_{SC} aims at compensating yield losses and, at the same time, accounts for the best trade-off between cost of over-production (c) and lost revenue by under-production (p) in the presence of yield randomness.

The interdependence of the multiplier K^*_{SC} and the cost/price parameters becomes evident from the integral function in (4). For values of K below 1, the integral is not defined as we assume the yield rate to be distributed only between 0 and 1. At a K-value of 1, the integral is exactly equal to the mean yield rate μ_Z. As the multiplier increases the integral decreases which results in an increased value for the production quantity.

By inserting the optimal decision (2) into the profit function (1) and further evaluating we find the maximum profit to be

$$\Pi^*_{SC} = \Pi_{SC}(Q^*_{SC}) = p \cdot \left[1 - \Phi(1/K^*_{SC})\right] \cdot D. \tag{5}$$

It turns out that the SC profit under an optimal production decision is always proportional to the demand level.

In the following, a two-member (i.e. decentralized) SC is considered in which two companies interact with each other and decide individually. Using the results of the previous analysis as a benchmark, the situation of decentralized decision making can be evaluated.

3.2 Decentralized decision making

Under decentralized decision making the buyer releases an order and the supplier decides on the (input) quantity for production to fulfill the buyer's request in a most profitable way. Depending on her overall production yield, the supplier will deliver as much as possible to satisfy the demand quantity ordered by the buyer. For all following analyses the Stackelberg game is applied as a game theoretic approach. According to the sequence of decisions the buyer is modeled as leader and the supplier as follower, i.e. the buyer anticipates the supplier's reaction to his own decision.

In this context we will investigate three different contract types the design of which results in a different way of risk sharing between the SC members. We consider the wholesale price contract as the most simple contract type first and then proceed with two more sophisticated ones which are proposed in literature for more flexible risk sharing policies.

3.2.1 Wholesale price contract

The simple wholesale price (WHP) contract states that only a constant wholesale price is charged by the supplier for each unit delivered to the buyer. No other financial transactions take place between the two parties.

Supplier's optimal decision

The supplier reacts to the buyer's order quantity X and seeks to maximize her own profit which is given as

$$\begin{aligned}\Pi_S^{WHP}(Q_S|X) &= w \cdot E\ [\min(z \cdot Q_S, X)] - c \cdot Q_S \\ &= w \cdot \left[\int_0^{X/Q_S} z \cdot Q_S \cdot \varphi(z)dz + \int_{X/Q_S}^{1} X \cdot \varphi(z)dz \right] - c \cdot Q_S \qquad (6)\end{aligned}$$

Thus, the profit function has the same structure as in (1) and, accordingly, from the FOC the optimal supplier decision can be derived as

$$Q_S^{WHP} \quad \text{from} \quad \int_0^{X/Q_S} z \cdot \varphi(z)dz = \frac{c}{w} \qquad (7)$$

which results in

$$Q_S^{WHP}(Y) = K_S^{WHP} \cdot X \text{ with } K_S^{WHP} > 1 \quad \text{for} \quad w > c/\mu_z. \qquad (8)$$

It turns out that also in the decentralized setting the optimal production decision results from inflating demand. In this case, however, the demand is not an external one but given by the buyer's order quantity. Additionally, the multiplier K_S^{WHP} is different from K_{SC}^* in the centralized supply chain setting. More precisely, it turns out that along with $p > w$ we find that $K_S^{WHP} < K_{SC}^*$ since from (7) K_S^{WHP} can be expressed as

$$K_{\mathrm{S}}^{\mathrm{WHP}} \quad \text{from} \quad \int_{0}^{1/K_{\mathrm{S}}} z \cdot \varphi(z) dz = \frac{c}{w}. \tag{9}$$

Inserting the optimal decision in the supplier's profit function from (6) yields

$$\Pi_{\mathrm{S}}^{\mathrm{WHP}}(Q_{\mathrm{S}}^{\mathrm{WHP}}|X) = w \cdot \left[1 - \Phi(1/K_{\mathrm{S}}^{\mathrm{WHP}})\right] \cdot X. \tag{10}$$

In case of $p \leq w$ it is obvious that $Q_{\mathrm{S}}^{\mathrm{WHP}}(X) = 0$ or, equivalently, $K_{\mathrm{S}}^{\mathrm{WHP}} = 0$ since the buyer cannot make any profit and consequently does not order anything.

Buyer's optimal decision

From the buyer's point of view, profit is random too because of an uncertain delivery quantity from the supplier which is given as $\min(z \cdot Q_{\mathrm{S}}, X)$. The buyer's expected revenue is equal to $p \cdot E\left[\min(z \cdot Q_{\mathrm{S}}^{\mathrm{WHP}}, X, D)\right]$ and thus in general depends on both, order quantity X and demand level D. For further evaluation we have to distinguish between the two cases $X \geq D$ and $X \leq D$.

I. Case $X \geq D$

In this case the buyer is ordering more than demanded by external customers to hedge against a possibly insufficiently inflated supplier's production quantity. Anticipating the supplier's reaction to what he orders the buyer's profit is the following:

$$\begin{aligned}
\Pi_{\mathrm{B}}^{\mathrm{WHP}}(X|Q_{\mathrm{S}}^{\mathrm{WHP}}) &= p \cdot E\left[\min(z \cdot Q_{\mathrm{S}}^{\mathrm{WHP}}, X, D)\right] - w \cdot E\left[\min(z \cdot Q_{\mathrm{S}}^{\mathrm{WHP}}, X)\right] \\
&= p \cdot \left[\int_{0}^{D/Q_{\mathrm{S}}^{\mathrm{WHP}}} (z \cdot Q_{\mathrm{S}}^{\mathrm{WHP}} - D) \cdot \varphi(z) dz + D\right] \\
&\quad - w \cdot \left[\int_{0}^{X/Q_{\mathrm{S}}^{\mathrm{WHP}}} (z \cdot Q_{\mathrm{S}}^{\mathrm{WHP}} - X) \cdot \varphi(z) dz + X\right].
\end{aligned}$$

As the buyer is anticipating the decision of the supplier which, specifically is $Q_{\mathrm{S}}^{\mathrm{WHP}} = K_{\mathrm{S}}^{\mathrm{WHP}} \cdot X$, this information can be used as an input to decision making and the profit function transforms to

$$\begin{aligned}
\Pi_{\mathrm{B}}^{\mathrm{WHP}}(X|Q_{\mathrm{S}}^{\mathrm{WHP}}) &= p \cdot \left[\int_{0}^{D/(K_{\mathrm{S}}^{\mathrm{WHP}} \cdot X)} (z \cdot K_{\mathrm{S}}^{\mathrm{WHP}} \cdot X - D) \cdot \varphi(z) dz + D\right] \\
&\quad - w \cdot \left[\int_{0}^{1/K_{\mathrm{S}}^{\mathrm{WHP}}} (z \cdot K_{\mathrm{S}}^{\mathrm{WHP}} \cdot X - X) \cdot \varphi(z) dz + X\right].
\end{aligned} \tag{11}$$

Exploiting the concavity of the above profit function (for proof see Appendix 1), from the FOC the buyer's optimal order quantity is

$$X^{\mathrm{WHP}} \qquad \text{from} \qquad \int_0^{D/(K_\mathrm{S}^{\mathrm{WHP}} \cdot X)} z \cdot \varphi(z) dz = \frac{c}{p} + \frac{w}{p} \cdot \frac{1 - \Phi(1/K_\mathrm{S}^{\mathrm{WHP}})}{K_\mathrm{S}^{\mathrm{WHP}}} \tag{12}$$

which results in

$$X^{\mathrm{WHP}} = K_\mathrm{B}^{\mathrm{WHP}} \cdot \frac{D}{K_\mathrm{S}^{\mathrm{WHP}}} \text{ with a multiplier } 1 < K_\mathrm{B}^{\mathrm{WHP}} < K_\mathrm{SC}^{*}. \tag{13}$$

This result means that it is optimal for the buyer to inflate the external demand with its own multiplier after deflating it with the supplier's multiplier. The buyer's multiplier $K_\mathrm{B}^{\mathrm{WHP}}$ is given from (12) as

$$K_\mathrm{B}^{\mathrm{WHP}} \quad \text{from} \quad \int_0^{1/K_\mathrm{B}} z \cdot \varphi(z) dz = \frac{c}{p} + \frac{w}{p} \cdot \frac{1 - \Phi(1/K_\mathrm{S}^{\mathrm{WHP}})}{K_\mathrm{S}^{\mathrm{WHP}}}. \tag{14}$$

The result from (14) only holds as long as $K_\mathrm{B}^{\mathrm{WHP}} > K_\mathrm{S}^{\mathrm{WHP}}$ is valid. Otherwise, according to (13) the buyer's order does not exceed his demand and the second case must be analyzed.

II. Case $X \leq D$

In this case the buyer anticipates that the supplier is inflating his order to such a high extent (above demand D) that he is better off to order at or below demand level. Under this condition his expected revenue no longer depends on the demand so that his profit function is different from the former case $X \geq D$ and can be written as

$$\Pi_\mathrm{B}^{\mathrm{WHP}}(X|Q_\mathrm{S}^{\mathrm{WHP}}) = p \cdot E\left[\min(z \cdot Q_\mathrm{S}^{\mathrm{WHP}}, X)\right] - w \cdot E\left[\min(z \cdot Q_\mathrm{S}^{\mathrm{WHP}}, X)\right].$$

After inserting $Q_\mathrm{S}^{\mathrm{WHP}} = K_\mathrm{S}^{\mathrm{WHP}} \cdot X$, the resulting profit function is

$$\Pi_\mathrm{B}^{\mathrm{WHP}}(X|Q_\mathrm{S}^{\mathrm{WHP}}) = (p-w) \cdot \left[\int_0^{1/K_\mathrm{S}^{\mathrm{WHP}}} (z \cdot K_\mathrm{S}^{\mathrm{WHP}} \cdot X - X) \cdot \varphi(z) dz + X \right], \tag{15}$$

where, in contrast to the situation in (11), the profit is a linear function of the order quantity X. Since the first-order derivative is positive due to $p > w$ (see Appendix 2) the buyer's optimal decision is to increase his order to the upper level D. So we find that in this case (as long as the profitability condition holds) the buyer's order will be equal to the external demand D.

Summarizing cases I and II, we find that the buyer will order a quantity equal to

$$X^{\mathrm{WHP}} = \begin{cases} \frac{K_{\mathrm{B}}^{\mathrm{WHP}}}{K_{\mathrm{S}}^{\mathrm{WHP}}} \cdot D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \geq K_{\mathrm{S}}^{\mathrm{WHP}} \\ D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \leq K_{\mathrm{S}}^{\mathrm{WHP}} \end{cases}, \tag{16}$$

with multipliers $K_{\mathrm{S}}^{\mathrm{WHP}}$and $K_{\mathrm{B}}^{\mathrm{WHP}}$ given from (9) and (14). With these order decisions the buyer's maximal profit in (17) can be derived as follows:

$$\begin{aligned} &\Pi_{\mathrm{B}}^{\mathrm{WHP}}(X^{\mathrm{WHP}}|Q_{\mathrm{S}}^{\mathrm{WHP}}) \\ &= \begin{cases} p \cdot \left[1 - \Phi(1/K_{\mathrm{B}}^{\mathrm{WHP}})\right] \cdot D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \geq K_{\mathrm{S}}^{\mathrm{WHP}} \\ (p - w) \cdot \left[\frac{c}{w} \cdot K_{\mathrm{S}}^{\mathrm{WHP}} + 1 - \Phi(1/K_{\mathrm{S}}^{\mathrm{WHP}})\right] \cdot D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \leq K_{\mathrm{S}}^{\mathrm{WHP}} \end{cases} \end{aligned} \tag{17}$$

Interaction of buyer and supplier decisions

Along with the supplier's decision function from (8) and the buyer's decision in (16), the following production decision within the decentralized SC setting under WHP contract turns out to be made:

$$Q_{\mathrm{S}}^{\mathrm{WHP}}(X^{\mathrm{WHP}}) = \begin{cases} K_{\mathrm{B}}^{\mathrm{WHP}} \cdot D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \geq K_{\mathrm{S}}^{\mathrm{WHP}} \\ K_{\mathrm{S}}^{\mathrm{WHP}} \cdot D & \text{if } K_{\mathrm{B}}^{\mathrm{WHP}} \leq K_{\mathrm{S}}^{\mathrm{WHP}} \end{cases} \tag{18}$$

As it is shown above that $K_{\mathrm{B}}^{\mathrm{WHP}} < K_{\mathrm{SC}}^{*}$ and $K_{\mathrm{S}}^{\mathrm{WHP}} < K_{\mathrm{SC}}^{*}$ it is obvious that $Q_{\mathrm{S}}^{\mathrm{WHP}}(X^{\mathrm{WHP}}) < Q_{\mathrm{SC}}^{*}$ will always hold under the general price/cost condition $p > w > c/\mu_z$. It follows that SC coordination is not possible when only a wholesale price is fixed in the parties' contract because this type of contract will always lead to an under-production decision. This results from the so-called double marginalization effect which states that when both parties aim for positive profits, each SC stage charges a mark-up on the cost it incurs when selling to successive stages. As a result the supplier inflates the buyer's order too low while the buyer does not compensate for this effect by raising his order sufficiently above the demand level.

From our analysis, there are two limiting cases where decentralized and centralized decision making result in the same production decision (i.e., $Q_{\mathrm{S}}^{\mathrm{WHP}}(X^{\mathrm{WHP}}) = Q_{\mathrm{SC}}^{*}$). First, when the wholesale price equals the expected production cost (i.e., $w = c/\mu_Z$), we find from (9) and (14) that $K_{\mathrm{S}}^{\mathrm{WHP}} = 1$ and $K_{\mathrm{B}}^{\mathrm{WHP}} = K_{\mathrm{SC}}^{*}$. This scenario, however, violates the business profitability condition for the supplier resulting in a supplier's unwillingness to participate in the interaction. Second, when the wholesale price equals the retail price (i.e., $w = p$), analogously we find that $K_{\mathrm{S}}^{\mathrm{WHP}} = K_{\mathrm{SC}}^{*}$ and $K_{\mathrm{B}}^{\mathrm{WHP}} < K_{\mathrm{S}}^{\mathrm{WHP}}$ so that according to (18) also in this case $Q_{\mathrm{S}}^{\mathrm{WHP}}(X^{\mathrm{WHP}}) = Q_{\mathrm{SC}}^{*}$ holds. However, this second scenario also violates the participation constraints for SC interaction since due to zero contribution margin this time the buyer will not make any profit.

Summarizing, under a WHP contract SC coordination cannot be achieved in the described random yield context. From (10) and (17) the total SC profit $\Pi_{\mathrm{SC}}^{\mathrm{WHP}}$in the case of decentralized decision making will sum up to

$$\Pi_{SC}^{WHP} = \begin{cases} \left[w \cdot \frac{K_B^{WHP}}{K_S^{WHP}} \cdot \left[1-\Phi(1/K_S^{WHP})\right] + p \cdot \left[1-\Phi(1/K_B^{WHP})\right]\right] \cdot D & \text{if } K_B^{WHP} \geq K_S^{WHP} \\ \left[(p-w) \cdot \frac{c}{w} \cdot K_S^{WHP} + p \cdot \left[1-\Phi(1/K_S^{WHP})\right]\right] \cdot D & \text{if } K_B^{WHP} \leq K_S^{WHP} \end{cases}$$

which for $p > w > c/\mu_z$ is always smaller than the SC optimal profit Π_{SC}^{*}in (5).

As described, the form of risk sharing inherent in the WHP contract does not result in a sufficiently high buyer's order and/or supplier's production volume to enable SC coordination. Next, it will be investigated if two other contract types from literature with different risk sharing properties are able to induce SC coordinating decisions.

3.2.2 Over-production risk sharing contract

The over-production risk sharing (ORS) contract ensures that, in case of random production yields, the risk of producing too many units (compared with the quantity ordered) will be shared among the two parties so that the supplier bears less risk and is motivated to respond to the buyer's order with a higher production quantity. Under this contract, the buyer commits to pay for all units produced by the supplier. While he pays the wholesale price w per unit for deliveries up to his actual order volume, quantities that exceed this amount are compensated at a lower price w_O. Thus, this contract type is characterized by two contract parameters w and w_O. In order to exclude situations where the supplier can generate unlimited profits from over-production the following parameter restrictions are set: $w_O < c/\mu_z < w$. As the supplier can salvage all units she has an incentive to produce a larger lot compared to the situation under the simple WHP contract. This increase might provide the potential to align the supplier's production decision with the SC optimal one.

In this context two contract variants have to be distinguished depending on the way a possible overproduction is handled by the parties. A first variant is characterized by the situation where the buyer just financially compensates the supplier for overproduction without accepting deliveries that exceed his order size. This Pull-ORS contract leaves him in a different risk position as when the parties agree that the supplier will deliver the whole production output irrespective of the buyer's order so that some kind of Push-ORS contract is given.

Supplier's optimal decision

The profit to optimize by the supplier is identical for both contract variants. It now also includes the rewards from over-production and is given by

$$\begin{aligned}\Pi_S^{ORS}(Q_S|X) &= w \cdot E\left[\min(z \cdot Q_S, X)\right] + w_O \cdot E\left[\max(z \cdot Q_S - X, 0)\right] - c \cdot Q_S \\ &= w \cdot \left[\int_0^{X/Q_S} z \cdot Q_S \cdot \varphi(z)dz + \int_{X/Q_S}^{1} X \cdot \varphi(z)dz\right] \\ &\quad + w_O \cdot \int_{X/Q_S}^{1} (z \cdot Q_S - X) \cdot \varphi(z)dz - c \cdot Q_S. \qquad (19)\end{aligned}$$

Given the above parameter restrictions, from the FOC (see Appendix 2 for first order derivative) the optimal production quantity can be obtained as

$$Q_S^{ORS} \quad \text{from} \quad \int_0^{X/Q_S} z \cdot \varphi(z)dz = \frac{c - w_O \cdot \mu_z}{w - w_O} \tag{20}$$

which results in

$$Q_S^{ORS} = K_S^{ORS} \cdot X \quad \text{with} \quad K_S^{ORS} > 1 \text{ and} \tag{21}$$

$$K_S^{ORS} \quad \text{from} \quad \int_0^{1/K_S} z \cdot \varphi(z)dz = \frac{c - w_O \cdot \mu_z}{w - w_O}. \tag{22}$$

Given the production policy in (21) with the multiplier in (22), the supplier's profit in (19) simplifies to

$$\Pi_S^{ORS} = (w - w_0) \cdot \left[1 - \Phi(1/K_S^{ORS})\right] \cdot X. \tag{23}$$

Buyer's optimal decision

The buyer's profit function depends on the specific type of ORS contract that is applied. Under a Pull-ORS type (exclusion of over-delivery) the buyer maximizes a profit which compared with the WHP contract is reduced by the supplier's reward for overproduced items

$$\Pi_B^{ORS}(X|Q_S^{ORS}) = p \cdot E\left[\min(z \cdot Q_S^{ORS}, X, D)\right] - w \cdot E\left[\min(z \cdot Q_S^{ORS}, X)\right]$$
$$-w_O \cdot E\left[\max(z \cdot Q_S^{ORS} - X, 0)\right]. \tag{24}$$

For evaluating this profit function we have to distinguish between the cases $X \geq D$ and $X \leq D$.

I. Case $X \geq D$

Here the buyer's profit function can be written as

$$\Pi_B^{ORS}(X|Q_S^{ORS}) = p \cdot \left[\int_0^{D/(K_S^{ORS} \cdot X)} (z \cdot K_S^{ORS} \cdot X - D) \cdot \varphi(z)dz + D\right]$$
$$-w \cdot \left[\int_0^{1/K_S^{ORS}} (z \cdot K_S^{ORS} \cdot X - X) \cdot \varphi(z)dz + X\right]$$
$$-w_O \cdot \int_{1/K_S^{ORS}}^{1} (z \cdot K_S^{ORS} \cdot X - X) \cdot \varphi(z)dz. \tag{25}$$

The buyer's optimal order decision can be derived from the FOC (see Appendix 2 for derivative) and results in a policy where the buyer's order quantity is proportional to his external demand like that for the WHP contract

$$X^{\mathrm{ORS}} = K_{\mathrm{B}}^{\mathrm{ORS}} \cdot \frac{D}{K_{\mathrm{S}}^{\mathrm{ORS}}} \text{with a multiplier } 1 < K_{\mathrm{B}}^{\mathrm{ORS}} < K_{\mathrm{SC}}^{*} \text{ and with} \tag{26}$$

$$K_{\mathrm{B}}^{\mathrm{ORS}} \text{ from } \int_{0}^{1/K_{\mathrm{B}}} z \cdot \varphi(z)dz = \frac{c}{p} + \frac{(w - w_O)}{p} \cdot \frac{(1 - \Phi(1/K_{\mathrm{S}}^{\mathrm{ORS}}))}{K_{\mathrm{S}}^{\mathrm{ORS}}}. \tag{27}$$

Note that for $w_O = 0$ the optimal decision is identical to that in the WHP contract, i.e. $K_{\mathrm{S}}^{\mathrm{ORS}} = K_{\mathrm{S}}^{\mathrm{WHP}}$ and thus, $K_{\mathrm{B}}^{\mathrm{ORS}} = K_{\mathrm{B}}^{\mathrm{WHP}}$.

This result only holds as long as $K_{\mathrm{B}}^{\mathrm{ORS}} > K_{\mathrm{S}}^{\mathrm{ORS}}$ is valid. Otherwise, the second case must be analyzed.

II. Case $X \leq D$

In this case, the demand level D is irrelevant and the buyer's profit function from (24) transforms to

$$\Pi_{\mathrm{B}}^{\mathrm{ORS}}(X|Q_{\mathrm{S}}^{\mathrm{ORS}}) = (p - w) \cdot \left[\int_{0}^{1/K_{\mathrm{S}}^{\mathrm{ORS}}} (z \cdot K_{\mathrm{S}}^{\mathrm{ORS}} \cdot X - X) \cdot \varphi(z)dz + X \right]$$
$$- w_O \cdot \int_{1/K_{\mathrm{S}}^{\mathrm{ORS}}}^{1} (z \cdot K_{\mathrm{S}}^{\mathrm{ORS}} \cdot X - X) \cdot \varphi(z)dz \tag{28}$$

which is linear in X. Inserting the integral expression from (22) the first-order derivative (see Appendix 2) is also linear in X. Thus, the buyer's optimal decision is to increase his order to the upper level D if the derivative is positive, and to order nothing if not. So, given our parameter restrictions, we find that

$$X^{\mathrm{ORS}} = D \quad \text{if} \quad p \cdot (c - w_0 \cdot \mu_Z) \geq c \cdot (w - w_0). \tag{29}$$

Interaction of buyer and supplier decisions

The investigation if the supplier-buyer interaction can result in SC coordination will be carried out separately for the two cases $X \geq D$ and $X \leq D$.

In **case I** $(X \geq D)$ the supplier's production quantity will be $Q_{\mathrm{S}}^{\mathrm{ORS}}(X^{\mathrm{ORS}}) = K_{\mathrm{S}}^{\mathrm{ORS}} \cdot X^{\mathrm{ORS}} = K_{\mathrm{B}}^{\mathrm{ORS}} \cdot D$ with $K_{\mathrm{B}}^{\mathrm{ORS}} > K_{\mathrm{S}}^{\mathrm{ORS}}$. From that it is obvious that the SC is coordinated only if $K_{\mathrm{B}}^{\mathrm{ORS}} = K_{\mathrm{SC}}^{*}$. From (27) and (4) it is easy to see that equality of multipliers is just given if the condition $(w - w_O) \cdot (1 - \Phi(1/K_{\mathrm{S}}^{\mathrm{ORS}})) = 0$ holds. This condition is only fulfilled if the following combination of parameters is given:

$$w = w_0 = c/\mu_Z. \tag{30}$$

A similar result is found in Yan et al. (2010). Since the parameter combination in (30) results in $K_{\mathrm{B}}^{\mathrm{ORS}} = K_{\mathrm{SC}}^{*} > K_{\mathrm{S}}^{\mathrm{ORS}} = 1$, it guarantees SC coordination and fulfills the condition of case I. However, inserting this parameter set in the supplier's profit function (23) makes evident that this solution is combined with a supplier's profit of zero so that the participation constraints of SC interaction are violated. Summarizing, it turns out that SC coordination cannot be achieved if the buyer orders at a level above his demand.

In **case II** $(X \leq D)$ when $X^{\mathrm{ORS}} = D$ it is obvious that SC coordination requires a supplier's multiplier that equals the SC's optimal one, i.e. $K_{\mathrm{S}}^{\mathrm{ORS}} = K_{\mathrm{SC}}^{*}$. From (22) and (4) one finds that this condition is fulfilled if the parameter equation $p \cdot (c - w_0 \cdot \mu_Z) = c \cdot (w - w_0)$ holds. Since according to (29) this equation also fulfills the condition for $X^{\mathrm{ORS}} = D$, the SC will be coordinated under this combination of contract parameters w and w_0 that can be written as

$$(w - w_0) \cdot \frac{c/\mu_Z}{(c/\mu_Z) - w_0} = p. \tag{31}$$

From (31) it follows that $w - w_0 > 0$ so that for the buyer's multiplier in (27) we get $K_{\mathrm{B}}^{\mathrm{ORS}} < K_{\mathrm{S}}^{\mathrm{ORS}} = K_{\mathrm{SC}}^{*}$. Thus, under coordinating contract parameters the condition of case II, namely $X \leq D$, is assured. The left-hand side of equation (31) can be interpreted as the overall unit price that the supplier can expect from serving the buyer when she marginally increases her production input. If this price equals the buyer's sales price, according to (2) the supplier's production quantity coincides with the SC optimal production input. Under these conditions the buyer just orders the demand size and the supplier chooses the SC optimal production level. The parameter choice combinations in (31) also enable an arbitrary split of the optimal SC profit under the two parties. From (23) it follows that the supplier's profit under coordination equals $\Pi_{\mathrm{S}}^{\mathrm{ORS}} = (w - w_0) \cdot \left[1 - \Phi(1/K_{\mathrm{SC}}^{*})\right] \cdot D$. If this profit is divided by the SC optimal profit from (5) we get a supplier's profit share, denoted by α_{S}, that is simply

$$\alpha_{\mathrm{S}}^{\mathrm{ORS}} = \frac{w - w_0}{p}. \tag{32}$$

From this ratio and parameter condition (31) we find as limiting cases where the participation constraints no longer hold, the combinations $w = p, w_0 = 0$ (with $\Pi_{\mathrm{B}}^{\mathrm{ORS}} = 0$) and $w = w_0 = c/\mu_Z$ (with $\Pi_{\mathrm{S}}^{\mathrm{ORS}} = 0$). The latter situation is identical to that which was found as condition for SC coordination in case I $(X \geq D)$.

When the SC parties agree upon a Push-ORS contract where over-production is connected with over-deliveries, the buyer is in the same situation as in case I $(X \geq D)$ since under this condition he will sell the minimum of the supplier's production output and external demand so that the profit function in (25) applies. Consequently, the outcome of the analysis for case I holds, namely that SC coordination cannot be achieved. So we see that the ability of the ORS contract to coordinate essentially depends on the specific variant of this contract type. This resembles the result that is also found in Yan and Liu (2009).

3.2.3 Penalty contract

If a penalty (PEN) contract is applied the supplier will bear a higher risk than under a simple WHP contract since she will be punished for under-delivery. The supplier is penalized by the buyer (in the amount of π) for each unit ordered that cannot be fulfilled because of insufficient production yield.

Given the potential penalty the supplier is motivated to produce more than under the simple WHP contract which again opens the chance of aligning decisions to achieve SC coordination.

Supplier's optimal decision

Under the PEN contract, the profit to optimize by the supplier in addition to the WHP contract includes the penalty for under-delivery and is given by

$$\Pi_{\mathrm{S}}^{\mathrm{PEN}}(Q_{\mathrm{S}}|X) = w \cdot E\left[\min(z \cdot Q_{\mathrm{S}}, X)\right] - \pi \cdot E\left[\max(X - z \cdot Q_{\mathrm{S}}, 0)\right] - c \cdot Q_{\mathrm{S}}. \quad (33)$$

From the FOC (see Appendix 2) the supplier's optimal production quantity can be obtained as

$$Q_{\mathrm{S}}^{\mathrm{PEN}} \quad \text{from} \quad \int_{0}^{X/Q_{\mathrm{S}}} z \cdot \varphi(z) dz = \frac{c}{w + \pi} \quad (34)$$

which results in

$$Q_{\mathrm{S}}^{\mathrm{PEN}} = K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X \quad \text{with} \quad K_{\mathrm{S}}^{\mathrm{PEN}} > 1 \text{ and} \quad (35)$$

$$K_{\mathrm{S}}^{\mathrm{PEN}} \quad \text{from} \quad \int_{0}^{1/K_{\mathrm{S}}} z \cdot \varphi(z) dz = \frac{c}{w + \pi}. \quad (36)$$

Under the production decision in (35) with the multiplier from (36) the supplier's profit in (33) simplifies to

$$\Pi_{\mathrm{S}}^{\mathrm{PEN}} = \left[w - (w + \pi) \cdot \Phi(1/K_{\mathrm{S}}^{\mathrm{PEN}})\right] \cdot X. \quad (37)$$

Buyer's optimal decision

The buyer, as Stackelberg leader, maximizes his profit which now is increased by the supplier's penalty payments for under-delivery

$$\Pi_{\mathrm{B}}^{\mathrm{PEN}}(X|Q_{\mathrm{S}}^{\mathrm{PEN}}) = p \cdot E\left[\min(z \cdot Q_{\mathrm{S}}^{\mathrm{PEN}}, X, D)\right] - w \cdot E\left[\min(z \cdot Q_{\mathrm{S}}^{\mathrm{PEN}}, X)\right]$$
$$+\pi \cdot E\left[\max(X - z \cdot Q_{\mathrm{S}}^{\mathrm{ORS}}, 0)\right]. \quad (38)$$

Like above, we have to distinguish the cases $X \geq D$ and $X \leq D$.

I. Case $X \geq D$

The buyer's profit function under anticipation of the supplier' decision can here be written as

$$\begin{aligned}\Pi_{\mathrm{B}}^{\mathrm{PEN}}(X|Q_{\mathrm{S}}^{\mathrm{PEN}}) &= p \cdot \left[\int_0^{D/(K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X)} (z \cdot K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X - D) \cdot \varphi(z) dz + D \right] \\ &\quad - w \cdot \left[\int_0^{1/K_{\mathrm{S}}^{\mathrm{PEN}}} (z \cdot K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X - X) \cdot \varphi(z) dz + X \right] \\ &\quad + \pi \cdot \int_0^{1/K_{\mathrm{S}}^{\mathrm{PEN}}} (X - z \cdot K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X) \cdot \varphi(z) dz. \end{aligned} \tag{39}$$

The buyer's optimal order decision can be derived from the FOC by using the respective derivative (see Appendix 2) and results in a policy where the buyer's order quantity is proportional to his external demand as in the WHP contract

$$X^{\mathrm{PEN}} = K_{\mathrm{B}}^{\mathrm{PEN}} \cdot \frac{D}{K_{\mathrm{S}}^{\mathrm{PEN}}} \quad \text{with a multiplier } 1 < K_{\mathrm{B}}^{\mathrm{PEN}} < K_{\mathrm{SC}}^{*} \text{ and} \tag{40}$$

$$K_{\mathrm{B}}^{\mathrm{PEN}} \quad \text{from} \quad \int_0^{1/K_{\mathrm{B}}} z \cdot \varphi(z) dz = \frac{c}{p} + \frac{w - (w + \pi) \cdot \Phi(1/K_{\mathrm{S}}^{\mathrm{PEN}})}{p \cdot K_{\mathrm{S}}^{\mathrm{PEN}}}. \tag{41}$$

Note that for $\pi = 0$ the optimal decisions from the WHP contract result as $K_{\mathrm{S}}^{\mathrm{PEN}} = K_{\mathrm{S}}^{\mathrm{WHP}}$ and thus, $K_{\mathrm{B}}^{\mathrm{PEN}} = K_{\mathrm{B}}^{\mathrm{WHP}}$.

The result for case I ($X \geq D$) only holds as long as $K_{\mathrm{B}}^{\mathrm{PEN}} > K_{\mathrm{S}}^{\mathrm{PEN}}$ is true. Otherwise, the second case must be analyzed.

II. Case $X \leq D$

In this case the buyer's profit function from (39) turns out to become linear in X :

$$\begin{aligned}&\Pi_{\mathrm{B}}^{\mathrm{PEN}}(X|Q_{\mathrm{S}}^{\mathrm{PEN}}) \\ &= (p - w - \pi) \cdot \left[\int_0^{1/K_{\mathrm{S}}^{\mathrm{PEN}}} (z \cdot K_{\mathrm{S}}^{\mathrm{PEN}} \cdot X - X) \cdot \varphi(z) dz + X \right] + (p - w) \cdot X. \end{aligned} \tag{42}$$

From the first-order derivative (see Appendix 2) the buyer's optimal decision is to increase his order to the upper level D if the derivative is positive, and to order nothing if not. So, under our general parameter restrictions, we find that

$$X^{\mathrm{PEN}} = D \quad \text{if} \quad p \geq w + \pi. \tag{43}$$

Interaction of buyer and supplier decisions

Like for the ORS contract, the supplier-buyer interaction and its impact on the potential for coordination will be carried out separately for the two cases $X \geq D$ and $X \leq D$.

In **case I** $(X \geq D)$ SC coordination can be achieved if $K_B^{PEN} = K_{SC}^*$. From (41) and (4) it is obvious that both multipliers are equal if the contract parameters fulfill the following condition

$$w = (w + \pi) \cdot \Phi(1/K_S^{PEN}) = 0. \tag{44}$$

Since this parameter combination results in $K_B^{PEN} = K_{SC}^* > K_S^{PEN} \geq K_S^{WHP}$, it guarantees SC coordination and follows the condition of case I. However, inserting the parameter combination from (44) in the supplier's profit function (37) under the PEN contract results in a profit of zero so that the participation constraints of SC interaction are violated. Summarizing, it turns out that just like under an ORS contract, SC coordination cannot be achieved if the buyer orders at a level above his demand. This is an insight that was also found by Gurnani and Gerchak (2007) where the following analysis of case II, however, is missing.

In **case II** $(X \leq D)$ when $X^{PEN} = D$, SC coordination is guaranteed if the supplier's multiplier equals the SC's optimal one, i.e. $K_S^{PEN} = K_{SC}^*$. From (36) and (4) one finds that this condition is fulfilled if the parameter equation $c/(w + \pi) = c/p$ holds. Since according to (43) this equation also fulfills the condition for $X^{PEN} = D$, the SC will be coordinated under this combination of contract parameters w and π that are simply related as follows:

$$w + \pi = p. \tag{45}$$

Like for the ORS contract condition in (31), the left-hand side term of equation (45) corresponds to the overall unit price the supplier gets (in the form of the wholesale price and saved penalty) for expected delivery to the buyer if she marginally increases her production input. If this price equals the buyer's sales price, condition (2) for global SC optimization is fulfilled. Under these circumstances coordination is achieved because the buyer just orders the demand volume and the supplier produces the SC optimal quantity. The parameter combinations in (45) also enable an arbitrary split of the optimal SC profit under the two parties. From (37) it follows that the supplier's profit under coordination equals $\Pi_S^{PEN} = \left[w - (w + \pi) \cdot \Phi(1/K_{SC}^*)\right] \cdot D$. If this profit is divided by the SC optimal profit from (5) we get a supplier's profit share α_S amounting to

$$\alpha_S^{PEN} = \frac{w - (w + \pi) \cdot \Phi(1/K_{SC}^*)}{p - p \cdot \Phi(1/K_{SC}^*)}. \tag{46}$$

From (46) in combination with (41) it is obvious that for a profit fraction $\alpha_S > 0$ the buyer's multiplier fulfills the condition $K_B^{PEN} < K_S^{PEN} = K_{SC}^*$ so that under coordination indeed case II, i.e. $X \leq D$, applies.

From the profit ratio in (46) and parameter condition in (45) we find as limiting cases, where the participation constraints of the parties no longer hold, the combinations

$w = p, \pi = 0$ (with $\Pi_B^{PEN} = 0$) and $w = p \cdot \Phi(1/K_{SC}^*), \pi = p \cdot \left[1 - \Phi(1/K_{SC}^*)\right]$ (with $\Pi_S^{PEN} = 0$). The latter case is identical to the situation we found as a condition for SC coordination in case I ($X \geq D$).

3.3 General insights

In a SC which is exposed to a supply situation with quantity uncertainty due to a production process with random yield, a specific risk situation occurs even if customer demand is deterministic. This situation is characterized by two types of risk stemming from under-production on the one hand and over-production on the other. Under conditions of central decision making both risks are coped with globally and are responded to by an optimal policy which determines the production quantity by inflating the external demand. The inflation factor or multiplier depends on the level of yield risk as well as on cost/price parameters.

In a decentralized SC with an independent supplier and buyer, both actors are exposed to the above risks. The SC interaction, however, strongly depends on how these risks are divided among the SC members. This division again depends on the type of contract that rules the terms of business. Concerning the policy structure it turns out that under all contracts and risk distributions considered a linear demand/order inflation rule is valid with contract-specific multipliers.

Under a simple WHP contract the risk distribution is such that the combination of buyer and supplier decisions does not inflate the external demand to a sufficient level to reach the SC optimal production quantity. This type of double marginalization effect holds for every wholesale price that is acceptable for the buyer and the supplier under their goal to make profits. Thus, the WHP contract fails to achieve SC coordination. In this context it is interesting that—in contrast to the corresponding SC situation with deterministic yield and random demand—the buyer-supplier interaction does not only come close to coordination if the wholesale price approaches the supplier's unit cost, but also if it gets near to the buyer's retail price. So, there exists some 'inner' wholesale price for which the coordination deficit reaches its maximum level.

SC coordination becomes possible if contracts more sophisticated than a WHP contract are installed. This results from an additional contract parameter which allows for a different sharing of risks. In particular, this is done by reducing the supplier's risk position under an ORS and by increasing it under a PEN contract. The flexibility of risk sharing built into both contracts can be used to motivate the supplier to deviate from the WHP under-production decision. Interestingly, SC coordination is only achievable if contract parameters are fixed in such a way that the buyer is not motivated to inflate his order above end customer demand. Additionally, by adequate parameter choice it is possible to generate an arbitrary split of the SC optimal profit between the two SC members. Concerning the ORS contract, however, this is only true if the contract is arranged in such a way that the over-produced items remain with the supplier so that a Pull-ORS type is exercised. If these items, notwithstanding that they were not ordered, are shipped to the buyer at the reduced price according to a Push-ORS contract, the split of SC risks changes in such a way that coordination can only be accomplished at price parameter values which reduce the supplier's profit to zero. So it turns out that an

Table 1 Closed-form results for contract specific multipliers and profits

	Supplier	Buyer
WHP contract	$K_S^{WHP} = \sqrt{w/(2 \cdot c)}$	$K_B^{WHP}/K_S^{WHP} = \sqrt{\frac{p}{(\sqrt{2 \cdot c \cdot w} - c)} \cdot \frac{c}{w}}$
	$\Pi_S^{WHP} = \left[w - \sqrt{2 \cdot c \cdot w}\right] \cdot X$	$\Pi_B^{WHP} = \left[p - \sqrt{p \cdot (\sqrt{8 \cdot c \cdot w} - 2 \cdot c)}\right] \cdot D$
Pull-ORS contract	$K_S^{ORS} = \sqrt{(w - w_0)/(2 \cdot c - w_0)}$	$K_B^{ORS}/K_S^{ORS} = 1$
	$\Pi_S^{ORS} = (w - w_0) \cdot \left[1 - \sqrt{2 \cdot c/p}\,\right] \cdot D$	$\Pi_B^{ORS} = \Pi_{SC}^* - \Pi_S^{ORS}$
PEN contract	$K_S^{PEN} = \sqrt{(w + \pi)/(2 \cdot c)}$	$K_B^{PEN}/K_S^{PEN} = 1$
	$\Pi_S^{PEN} = \left[w - (w + \pi) \cdot \sqrt{2 \cdot c/p}\,\right] \cdot D$	$\Pi_B^{PEN} = \Pi_{SC}^* - \Pi_S^{PEN}$

ORS contract needs a non-over-delivery condition to facilitate SC coordination while a PEN contract does not require this.

These insights become more evident when the results of the preceding analysis are demonstrated by means of a numerical example. To this end, we assume that the yield rate z is uniformly distributed between 0 and 1, resulting in a density $\varphi(z) = 1$, distribution $\Phi(z) = z$, and a mean yield rate of $\mu_z = 0.5$. The condition for profitability translates to $p > w > 2 \cdot c$. For this case we can derive simple closed-form expressions for all multipliers K that describe the production and order policies within the SC. So, from (4) and (5) we get

$$K_{SC}^* = \sqrt{p/(2 \cdot c)} \quad \text{and} \quad \Pi_{SC}^* = \left[p - \sqrt{2 \cdot c \cdot p}\right] \cdot D$$

as results for the optimal demand multiplier and maximum SC profit in the case of *centralized decision making*. The respective results for *decentralized decision making* under all contracts are summarized in Table 1.

The actors' profits for the Pull-ORS and PEN contract hold if the conditions for coordinating contract parameters w and w_0 from (31) and w and π from (45) are met, respectively.

When we fix demand and cost/price data to be $D = 100$, $c = 1$ and $p = 14$ the SC optimal production quantity yields $Q_{SC}^* = 265$ with a respective profit of $\Pi_{SC}^* = 871$. The Table 2 contains numerical results for all three introduced contract types for various parameter combinations.

Under the **WHP contract** (Sect. 1 in Table 2) the impact of different values of the wholesale price (in the interval $2 \cdot c \le w \le p$) is presented. The interplay of production and order sizes for different wholesale price levels becomes visible, and it can be seen how the SC loses efficiency (represented by Δ_{SC}^{WHP}, the relative deviation of the sum of supplier and buyer profits from the SC optimal profit) if this SC internal price deviates from both its minimum and maximum feasible levels. The highest efficiency loss of over 5 % occurs if the wholesale price w is fixed such that the buyer just loses

Table 2 Numerical results of all contract types for various values of the wholesale price and the respective contract parameter

(1) WHP contract							**(2) Pull-ORS contract**					**(3) PEN contract**				
w	Q_S^{WHP}	X^{WHP}	Π_S^{WHP}	Π_B^{WHP}	Δ_{SC}^{WHP}	α_S^{WHP}	w	w_o	Π_S^{ORS}	Π_B^{ORS}	α_S^{ORS}	w	π	Π_S^{PEN}	Π_B^{PEN}	α_S^{PEN}
2	265	265	0	871	0%	0%	**2**	2	0	871	0%	**2**	-	-	-	-
3	220	179	99	763	1%	11%	**3**	1,83	73	798	8%	**5,3**	8,7	0	871	0%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	**⋮**	⋮	⋮	⋮	⋮	**⋮**	⋮	⋮	⋮	⋮
6	173	100	254	569	5,5%	31%	**6**	1,33	290	581	33%	**6**	8	71	800	8%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	**⋮**	⋮	⋮	⋮	⋮	**⋮**	⋮	⋮	⋮	⋮
13	255	100	790	80	0%	91%	**13**	0,17	798	73	92%	**13**	1	771	100	89%
14	265	100	871	0	0%	100%	**14**	0	871	0	100%	**14**	0	871	0	100%

his incentive to increase his order size above the demand level of 100. As soon as the wholesale price w reaches a level that no longer motivates the buyer to order more than externally demanded, raising w further results in an increased supplier's production so that the SC coordination deficit diminishes.

For a **non-over-delivery ORS contract** with parameters w and w_0 Sect. (2) of Table 2 presents the coordinating contract parameter combinations from (31) when the wholesale price is varied in its feasible range. Under coordination the supplier's production decision equals the SC optimum, i.e. $Q_S^{ORS} = Q_{SC}^* = 265$, whereas the buyer is motivated to order exactly at demand level, i.e. $X^{ORS} = D = 100$. The results give evidence about how the total SC profit is split among buyer and supplier for different contract parameter sets.

It is illustrated that, as shown in the previous analysis, either party yields zero profit when it faces price/cost parameters of $w = w_0 = c/\mu_Z$ with $\Pi_S^{ORS} = 0$ and $w = p, w_0 = 0$ with $\Pi_B^{ORS} = 0$. Between those boundaries the profit can be split arbitrarily. Furthermore, it becomes evident that coordination is only possible if the buyer orders at the demand level and the supplier alone accounts for the yield uncertainty by equivalently raising her production quantity.

Finally, considering the **PEN contract** with parameters w and π Sect. (3) in Table 2 reveals some numerical results. As for the ORS contract, the coordinating parameter sets guarantee that the supplier produces at the SC optimal level and the buyer orders at demand level, i.e. $Q_S^{PEN} = Q_{SC}^* = 265$ and $X^{PEN} = D = 100$, respectively. The above results can be used to demonstrate how the profit split depends on choice of the wholesale price and the coordinating penalty charge.

Different from the ORS contract, a wholesale price $w < 5.3$ is not feasible, since according to condition (46) this would result in a negative profit for the supplier. Similar to the results of the ORS contract, the parties yield zero profits at the limiting parameter combinations of w and π: (i) $w = p \cdot \Phi(1/K_{SC}^*), \pi = p \cdot \left[1 - \Phi(1/K_{SC}^*)\right]$ resulting in $\Pi_S^{PEN} = 0$ and (ii) $w = p, \pi = 0$ with $\Pi_B^{PEN} = 0$. In between these boundaries an arbitrary profit split is possible. Again, it becomes evident that coordination is only enabled if the buyer orders according to the demand level and the supplier appropriately raises the production quantity to account for the yield uncertainty.

In the next section it is evaluated to which extent the existence of an emergency source for procurement will influence the SC decisions and interaction in connection with yield uncertainty.

4 Contract analyses with emergency procurement

In this section, we deviate from the assumption that only a single opportunity exists to supply the quantities demanded by the buyer. Instead, we consider the case that in addition to regular production with random yield a second source of supply exists which can be used after realization of production yields. As mentioned previously, this emergency source is not subject to risk or capacity restrictions. To restrict the analysis to meaningful sourcing problems, the unit cost of this option, denoted by c_E, is assumed to be higher than the expected cost of regular production, i.e. $c_E > c/\mu_Z$.

We will consider two different situations, namely (i) the case where the supplier has the emergency option at her disposal and (ii) the case where this procurement source is under the control of the buyer. In both cases we face a sequential procurement decision. First, a regular production quantity is determined and then, after yield realization, an emergency procurement decision is made. As a result, under-production can be compensated, and external demand can be fulfilled completely. In this section's problem setting, the relevant decisions and profits are indicated by a hat ($\hat{Q}$, $\hat{X}$, $\hat{\Pi}$).

4.1 Centralized decision making

The centralized decision maker in this sequential decision making process first decides on the production input quantity $\hat{Q}_{\text{SC}}$ and then on the amount to be procured from the emergency source, denoted by $\hat{Q}^E_{\text{SC}}$. As long as the retail price is sufficiently high, namely $p > c_E$, the optimal emergency decision will always be to increase the available production output to the demand level if necessary, i.e. $\hat{Q}^E_{\text{SC}} = \max(D - z \cdot \hat{Q}_{\text{SC}}, 0)$. Anticipating this second-step quantity adjustment, the decision regarding regular production is made in order to maximize the following profit function $\hat{\Pi}_{\text{SC}}$ which accounts for fulfillment of total demand D

$$\hat{\Pi}_{\text{SC}}(\hat{Q}_{\text{SC}}) = p \cdot D - c \cdot \hat{Q}_{\text{SC}} - c_E \cdot E\left[\max(D - z \cdot \hat{Q}_{\text{SC}}, 0)\right]. \tag{47}$$

The profit maximizing production input quantity can then be derived from the FOC as

$$\hat{Q}^*_{\text{SC}} \quad \text{from} \quad \int_0^{D/\hat{Q}^{SC}} z \cdot \varphi(z) dz = \frac{c}{c_E} \quad \text{so that}$$

$$\hat{Q}^*_{\text{SC}} = \hat{K}^*_{\text{SC}} \cdot D \quad \text{with} \quad \hat{K}^*_{\text{SC}} > 1 \quad \text{and} \tag{48}$$

$$\hat{K}^*_{\text{SC}} \quad \text{from} \quad \int\limits_0^{1/\hat{K}_{\text{SC}}} z \cdot \varphi(z)dz = \frac{c}{c_E}. \tag{49}$$

In case of $c_E < c/\mu_z$, the regular production quantity would obviously be equal to zero, and only the emergency source would be utilized, as alluded to previously.

The first interesting result from this analysis is that the SC optimal production input quantity is not dependent on the retail price anymore, given that $p > c_E$ holds. This reflects the fact that under these circumstances the problem under yield risk turns into a problem with cost uncertainty. But still, demand is inflated by a factor which now depends solely on the costs for regular production and emergency procurement. Having analyzed the centralized SC problem, this result can again be used as a benchmark for decentralized decision making in a SC. For the decentralized case we will restrict our analysis to the WHP contract and show that this basic contract type has completely different coordination properties depending on who in the SC has access to the emergency option.

4.2 Decentralized decision making under a supplier's emergency source

If the supplier has the opportunity to use an emergency procurement source, she first chooses her production input level and then the emergency procurement quantity, if necessary. Under the simple WHP contract, given the buyer's order $\hat{X}$ emergency procurement will be $\hat{Q}^E_{\text{S}} = \max(\hat{X} - z \cdot \hat{Q}_{\text{S}}, 0)$ if the wholesale price permits profitability for the supplier, i.e. if $c_E < w$. Prior to utilizing the reliable source, the supplier chooses the quantity for the less expensive regular production process. Here, the profit to maximize for the supplier is

$$\hat{\Pi}^{\text{WHP}}_{\text{S}}(\hat{Q}_{\text{S}} \left| \hat{X} \right.) = w \cdot \hat{X} - c \cdot \hat{Q}_{\text{S}} - c_E \cdot E\left[\max(\hat{X} - z \cdot \hat{Q}_{\text{S}}, 0)\right]. \tag{50}$$

The optimal decision with respect to regular production is then derived from the FOC as

$$\hat{Q}^{\text{WHP}}_{\text{S}} \qquad \text{from} \qquad \int\limits_0^{X/\hat{Q}_{\text{S}}} z \cdot \varphi(z)dz = \frac{c}{c_E}, \quad \text{resulting in a linear production policy}$$

$\hat{Q}^{\text{WHP}}_{\text{S}} = \hat{K}^{\text{WHP}}_{\text{S}} \cdot \hat{X}$ with a multiplier $\hat{K}^{\text{WHP}}_{\text{S}} = \hat{K}^*_{\text{SC}}$ being equal to that of the centralized solution in (49). In contrast to the case without emergency procurement, the optimal production input choice of the supplier is independent of the wholesale price and identical to the SC optimal production quantity if the buyer's order $\hat{X}$ equals the external demand D.

This turns out to be just the case if an emergency procurement option exists which is profitable for the supplier. Due to the fact that the supplier will procure missing quantities after regular production, the delivery quantity received by the buyer is not random anymore. Under the WHP contract, this motivates the buyer to order exactly the demand size in order to maximize his profit, i.e. $\hat{X}^{\text{WHP}} = D$. Now, it is clear that $\hat{Q}_{\text{S}}^{\text{WHP}} = \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X}^{\text{WHP}} = \hat{K}_{\text{SC}}^{*} \cdot D$ which is exactly the SC optimal decision from (48). Accordingly, the decisions on emergency procurement are also identical (i.e. $\hat{Q}_{\text{S}}^{E} = \hat{Q}_{\text{SC}}^{E}$). From the production decision in (48) and the multiplier in (49) the profit of the supplier results in

$$\hat{\Pi}_{\text{S}}^{\text{WHP}} = \left[w - c_E \cdot \Phi(1/\hat{K}_{\text{SC}}^{*}) \right] \cdot D. \tag{51}$$

Thus, we come to the interesting conclusion that the simple WHP contract is sufficient to guarantee SC coordination if a non-random profitable emergency procurement option exists for the supplier. The reason behind this outcome is the fact that in a setting with a supplier's emergency source the SC risk sharing is such that the buyer bears no risk at all and the complete quantity and cost risk is with the supplier. The analyses provided by He and Zhao (2012) and Ma et al. (2012) for the case of additional demand randomness confirm this result.

From the above analysis it is evident that the emergency procurement option will only be exercised if its unit cost does not exceed the respective sales price. For $c_E > p > w$, utilizing the emergency source is not profitable for the supplier and the whole SC so that the resulting situation is identical to that without emergency procurement where a simple WHP contract will not enable SC coordination. Nevertheless, it can be shown that it might make sense to utilize an emergency procurement opportunity in case of $c_E > p > w$ when the SC actors abstain from complex multi-parameter contracts and only agree on implementing a simple WHP contract. In this context, we consider a SC situation where the supplier is fixing a wholesale price under anticipation of a fixed minimum profit requirement by the buyer. The respective contract type where the supplier applies this emergency option (despite of $c_E > p$) will be denoted a WHP+ contract.

Under decentralized decision making, a benefit can arise from utilizing the emergency option even under $c_E > p$ if the SC profit deficit resulting from individual optimization under the WHP contract without emergency procurement can be reduced. Such a reduction is always possible if the emergency cost c_E does not exceed the retail price p by too large a margin such that the loss in SC efficiency caused by diseconomies from emergency procurement is smaller than the SC deficit from implementing a simple WHP contract. If the supplier chooses her wholesale price such that the potential deficit reduction by using the emergency option is shared among the SC actors, a Pareto improvement (as a second best solution) can be achieved, i.e. the interaction can improve all parties' profits without being first best. However, the profitability of the interaction as well as the distribution of profits depends on the parameter setting applied.

Under a yield rate distribution and parameter combination as for Table 2 in Sect. 3, for instance, it turns out that for $c_E = 15$ and $p = 14$ (i.e., $c_E > p$) the total SC profit under a WHP+ contract is 3.5 % higher than under a simple WHP contract with a price of $w = 6$. By appropriately fixing the wholesale price in the WHP+ contract, this extra profit can be shared by both parties so that a win–win situation can be enabled (see Appendix 3 for detailed numerical results).

4.3 Decentralized decision making under a buyer's emergency source

The WHP contract consequences are completely different when the buyer, as opposed to the supplier, has an emergency option at his disposal. Then he bears a cost risk as he will sequentially decide upon the order quantity and, after receipt of the supplier's delivery quantity, upon a potential emergency order. In contrast to the case where the emergency source is available to the supplier, the necessary cost condition now is that the emergency cost must be larger than the wholesale price (i.e. $c_E > w$); otherwise, the buyer would not order from the unreliable supplier at all but procure all units from the emergency source at a lower price.

Under a buyer's emergency source, the supplier is in a position like in the WHP case without emergency procurement and produces according to (8). Thus, the production decision is affected by the wholesale price w in form of

$$\hat{Q}_{\text{S}}^{\text{WHP}}(\hat{X}) = \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X} \quad \text{with} \quad \hat{K}_{\text{S}}^{\text{WHP}} > 1 \quad \text{from} \quad \int\limits_{0}^{1/\hat{K}_{\text{S}}^{\text{WHP}}} z \cdot \varphi(z) dz = \frac{c}{w}. \tag{52}$$

Due to the cost condition $c_E > w$ the buyer will never order below the demand level because missing units needed to satisfy end customer demand would have to be procured at a higher cost from the emergency source, i.e. the case $\hat{X} \leq D$ is not relevant. Therefore, only the case of $\hat{X} \geq D$ has to be considered which accounts for complete fulfillment of demand D and results in the buyer's profit to be

$$\begin{aligned} \hat{\Pi}_{\text{B}}^{\text{WHP}}(\hat{X}|\hat{Q}_{\text{S}}^{\text{WHP}}) = p \cdot D - w \cdot E\left[\min(z \cdot \hat{Q}_{\text{S}}^{\text{WHP}}, \hat{X})\right] \\ -c_E \cdot E\left[\max(D - z \cdot \hat{Q}_{\text{S}}^{\text{WHP}}, 0)\right]. \end{aligned} \tag{53}$$

After inserting $\hat{Q}_{\text{S}}^{\text{WHP}} = \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X}$, the profit function transforms to

$$\begin{aligned} \hat{\Pi}_{\text{B}}^{\text{WHP}}(\hat{X}|\hat{Q}_{\text{S}}^{\text{WHP}}) = p \cdot D - w \cdot \int\limits_{0}^{1/\hat{K}_{\text{S}}^{\text{WHP}}} z \cdot \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X} \cdot \varphi(z) dz - w \cdot \int\limits_{1/\hat{K}_{\text{S}}^{\text{WHP}}}^{1} \hat{X} \cdot \varphi(z) dz \\ -c_E \cdot \int\limits_{0}^{D/(\hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X})} (D - z \cdot \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X}) \cdot \varphi(z) dz. \end{aligned}$$

Exploiting the concavity of the above profit function, (for the first order derivative see Appendix 2) the buyer's optimal order quantity results in

$$\hat{X}^{\text{WHP}} = \hat{K}_{\text{B}}^{\text{WHP}} \cdot \frac{D}{\hat{K}_{\text{S}}^{\text{WHP}}} \quad \text{with a multiplier } \hat{K}_{\text{B}}^{\text{WHP}} \quad \text{that fulfills } 1 < \hat{K}_{\text{B}}^{\text{WHP}} < \hat{K}_{\text{SC}}^{*}$$

$$\text{and is calculated from} \int_{0}^{1/\hat{K}_{\text{B}}^{\text{WHP}}} z \cdot \varphi(z) dz = \frac{c}{c_E} + \frac{w}{c_E} \cdot \frac{1 - \Phi(1/\hat{K}_{\text{S}}^{\text{WHP}})}{\hat{K}_{\text{S}}^{\text{WHP}}}. \tag{54}$$

Inserting the buyer's order quantity into the supplier's production decision yields

$$\hat{Q}_{\text{S}}^{\text{WHP}}(\hat{X}^{\text{WHP}}) = \hat{K}_{\text{B}}^{\text{WHP}} \cdot D < \hat{Q}_{\text{SC}}^{*} \quad (\text{due to } \hat{K}_{\text{B}}^{\text{WHP}} < \hat{K}_{\text{SC}}^{*}).$$

Thus, it turns out that the WHP contract loses its coordinating property if the emergency procurement option is on the buyer's instead of the supplier's side. This is a consequence of the fact that under these circumstances the buyer has to take over some SC risk in form of the price risk from exercising the emergency option. Furthermore, it is evident that in a decentralized setting under a buyer's emergency source it will never make sense to utilize emergency procurement within a WHP contract if $c_E > p$.

4.4 General insights

It is surprising to realize that the mere existence of a second procurement option for the supplier without yield randomness makes the design of sophisticated contracts for facilitating SC coordination unnecessary. This, at least, holds as long as the emergency option is economically viable for the SC or the supplier, respectively. The reason behind this result is that the double marginalization effect vanishes as both actors' decisions do not depend on the wholesale price. It is interesting to see that, like under the coordinating ORS and PEN contract, SC coordination is achieved along with contract conditions that generate an incentive for the buyer to order exactly the firm demand size, thus preventing a distortion of customer demand information.

The coordinating property of the WHP contract in case of reliable emergency procurement relies on its ability to shift the whole SC risk to the supplier who is in a position to guarantee complete delivery of the buyer's order volume. This can even be exploited if the emergency option is not economically attractive for the SC. Utilizing emergency procurement in this case can help the supplier to offer a contract which is beneficial for all parties compared with a WHP contract with a single (unreliable) procurement source.

Interestingly, the option of emergency procurement no longer enables SC coordination if the buyer is the SC actor who has this option at hand. In this case the cost risk is shifted to the buyer, and the wholesale price matters for SC decisions so that a double marginalization effect occurs. So it is evident that it essentially matters for SC coordination which one of the SC actors has access to an emergency procurement option. Summarizing, we learn that in general an emergency procurement option enlarges the decision space and profitability in a SC. From an overall SC perspective, however, it is more advantageous that this option is available to the supplier than to the buyer. Furthermore, it turns out that even in the case of a supplier-owned emergency option the coordination property of the WHP contract is closely associated with the existence of certainty regarding external customer demand. If in addition to production yield also demand is random, it is easy to show (without going into details) that a WHP contract loses its coordinating power even if an emergency procurement option exists. If demand is stochastic with $cdf\, F(\cdot)$, the buyer's decision problem in a decentralized setting is completely identical to that of a simple newsvendor so that $\hat{X}^{\text{WHP}} = F^{-1}[(p-w)/p]$ holds. The supplier herself has to solve exactly the problem described in Sect. 4.2 so that the production decision equals $\hat{Q}_{\text{S}}^{\text{WHP}} = \hat{K}_{\text{S}}^{\text{WHP}} \cdot \hat{X}^{\text{WHP}}$ with $\hat{K}_{\text{S}}^{\text{WHP}}$ from $\int_0^{1/\hat{K}_{\text{S}}} z \cdot \varphi(z) dz = c/c_E$ [for a detailed analysis see He and Zhang 2008]. From this interaction it becomes clear that under a WHP contract the buyer's order size depends on the wholesale price, the same holds for the supplier's production quantity. Since under centralized planning the optimal production decision will not be influenced by the (SC internal) wholesale price it is evident that $\hat{Q}_{\text{SC}}^{*}$ and $\hat{Q}_{\text{S}}^{\text{WHP}}$ will not coincide. As a consequence, the coordinating property of the WHP contract with emergency procurement option will not hold in the case of stochastic customer demand.

5 Conclusion and further research

The previous analysis revealed interesting insights into the field of SC coordination through contracts in the case of random production yields and deterministic demand. Without an option to utilize an emergency source the simple WHP contract fails to coordinate the SC due to the double marginalization effect. Interestingly, for the considered ORS contract it depends on the definition of the contract whether coordination can be achieved or not. Under a Pull-variant of the contract (no over-deliveries) coordination is possible and an arbitrary profit split can be generated. In contrast, under a Push-ORS contract (with over-deliveries), coordination cannot be enabled as the coordinating parameter setting violates the supplier's participation constraint.

For the PEN contract, however, it can be shown that the design enables SC coordination and, depending on the parameter setting, guarantees an arbitrary distribution of profits among the actors. For both contract types, Pull-ORS and PEN, it has been illustrated that only in cases where the buyer orders exactly at demand level coordination is achieved.

The situation is different when an emergency source with perfect yield is available to the supplier. Here, the WHP contract can achieve coordination as the buyer always orders what is demanded and the supplier's production decision does not depend on the wholesale price. Under these circumstances, it is not necessary to employ more complex contracts because the simple WHP contract suffices. Things are different, however, if the emergency source is available to the buyer.

As coordination can be achieved by the analyzed contracts in the considered SC settings, further research should focus on the question to which extent the above results carry over to modified settings. An important aspect in this context is the extension from a serial to a converging SC. As mentioned in Sect. 1, for settings with such a SC structure as well as with stochastic customer demand some limited results are available from the literature. Concerning the effect of demand randomness appearing in addition to yield risk, we know from Yan and Liu (2009) that both the ORS and PEN contact are not able to coordinate under general conditions. Only more complex contract types have a coordination property in this case. This might be due to the fact that only contracts which under random yield induce the buyer to order exactly the demand quantity turned out to guarantee supply chain coordination with an ORS or PEN contract as our analysis revealed. Under stochastic demand this buyer's behavior naturally cannot be achieved. For the case of a supplier's emergency source it was already discussed in Sect. 4 the WHP contract will no longer coordinate if demand is uncertain. In this case we face a newsvendor problem, and the well-studied contracts that coordinate in this context will apply (see Cachon 2003).

Almost no research is available for different types of yield processes that could be addressed like an all-or-nothing type of yield realization, also known as disruption risk (see Xia et al. 2011), which can be seen as a special case of a stochastically proportional yield model. Furthermore, the assumption of stochastically proportional yield itself might be questioned. In some cases it is more realistic to suppose that a binomial yield process applies (see Yano and Lee 1995). A further promising field for future research would be to investigate if the theoretical results from optimizing supplier's and buyer's SC decisions coincide with real-world behavior. In other fields of SC management there has been much attention paid to behavioral operations (see Bendoly et al. 2010). Up to now, Gurnani et al. (2011) are the only ones to present insights into actual human behavior in stochastic yield scenarios by experimental research. SC interaction under random yield, however, has not yet been investigated by this type of research.

Appendix 1

Proof of concavity of $\Pi_{SC}(Q_{SC})$ in (1):[1]

For the first-order and second-order derivative of $\Pi_{SC}(Q_{SC})$ we get

$$\frac{\partial}{\partial Q_{SC}}\Pi_{SC}(Q_{SC}) = p \cdot \int_0^{D/Q_{SC}} z \cdot \varphi(z)dz - c \quad \text{and}$$

$$\frac{\partial^2}{\partial Q_{SC}^2}\Pi_{SC}(Q_{SC}) = -p \cdot \frac{D^2}{Q_{SC}^3} \cdot \varphi\left(\frac{D}{Q_{SC}}\right) < 0.$$

Thus, Π_{SC} is concave in Q_{SC}. Setting the first-order derivative equal to zero, i.e. $\partial \Pi_{SC}/\partial Q_{SC} = 0$, equation (2) can immediately be derived.
Proof of concavity of $\Pi_B^{WHP}(X|Q_S^{WHP})$ in (11):[2]

[1] The profit function structure and the way of proving concavity and exploiting the FOC is identical for supplier's profit functions under all following contracts. For that reason the respective derivations are not repeated hereafter.

[2] The profit function structure and the way of proving concavity and exploiting the FOC is identical for buyer's profit functions under all following contracts. For that reason the respective derivations are not further repeated.

For the first-order and second-order derivative of $\Pi_B^{WHP}(X|Q_S^{WHP})$ we get $\frac{\partial}{\partial X}\Pi_B^{WHP}(X|Q_S^{WHP}) = p \cdot K_S^{WHP} \cdot \int_0^{D/(K_S^{WHP} \cdot X)} z \cdot \varphi(z)dz - w \cdot \left[K_S^{WHP} \cdot \int_0^{1/K_S^{WHP}} z \cdot \varphi(z)dz - \Phi(1/K_S^{WHP}) + 1\right]$ and
$\frac{\partial^2}{\partial X^2}\Pi_B^{WHP}(X|Q_S^{WHP}) = -p \cdot \frac{D^2}{(K_S^{WHP})^2 \cdot X^3} \cdot \varphi(\frac{D}{K_S^{WHP} \cdot X}) < 0$.
Thus, $\Pi_B^{WHP}(X|Q_S^{WHP})$is concave in X. Setting the first-order derivative equal to zero, i.e. $\frac{\partial}{\partial X}\Pi_B^{WHP}(X|Q_S^{WHP}) = 0$, and exploiting $\int_0^{1/K_S^{WHP}} z \cdot \varphi(z)dz = \frac{c}{w}$ from equation (9) yields the respective optimality condition for X^{WHP} in (12).

Appendix 2

First-order derivatives to following profit functions:

(15) $$\frac{\partial}{\partial X}\Pi_B^{WHP}(X|Q_S^{WHP}) = (p - w) \cdot \left[\frac{c}{w} \cdot K_S^{WHP} + 1 - \Phi(1/K_S^{WHP})\right]$$

(19) $$\frac{\partial}{\partial Q_S}\Pi_S^{ORS}(Q_S|X) = (w - w_O) \cdot \int_0^{X/Q_S} z \cdot \varphi(z)dz + w_O \cdot \mu_z - c$$

(25) $$\begin{aligned}\frac{\partial}{\partial X}\Pi_B^{ORS}(X|Q_S^{ORS}) = {} & p \cdot K_S^{ORS} \cdot \int_0^{D/(K_S^{ORS} \cdot X)} z \cdot \varphi(z)dz \\ & -(w - w_O) \cdot \left[(1 - \Phi(1/K_S^{ORS})) + K_S^{ORS} \cdot \int_0^{1/K_S^{ORS}} z \cdot \varphi(z)dz\right] \\ & -w_O \cdot \mu_z \cdot K_S^{ORS}\end{aligned}$$

(28) $$\begin{aligned}\frac{\partial}{\partial X}\Pi_B^{ORS}(X|Q_S^{ORS}) = {} & \left[\frac{p \cdot (c - w_0 \cdot \mu_Z)}{w - w_0} - c\right] \cdot K_S^{ORS} \\ & +(p - w + w_O) \cdot [1 - \Phi(1/K_S^{ORS})]\end{aligned}$$

(33) $$\frac{\partial}{\partial Q_S}\Pi_S^{PEN}(Q_S|X) = (w + \pi) \cdot \int_0^{X/Q_S} z \cdot \varphi(z)dz - c$$

(39) $$\begin{aligned}\frac{\partial}{\partial X}\Pi_B^{PEN}(X|Q_S^{PEN}) = {} & p \cdot K_S^{PEN} \cdot \int_0^{D/(K_S^{PEN} \cdot X)} z \cdot \varphi(z)dz \\ & -\left[w + c \cdot K_S^{PEN} - (w + \pi) \cdot \Phi(1/K_S^{PEN})\right]\end{aligned}$$

(43) $$\begin{aligned}\frac{\partial}{\partial X}\Pi_B^{PEN}(X|Q_S^{PEN}) = {} & (p - w) \cdot [1 - \Phi(1/K_S^{PEN})] \\ & + \left[\frac{(p - w - \pi) \cdot c}{w + \pi}\right] \cdot K_S^{PEN} + \pi \cdot \Phi(1/K_S^{PEN})\end{aligned}$$

$$(53)\quad \frac{\partial}{\partial \hat{X}} \hat{\Pi}_{\mathrm{B}}^{\mathrm{WHP}}(\hat{X}|\hat{Q}_{\mathrm{S}}^{\mathrm{WHP}}) = -w \cdot \hat{K}_{\mathrm{S}}^{\mathrm{WHP}} \cdot \int_0^{1/\hat{K}_{\mathrm{S}}^{\mathrm{WHP}}} z \cdot \varphi(z)dz$$

$$-w \cdot \left[1 - \Phi(1/\hat{K}_{\mathrm{S}}^{\mathrm{WHP}})\right] + c_E \cdot \hat{K}_{\mathrm{S}}^{\mathrm{WHP}} \cdot \int_0^{D/(\hat{K}_{\mathrm{S}}^{\mathrm{WHP}} \cdot \hat{X})} z \cdot \varphi(z)dz$$

Appendix 3

The described potential for improving SC coordination under a WHP contact even for $c_E > p$ will be illustrated by a numerical example. In order to exploit closed-form solutions we again assume a uniformly distributed yield rate in [0,1] with mean $\mu_z = 0.5$ like in Sect. 3.

Under *centralized decision making* the SC optimal demand multiplier and profit are given as

$$\hat{K}_{\mathrm{SC}}^{*} = \sqrt{c_E/(2 \cdot c)} \quad \text{and} \quad \hat{\Pi}_{\mathrm{SC}}^{*} = \left[p - \sqrt{2 \cdot c \cdot c_E}\right] \cdot D, \text{ respectively.}$$

When decision making is *decentralized* and the parties agree on the WHP contract, the supplier's multiplier and profit are

$$\hat{K}_{\mathrm{S}}^{\mathrm{WHP}} = \sqrt{c_E/(2 \cdot c)} = \hat{K}_{\mathrm{SC}}^{*} \quad \text{and} \quad \hat{\Pi}_{\mathrm{S}}^{\mathrm{WHP}} = \left[w - \sqrt{2 \cdot c \cdot c_E}\right] \cdot D, \text{ respectively.} \tag{55}$$

As coordination is achieved the buyer's profit is simply

$$\hat{\Pi}_{\mathrm{B}}^{\mathrm{WHP}} = \hat{\Pi}_{\mathrm{SC}}^{*} - \hat{\Pi}_{\mathrm{S}}^{\mathrm{WHP}}.$$

In Table 3, we provide a numerical example to illustrate the development of profits with increasing values for the emergency cost c_E. Assuming demand and price/cost data as for Table 2 in Sect. 3.3, we proceed from the parameter combination which yielded the largest profit deficit under the WHP contract without emergency option (i.e. $w = 6$). From Table 2, recall the accompanying profits of $\Pi_{\mathrm{S}}^{\mathrm{WHP}} = 254$ and $\Pi_{\mathrm{B}}^{\mathrm{WHP}} = 569$ which result in a SC deficit of $\Pi_{\mathrm{SC}}^{*} - (\Pi_{\mathrm{S}}^{\mathrm{WHP}} + \Pi_{\mathrm{B}}^{\mathrm{WHP}}) = 871 - 823 = 48$.

Table 3 now displays the results emerging from a supplier's offer of a WHP contract under emergency procurement with different c_E cost levels where the buyer will be guaranteed to receive complete delivery of all ordered units so that his order equals external demand, i.e. $\hat{X}^{\mathrm{WHP}} = D = 100$.

The results in Table 3 first illustrate that for an emergency cost c_E which is smaller than the retail price $p = 14$, the SC profit is always higher than the sum of profits under the WHP contract without emergency production, which from Table 1 equals 823 for $w = 6$. Second, it turns out that, even if the retail price is exceeded (i.e. $c_E > 14$), utilizing the emergency option can be reasonable. For $c_E = 15$ and $c_E = 16$ the SC faces a reduction in the maximal profit $\Pi_{\mathrm{SC}}^{*} = 871$ without emergency procurement. This loss, however, is smaller than the coordination deficit of

Table 3 Impact of emergency cost values on profits

c_E	$\hat{Q}_S^{WHP} = \hat{Q}_{SC}^*$	$\hat{X}^{WHP} = D$	$\hat{\Pi}_S^{WHP}$	$\hat{\Pi}_B^{WHP}$	$\hat{\Pi}_S^{WHP} + \hat{\Pi}_B^{WHP} = \hat{\Pi}_{SC}^*$	$\hat{\alpha}_S^{WHP}$ (%)
6	173	100	254	800	1.054	24
7	187	100	226	800	1.026	22
8	200	100	200	800	1.000	20
⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	265	100	71	800	871	8
15	274	100	52	800	852	6
16	283	100	34	800	834	4
17	292	100	17	800	817	2

Table 4 Profit split for different WHP values under emergency option

w	$\hat{Q}_S^{WHP} = \hat{Q}_{SC}^*$	$\hat{X}^{WHP} = D$	$\hat{\Pi}_S^{WHP}$	$\hat{\Pi}_B^{WHP}$	$\hat{\Pi}_S^{WHP} + \hat{\Pi}_B^{WHP} = \hat{\Pi}_{SC}^*$	$\hat{\alpha}_S^{WHP}$ (%)
8.10	274	100	262	590	852	31
8.20	274	100	272	580	852	32
8.30	274	100	282	570	852	33

48. Hence, from a SC perspective it is profitable to utilize emergency procurement even if the respective cost exceeds the price gained per unit. Furthermore, it can be shown that exercising the emergency option is not profitable for the supplier under all circumstances. From (55) it follows that the supplier's profit will only be positive if $c_E < w^2/(2 \cdot c)$ holds. This means that, if $c_E \geq 18$ in our numerical example, the supplier's participation constraint no longer holds as $\hat{\Pi}_S^{WHP} \leq 0$.

Nevertheless, the results also show that for some cases the supplier is worse off than under the WHP contract without emergency production while the buyer always benefits. In order to guarantee beneficial profit sharing for all parties, contract terms can be formulated appropriately. If the supplier is in the position to determine the wholesale price and has to guarantee the buyer a minimum profit of, e.g. 569, parameter combinations can be found which assure higher profits for all parties compared with the situation without emergency production. For $p = 14$ and $c_E = 15$ (i.e. $c_E > p = 14$), Table 4 illustrates under which wholesale prices both buyer and supplier benefit from the emergency option and how the profit is split among the actors.

Note that from Table 4 we find that the starting point for our consideration (based on $w = 6$) is $\Pi_S^{WHP} = 254$, $\Pi_B^{WHP} = 569$ and $\Pi_{SC}^{WHP} = 823$. Thus, Table 4 describes contract terms that result in a win–win situation.

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3. Supply chain coordination by contracts under binomial production yield

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ORIGINAL RESEARCH

Supply chain coordination by contracts under binomial production yield

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Abstract Supply chain coordination is enabled by adequately designed contracts so that decision making by multiple actors avoids efficiency losses in the supply chain. From the literature it is known that in newsvendor-type settings with random demand and deterministic supply the activities in supply chains can be coordinated by sophisticated contracts while the simple wholesale price contract fails to achieve coordination due to the double marginalization effect. Advanced contracts are typically characterized by risk sharing mechanisms between the actors, which have the potential to coordinate the supply chain. Regarding the opposite setting with random supply and deterministic demand, literature offers a considerably smaller spectrum of solution schemes. While contract types for the well-known stochastically proportional yield have been analyzed under different settings, other yield distributions have not received much attention in the literature so far. However, practice shows that yield types strongly depend on the industry and the production process that is considered. As consequence, they can deviate very much from the specific case of a stochastically proportional yield. This paper analyzes a buyer–supplier supply chain in a random yield, deterministic demand setting with production yield of a binomial type. It is shown how under binomially distributed yields risk sharing contracts can be used to coordinate buyer's ordering and supplier's production decision. Both parties are exposed to risks of overproduction and under-delivery. In contrast to settings with stochastically proportional yield, however, the impact of yield uncertainty can be quite different in the binomial yield case. Under binomial yield, the output uncertainty decreases with larger production quantities while it is independent from lot sizes under stochastically proportional yield. Consequently, the results from previous contract analyses on other yield types

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may not hold any longer. The current analytical study reveals that, like under stochastically proportional yield, coordination is impeded by double marginalization if a simple wholesale price contract is applied. However, more sophisticated contracts which penalize or reward the supplier can change the risk distribution so that supply chain coordination is possible also under binomial yield. In this context, many contract properties from planning under stochastically proportional yield carry over. Nevertheless, numerical examples reveal that a misspecification of the yield type can considerably downgrade the extent of supply chain coordination.

Keywords Supply chain coordination · Contracts · Binomial yield · Risk sharing

1 Introduction

Uncertainties are widely spread in supply chains with demand and supply uncertainties being the most common types. Regarding the supply side, business risks primarily result from yield uncertainty which is typical for a variety of business sectors. It frequently occurs in the agricultural sector or in the chemical, electronic and mechanical manufacturing industries (see Gurnani et al. 2000; Jones et al. 2001; Kazaz 2004; Nahmias 2009). Here, random supply can appear due to different reasons such as weather conditions, production process risks or imperfect input material. In a supply chain context, yield or supply randomness obviously influences the risk position of the actors and, therefore, has an effect on the buyer–supplier relationship in a supply chain. The question that arises is to what extent random yields affect the decisions of the single supply chain actors and the performance of the whole supply chain. In this study, we limit ourselves to a problem setting with deterministic demand. This is to focus the risk analysis of contracting on the random yield aspect which is of practical relevance for production planning in some industries (see Bassok et al. 2002). Except for papers that address disruption risks (e.g., Asian 2014; Hou et al. 2010), all contributions in the field of contract analysis under yield randomness restrict to situations where the yield type is characterized by stochastically proportional random yields. This also holds for a prior work of Inderfurth and Clemens (2014) which considers the coordination properties of various risk-sharing contracts under this type of yield randomness.

The preference for the assumption of stochastically proportional yield is mainly due to the fact that this yield type is relatively easy to handle analytically in standard yield models where only a single production run per period is used for demand fulfillment. In this model context, already the basic analytical studies by Gerchak et al. (1988) and Henig and Gerchak (1990) which investigate the optimal policy structure in a centralized supply chain setting with random yield environment refer to the stochastically proportional yield type. In practice, this form of production yield is only observed if yield losses are caused by an external effect that has a joint impact on a complete production batch so that the yield of each unit in the batch is perfectly correlated. Often, however, other yield types are found (see Yano and Lee

1995) which are of greater practical relevance and demand for specific consideration in decision making and contract analysis. Literature contributions which refer to a larger variety of yield models concentrate on planning situations where multiple production lots within a single period can be released [see (Grosfeld-Nir and Gerchak 2004) for an overview]. These studies, however, only address centralized decision making problems.

In our study, we focus on problems with a single production run and deviate from the assumption of stochastically proportional yield. Instead, we study a framework with binomially distributed yield which is characterized by a zero yield correlation of units within a production batch. This yield property is observed if failures in manufacturing operations or if material defectives occur independently in a production process. Since the properties of stochastically proportional and binomial yield are contrary (perfect vs. zero yield correlation), it is by no means straightforward if the coordination properties of contracts hold for both yield types in the same way. This paper is the first one that addresses the analysis of coordination by contracts under binomial yield conditions and investigates to which extent the results for stochastically proportional yields in Inderfurth and Clemens (2014) carry over to a situation where yields are binomially distributed.

In this context, the main purpose of this paper is to study how contracts can be used to diminish profit losses which are driven by uncoordinated behavior. Therefore, three different contracts are applied and analyzed regarding their coordination ability, namely the simple wholesale price contract, a reward contract [overproduction risk-sharing contract, first introduced by He and Zhang (2008)] and a penalty contract (compare Gurnani and Gerchak 2007). Comparable to the newsvendor setting with stochastic demand but reliable supply, the double marginalization effect of the wholesale price contract is found in our setting. Both advanced contract types can be shown to facilitate supply chain coordination if contract parameters are chosen appropriately.

The rest of this paper is organized as follows. In Sect. 2 the supply chain model and the yield distribution are introduced. In part 3 the centralized supply chain is analyzed in a binomial yield setting to generate a benchmark for decisions and objective values in the following contract analyses. Section 4 describes three contract designs, namely the wholesale price contract, the overproduction risk sharing contract, and the penalty contract and analyzes them with respect to their supply chain coordination potential. Section 5 summarizes main results, highlights problems caused by yield misspecification and suggests aspects of further research.

2 Model and assumptions

This paper considers a basic single-period interaction within a serial supply chain with one buyer (indicated by B) and one supplier (indicated by S). It is assumed that all cost, price, and yield information is common knowledge. In contrast to that, deterministic end-customer demand is not common knowledge but only known to the buyer. As the supplier decision is totally independent from end-customer demand, this is a reasonable assumption. This setting connects to the field of

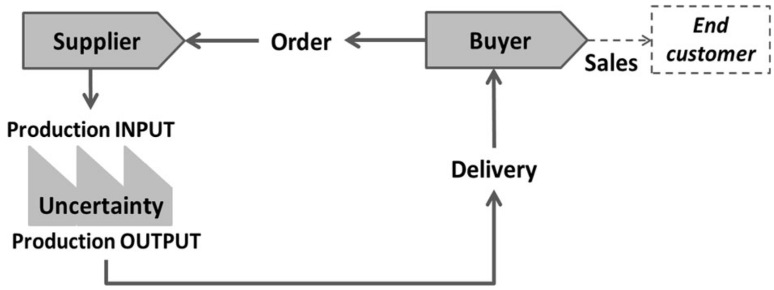


Fig. 1 Serial supply chain and course of interaction

contracting in a principal-agent context with information asymmetry [see (Corbett and Tang 1999) or (Burnetas et al. 2007)] where the principal (buyer) is better informed than the agent (supplier). Nevertheless, this property has no effect on the agent's profit because it is not a direct function of the principal's information on demand (compare Maskin and Tirole 1990). The supply chain and the course of interaction (explained below) are depicted in Fig. 1.

Assume the above two-member supply chain (indexed by SC). End-customer demand is denoted by D. The buyer orders from the supplier an amount of X units. The production process of the supplier, however, underlies risks which lead to random production yields, i.e., although the production input is fixed the output quantity in a specific production run is uncertain. The supplier can, due to production lead times, realize only a single production run.

In the following, production yield is denoted by $Y(Q)$ where Q is the production input chosen by the supplier. The quantity delivered to the buyer is the minimum of order quantity and production output. Hence, the supplier faces the risk of losing sales in case of too low production yield. However, it is a reasonable assumption that, given a simple wholesale price contract, the supplier is not further penalized (in addition to losing potential revenue) if end-customer demand cannot be satisfied due to under-delivery. In typical business transactions the supplying side is usually measured in terms of its ability to deliver to the buyer and not to the end customer. As the mechanism to satisfy end-customer demand is not in the control of the supplier, she cannot be held responsible for potential sales losses. However, both actors face the risk of lost sales because under-delivery by the supplier can cause unsatisfied demand at the buyer as stated above. Consequently, both parties may have incentives to inflate demand (from the buyer's perspective) or order quantity (from the supplier's perspective) to account for the yield risk and avoid lost sales. In case production output is larger than order quantity, excess units are worthless and cannot generate any revenue even though they incurred production cost. Sales at the buyer are the minimum of delivery quantity and end-customer demand. If the buyer's order and delivery quantity exceed demand, excess units are also of no value and cannot be turned into revenues.

Production yields are assumed to be binomially distributed, i.e., a unit turns out 'good' (or usable) with success probability θ $(0 \leq \theta \leq 1)$ and it is unusable with counter probability $1 - \theta$.

Thus, the probabilities for possible yields from a production batch Q are given by

$$\Pr\{Y(Q)=k\} = \binom{Q}{k} \cdot \theta^k (1-\theta)^{Q-k} \quad \forall k = 0, 1, \ldots, Q$$

Mean production yield amounts to

$$\mu_{Y(Q)} = \theta \cdot Q \tag{1}$$

with a standard deviation of

$$\sigma_{Y(Q)} = \sqrt{\theta \cdot (1-\theta) \cdot Q} \tag{2}$$

Note that the coefficient of variation $(\sigma_{Y(Q)} \big/ \mu_{Y(Q)})$ decreases as the input quantity grows, i.e., the risk diminishes with increasing production quantity. This is different from a situation with stochastically proportional yield where production yield is a random fraction of production input and neither mean nor variance of the yield rate depends on the batch size. Thus, a reasonable conjecture is that under binomially distributed yields, the risk allocation between the single actors is different from that under stochastically proportional yields. Hence, contract schemes with different risk-sharing mechanisms may perform differently when the lot size influences the "amount of risk" in the supply chain and may change the proposed contract types' coordination efficiency. The subsequent analyses will shed light on this issue.

For large values of demand (like for most consumer goods) and the respective production quantity, i.e., if the sample of the binomial distribution is sufficiently large, according to the De Moivre–Laplace theorem[1] the binomial distribution can be approximated through a normal distribution. This approximation will be used in the sequel with parameters which are fitted according to (1) and (2).[2] This deviation from the exact binomial distribution is motivated by the fact that it facilitates the contract analysis by modeling the decision problem with continuous instead of discrete variables so that general analytic results with closed-form expressions can be derived. Furthermore, the respective numerical results are very close to optimal under fairly high demand levels.

Further notation is as follows:

c	Production cost (per unit input)
w	Wholesale price (per unit)
p	Retail price (per unit)
$f_S(\cdot)$	*pdf* of standard normal distribution
$F_S(\cdot)$	*cdf* of standard normal distribution
$f_{Y(Q)}(\cdot)$	*pdf* of random variable $Y(Q)$ (yield)
$F_{Y(Q)}(\cdot)$	*cdf* of random variable $Y(Q)$ (yield)

The problem which arises is how to determine quantities for ordering on the one hand (by the buyer) and choosing a production input quantity on the other hand (by the supplier) given the risks mentioned above. The general underlying

[1] Compare Feller (1968) pp. 174 ff.

[2] The condition which justifies the use of the Normal distribution is the following: $Q \cdot \theta \cdot (1-\theta) > 5$ for $0.1 \le \theta \le 0.9$ (compare Evans et al. 2000 p. 45).

assumption in this analysis is that profitability of the business for both parties is assured, i.e., the retail price p exceeds the wholesale price w which in turn exceeds the expected production cost c/θ, i.e., $p > w > c/\theta$. As is common in the field of contract analysis, the behavior of the actors in a supply chain is investigated under the assumption that decentralized decision making can be modeled as a Stackelberg game. Before we come to the respective analyses, first the optimal decisions will be evaluated for a centralized supply chain setting to provide a benchmark solution.[3]

3 Analysis for a centralized supply chain

Under centralized decision making, the planner has only one decision to make, namely the production input quantity Q. Revenues are generated from selling the available quantity, i.e., the minimum of production output and demand, to the end customer. Production cost, however, is incurred for every produced unit. Thus, the total supply chain profit is given by

$$\Pi_{\mathrm{SC}}(Q) = p \cdot E[\min(D, Y(Q))] - c \cdot Q. \tag{3}$$

The first term in (3) describes the expected revenue from selling usable units; the second part constitutes the costs which are incurred by the respective production quantity. For deriving the optimal decision on production input, two cases have to be analyzed separately: $Q \leq D$ and $Q \geq D$.

Case SC(I)

Under case SC(I) ($Q \leq D$) it is obvious that $Y(Q) \leq Q \leq D$, due to $0 \leq \theta \leq 1$. Thus, the supply chain profit transforms to

$$\Pi_{\mathrm{SC}}(Q) = p \cdot E[Y(Q)] - c \cdot Q = (p \cdot \theta - c) \cdot Q$$

Taking the first-order derivative yields

$$\frac{\mathrm{d}\Pi_{\mathrm{SC}}(Q)}{\mathrm{d}Q} = p \cdot \theta - c \begin{cases} > 0 & \text{for } p > c/\theta \\ \leq 0 & \text{else} \end{cases}$$

For case SC(I), it follows that the supply chain produces the following quantity

$$Q_{\mathrm{SC(I)}} = \begin{cases} D & \text{for } p > c/\theta \\ 0 & \text{else} \end{cases}. \tag{4}$$

If the condition for profitability of the business holds, i.e., $p > c/\theta$, it has to be evaluated whether an input quantity $Q \geq D$ is preferable.

Case SC(II)

In this case ($Q \geq D$) the supply chain profit to maximize is given in (3). In this function the expected sales quantity of the supply chain will be denoted by L (D, Q) and can be expressed by

[3] More details of the analyses and all respective proofs can be found in a working paper version of Clemens and Inderfurth (2014) under http://www.fww.ovgu.de/fww_media/femm/femm_2014/2014_11.pdf.

$$L(D,Q) := E[\min(D, Y(Q))] = D - \int_0^D (D-y) \cdot f_{Y(Q)}(y)\mathrm{d}y.$$

Transforming this expression under the normality assumption for *Y(Q)* yields

$$L(D,Q) := D - \sigma_{Y(Q)} \cdot \left(F_S\left(z_{D,Q}\right) \cdot z_{D,Q} + f_S\left(z_{D,Q}\right)\right) \tag{5}$$

Here we define $z_{D,Q} := \frac{D-\mu_{Y(Q)}}{\sigma_{Y(Q)}}$. Note that $z_{D,Q}$ depends on demand D as well as on production input Q through mean and standard deviation of the yield $Y(Q)$. Thus, the above supply chain profit transforms to

$$\Pi_{\mathrm{SC}}(Q) = p \cdot L(D,Q) - c \cdot Q \tag{6}$$

Taking the first-order derivative yields

$$\begin{aligned}\frac{\mathrm{d}\Pi_{\mathrm{SC}}(Q)}{\mathrm{d}Q} &= p \cdot \frac{\partial L(D,Q)}{\partial Q} - c\\ &= p \cdot \frac{\theta}{2} \cdot \left(2 \cdot F_S\left(z_{D,Q}\right) - \frac{\sigma_{Y(Q)}}{\mu_{Y(Q)}} \cdot f_S\left(z_{D,Q}\right)\right) - c.\end{aligned}$$

The second-order derivative turns out to be negative so that the profit function in (6) is concave. Thus, we can utilize the first-order condition $\mathrm{d}\Pi_{\mathrm{SC}}(Q)/\mathrm{d}Q \overset{!}{=} 0$ to derive the optimal input decision for case SC(II). The respective production quantity results implicitly from the following optimality condition

$$\frac{c}{p} = \frac{\theta}{2} \cdot \left(2 \cdot F_S\left(z_{D,Q}\right) - \frac{\sigma_{Y(Q)}}{\mu_{Y(Q)}} \cdot f_S\left(z_{D,Q}\right)\right)$$

and is denoted by $Q_{\mathrm{SC(II)}}$. If we define

$$M(D,Q) := \frac{\theta}{2} \cdot \left(2 \cdot F_S\left(z_{D,Q}\right) - \frac{\sigma_{Y(Q)}}{\mu_{Y(Q)}} \cdot f_S\left(z_{D,Q}\right)\right) = \frac{\partial L(D,Q)}{\partial Q} \tag{7}$$

and $z_{D,Q}$ as above, the optimality condition for $Q_{\mathrm{SC(II)}}$ can be re-formulated as

$$\frac{c}{p} = M\left(D, Q_{\mathrm{SC(II)}}\right) \tag{8}$$

3.1 Overall solution

Since the solution space of case SC(II) includes the solution from (4) for $p > c/\theta$, the overall production decision of the supply chain is given by

$$Q^* = \begin{cases} Q_{\mathrm{SC(II)}} & \text{for } p > c/\theta \\ 0 & \text{else} \end{cases} \tag{9}$$

The corresponding optimal profit of the supply chain results from (6) and takes the following form:

$$\Pi^*_{SC} = \Pi_{SC}(Q^*)$$
$$= p \cdot D - p \cdot \left(F_S\left(z^*_{D,Q}\right) \cdot \left(D - \mu^*_{Y(Q)}\right) + \sigma^*_{Y(Q)} \cdot f_S\left(z^*_{D,Q}\right)\right) - c \cdot Q^*$$

with $\mu^*_{Y(Q)} = \mu_{Y(Q^*)}$, $\sigma^*_{Y(Q)} = \sigma_{Y(Q^*)}$, and $z^*_{D,Q} = \frac{D - \mu^*_{Y(Q)}}{\sigma^*_{Y(Q)}}$.

Inserting $\sigma^*_{Y(Q)} \cdot f_S\left(z^*_{D,Q}\right) = 2 \cdot F_S\left(z^*_{D,Q}\right) \cdot \mu^*_{Y(Q)} - \frac{2 \cdot c}{p \cdot \theta} \cdot \mu^*_{Y(Q)}$ which is given from (7) and (8) and exploiting $\mu^*_{Y(Q)} = \theta \cdot Q^*$ yields the optimal supply chain profit

$$\Pi^*_{SC} = p \cdot \left(1 - F_S\left(z^*_{D,Q}\right)\right) \cdot D - \left(p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c\right) \cdot Q^*. \tag{10}$$

To analyze the relationship between production quantity and demand, the derivative $\mathrm{d}Q(D)/\mathrm{d}D$ is evaluated. The relation between Q and D is given by

$$\frac{\mathrm{d}Q(D)}{\mathrm{d}D} = -\frac{\partial M(D,Q)}{\partial D} \Big/ \frac{\partial M(D,Q)}{\partial Q}$$
$$= \frac{2 \cdot \mu_{Y(Q)} \cdot \left(\mu_{Y(Q)} + D\right)}{\theta \cdot \left(\mu_{Y(Q)} + D + \sigma_{Y(Q)}\right)\left(\mu_{Y(Q)} + D - \sigma_{Y(Q)}\right)} > 0 \tag{11}$$

which shows that larger demand leads to larger production quantities which is intuitive. Interestingly, the production/demand ratio (Q/D) converges to a constant the larger demand gets. Assuming that demand approaches infinity, it can be shown that the production quantity approaches demand multiplied by $1/\theta$. This means that production is only inflated to compensate for expected yield losses, but no further adjustment is made to account for the yield risk. This is reasonable as binomially distributed yields decrease in risk as the input quantity rises (noting that $\lim_{Q\to\infty}\left(\sigma_{Y(Q)} \big/ \mu_{Y(Q)}\right) = 0$). Generally, we can formulate the following Lemma:

Lemma *If demand approaches infinity, the inflation factor of demand for the production input, i.e.,* Q/D*, approaches* $1/\theta$.

However, there is no unique way how the Q/D ratio is approaching $1/\theta$ as demand grows. Rather, it depends on the value of demand, production cost, retail price, and success probability whether the ratio is increasing from below $1/\theta$, decreasing from above $1/\theta$ or takes a combination of both. "Examples for the development of the production/demand ratio" in Appendix shows respective numerical examples.

4 Contract analysis for a decentralized supply chain

A decentralized supply chain consists of more than one decision maker. In our setting, a single buyer decides on the order quantity to fill end-customer demand and a single supplier produces to satisfy the order from the buyer as described in the beginning. The decentralized supply chain is modelled as a Stackelberg game with the buyer being the leader and the supplier being the follower, i.e., the buyer anticipates the production decision by the supplier in reaction to his order. In this context, it is assumed that the buyer has knowledge of the supplier's yield distribution and production cost.

Following the above decision making process, each of the considered contract types is analyzed in three steps. First, the supplier's optimal production decision for a given buyer's order volume is analyzed. Second, the buyer's decision is evaluated that maximizes his profit under anticipation of the supplier's production response. Third, it is investigated if and under which specific conditions the interaction of buyer and supplier is able to lead to the first-best result from the centralized supply chain so that coordination is achieved. This three-step analysis will first be carried out for the standard wholesale price contract before it is extended to two contracts (overproduction risk sharing contract and penalty contract) which are known to coordinate the supply chain in the case of stochastically proportional production yield.

4.1 Wholesale price contract

Under a simple wholesale price (WHP) contract the buyer orders some quantity X, and the supplier releases a production batch Q. The output from this batch is used to satisfy the buyer's order to a maximum extent. Delivered units are sold to the buyer at a per unit wholesale price w. In the context of this analysis the price w which rules the distribution of supply chain profits is a given parameter. In the following, the decisions made by the supplier and by the buyer are analyzed separately.

4.1.1 Supplier decision

Given the buyer's order quantity X, the supplier maximizes the following expected profit[4]:

$$\Pi_S^{\mathrm{WHP}}(Q|X) = w \cdot E[\min(X, Y(Q))] - c \cdot Q \tag{12}$$

The first term in (12) describes the expected revenue from selling usable units to the buyer; the second term represents the corresponding production cost. According to their implication for the supplier's profit function, two cases ($Q \leq X$ and $Q \geq X$) are considered separately.

Case S(I)

Under case S(I) ($Q \leq X$) it holds that $Y(Q) \leq Q \leq X$ due to $0 \leq \theta \leq 1$, and the supplier faces a profit of

$$\Pi_S^{\mathrm{WHP}}(Q|X) = w \cdot E[Y(Q)] - c \cdot Q = (w \cdot \theta - c) \cdot Q \tag{13}$$

The first-order derivative

$$\frac{\mathrm{d}\Pi_S^{\mathrm{WHP}}(Q|X)}{\mathrm{d}Q} = w \cdot \theta - c$$

is positive if w $> c/\theta$ and zero or negative otherwise. This implies the following production decision

[4] The following analysis is identical to the centralized case with X instead of D and w instead of p.

$$Q_{\mathrm{S(I)}}^{\mathrm{WHP}}(X) = \begin{cases} X & \text{for } w > c/\theta \\ 0 & \text{else} \end{cases} \tag{14}$$

If the condition for profitability of the business holds, i.e., $w > c/\theta$, it has to be evaluated whether $Q \geq X$ is preferable for the supplier.

Case S(II)

In this case ($Q \geq X$) the supplier's profit to maximize is the one in (12) which after some transformation is given by

$$\Pi_S^{\mathrm{WHP}}(Q|X) = w \cdot L(X,Q) - c \cdot Q \tag{15}$$

Here, we define the delivery quantity from the supplier to the buyer as

$$L(X,Q) = X - \sigma_{Y(Q)} \cdot \left(F_S\left(z_{X,Q}\right) \cdot z_{X,Q} + f_S\left(z_{X,Q}\right)\right) \tag{16}$$

and $z_{X,Q} := \frac{X-\mu_{Y(Q)}}{\sigma_{Y(Q)}}$. The optimal production input for case S(II) results from the first-order condition below:

$$\frac{\mathrm{d}\Pi_S^{\mathrm{WHP}}(Q|X)}{\mathrm{d}Q} = w \cdot \frac{\partial L(X,Q)}{\partial Q} - c \overset{!}{=} 0$$

with

$$\frac{\partial L(X,Q)}{\partial Q} = \frac{\theta}{2} \cdot \left(2 \cdot F_S\left(z_{X,Q}\right) - \frac{\sigma_{Y(Q)}}{\mu_{Y(Q)}} \cdot f_S\left(z_{X,Q}\right)\right) = M(X,Q) \tag{17}$$

which is independent from any cost or price parameter. The optimal input quantity under case S(II) is denoted by $Q_{\mathrm{S(II)}}^{\mathrm{WHP}}$ and satisfies the optimality condition below

$$\frac{c}{w} = M\left(X, Q_{\mathrm{S(II)}}^{\mathrm{WHP}}\right) \tag{18}$$

Theoretically, the supplier can choose a production quantity which is smaller than the order quantity and generate positive profits. However, in this case the optimization will follow case S(I), the solution of which is included in the solution space of S(II). Summarizing, the supplier's production decision under the simple WHP contract is given by

$$Q^{\mathrm{WHP}}(X) = \begin{cases} Q_{\mathrm{S(II)}}^{\mathrm{WHP}} & \text{for } w > c/\theta \\ 0 & \text{else} \end{cases}. \tag{19}$$

The supplier's profit is concave as the second-order derivative is negative[5]:

[5] The result is identical to the second-order derivative of the supply chain profit with X instead of D and w instead of p.

$$\frac{\mathrm{d}^2 \Pi_S^{\mathrm{WHP}}(Q|X)}{\mathrm{d}Q^2} = w \cdot \frac{\partial M(X,Q)}{\partial Q}$$
$$= -f_S(z_{X,Q}) \cdot \frac{w \cdot \theta^2}{4}$$
$$\cdot \frac{\left(X + \mu_{Y(Q)} + \sigma_{Y(Q)}\right) \cdot \left(X + \mu_{Y(Q)} - \sigma_{Y(Q)}\right)}{\sigma_{Y(Q)} \cdot \mu_{Y(Q)}^2} < 0.$$

Analogously to the centralized supply chain analysis, the relation between Q and X is given by[6]

$$\frac{\mathrm{d}Q(X)}{\mathrm{d}X} = -\frac{\partial M(X,Q)}{\partial X} \Big/ \frac{\partial M(X,Q)}{\partial Q}$$
$$= \frac{2 \cdot \mu_{Y(Q)} \cdot \left(\mu_{Y(Q)} + X\right)}{\theta \cdot \left(\mu_{Y(Q)} + X + \sigma_{Y(Q)}\right)\left(\mu_{Y(Q)} + X - \sigma_{Y(Q)}\right)} > 0. \qquad (20)$$

4.1.2 Buyer decision

The buyer as the leader in this Stackelberg game anticipates the supplier's decision from (19). As first mover, under a simple WHP contract the buyer maximizes the following expected profit:

$$\Pi_B^{\mathrm{WHP}}(X) = p \cdot E[\min(D, X, Y(Q))] - w \cdot E[\min(X, Y(Q))] \qquad (21)$$

The first term of this profit function is the expected revenue from selling to the end customer; the second term describes the expected cost from procuring units from the supplier. Also for the buyer decision, depending on the order/demand relationship ($X \leq D$ or $X \geq D$), two cases for the profit function have to be distinguished.

Case B(I)

Under case B(I) ($X \leq D$) the buyer's profit is given by

$$\Pi_B^{\mathrm{WHP}}(X) = (p - w) \cdot E[\min(X, Y(Q))] = (p - w) \cdot L(X, Q) \qquad (22)$$

The first-order derivative is rather complex as the buyer is the leader in this Stackelberg game and accounts for the supplier's reaction to his decision, i.e., $Q = Q^{\mathrm{WHP}}(X)$. Therefore, the total first-order derivative of this function includes the relation $\mathrm{d}Q(X)/\mathrm{d}X$ from (20) which describes the change in production input given a change in order quantity. The total first-order derivative is given by

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X} = \frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial X} + \frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial Q} \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X} \qquad (23)$$

Given the partial first-order derivative $\partial L(X,Q)/\partial X$ [with $L(X,Q)$ from (16)] as

[6] The result is identical to (11) with X instead of D.

$$\begin{aligned}\frac{\partial L(X,Q)}{\partial X} &= 1-\sigma_{Y(Q)}\\ &\quad\cdot\left(f_S\left(z_{X,Q}\right)\cdot z_{X,Q}\cdot\frac{1}{\sigma_{Y(Q)}}+F_S\left(z_{X,Q}\right)\cdot\frac{1}{\sigma_{Y(Q)}}-f_S\left(z_{X,Q}\right)\cdot z_{X,Q}\cdot\frac{1}{\sigma_{Y(Q)}}\right)\\ &= 1-F_S\left(z_{X,Q}\right)\end{aligned} \tag{24}$$

the total first-order derivative of the buyer's profit is derived from the following partial derivatives below

$$\frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial X}=(p-w)\cdot\frac{\partial L(X,Q)}{\partial X}=(p-w)\cdot\left(1-F_S\left(z_{X,Q}\right)\right)$$

$$\frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial Q}\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}=(p-w)\cdot\frac{\partial L(X,Q)}{\partial Q}\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}=(p-w)\cdot M(X,Q)\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}$$

with $\partial L(X,Q)/\partial Q$ from (17).

After inserting these terms, the total first-order derivative turns out to be

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X}=(p-w)\cdot\left(1-F_S\left(z_{X,Q}\right)\right)+(p-w)\cdot M(X,Q)\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X} \tag{25}$$

Due to $M(X,Q)>0$, $\mathrm{d}Q(X)/\mathrm{d}X>0$, and the profitability assumption $p>w$ it follows that $X^{\mathrm{WHP}}=D$ because

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X}\begin{cases}>0 & \text{for } p>w\\ \leq 0 & \text{else}\end{cases}$$

The order decision under case B(I) is formulated below

$$X_{\mathrm{B(I)}}^{\mathrm{WHP}}=\begin{cases}D & \text{for } p>w\\ 0 & \text{else}\end{cases}$$

Case B(II)

Analyzing the second case B(II) ($X\geq D$), the buyer's profit is given by $\Pi_B^{\mathrm{WHP}}(X)=p\cdot E[\min(D,Y(Q))]-w\cdot E[\min(X,Y(Q))]$ or, equivalently,

$$\Pi_B^{\mathrm{WHP}}(X)=p\cdot L(D,Q)-w\cdot L(X,Q). \tag{26}$$

As under case B(I), the first-order derivative is given by

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X}=\frac{\partial \Pi_B^{WHP}(X)}{\partial X}+\frac{\partial \Pi_B^{WHP}(X)}{\partial Q}\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}.$$

The single terms can be expressed as

$$\frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial X}=-w\cdot\frac{\partial L(X,Q)}{\partial X}=-w\cdot\left(1-F_S\left(z_{X,Q}\right)\right)$$

and

$$\frac{\partial \Pi_B^{\mathrm{WHP}}(X)}{\partial Q} \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X} = \left(p \cdot \frac{\partial L(D,Q)}{\partial Q} - w \cdot \frac{\partial L(X,Q)}{\partial Q}\right) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X}$$
$$= (p \cdot M(D,Q) - w \cdot M(X,Q)) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X}$$

with $\partial L(X,Q)/\partial X$ from (24) and $\partial L(X,Q)/\partial Q$ from (17).

Finally, the total first-order derivative is given by

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X} = -w \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + (p \cdot M(D,Q) - w \cdot M(X,Q)) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X} \quad (27)$$

Exploiting this derivative, the buyer decision under case B(II), denoted by $X_{\mathrm{B(II)}}^{\mathrm{WHP}}$, is implicitly given from the first-order condition $\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)/\mathrm{d}X \overset{!}{=} 0$. Hence, as the order decision under case B(II) includes the solution of case B(I), the overall order decision under the WHP contract is formulated below

$$X^{\mathrm{WHP}} = \begin{cases} X_{\mathrm{B(II)}}^{\mathrm{WHP}} & \text{for } p > w \\ 0 & \text{else} \end{cases} \quad (28)$$

4.1.3 Interaction of buyer and supplier

To evaluate the coordination ability of the WHP contract it has to be analyzed whether a wholesale price value exists which induces the supplier to produce the supply chain optimal quantity Q^* chosen in the centralized setting. In a second step it must be checked if a coordinating wholesale price leaves each supply chain actor with a positive profit so that both of them have an incentive to participate in the business.

The following analysis shows that two extreme wholesale price values ($w = p$ and $w = c/\theta$) exist which formally meet the coordination condition but violate the participation constraints.

(I) *Wholesale price* $w = p$

From the supply chain's and the supplier's optimality conditions in (8) and (18) we know that $\frac{c}{p} = M(D,Q^*)$ and $\frac{c}{w} = M(X,Q^{\mathrm{WHP}})$, respectively, if $p > w > c/\theta$.

Coordination is achieved if $Q^{\mathrm{WHP}} = Q^*$. Obviously, this is guaranteed if the following two conditions hold: (i) the buyer orders at demand level ($X^{\mathrm{WHP}} = D$) which yields $M(X,Q^{\mathrm{WHP}}) = M(D,Q^*)$ and (ii) the wholesale price is equal to the retail price which guarantees that $c/p = c/w$. Given $w = p$, the effect on the buyer's profit has to be evaluated. Under case B(II) ($X \geq D$), the first-order derivative of the buyer profit in (27) transforms to

$$\frac{\mathrm{d}\Pi_B^{\mathrm{WHP}}(X)}{\mathrm{d}X} = -p \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + \left(p \cdot \frac{c}{p} - p \cdot \frac{c}{p}\right) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X}$$
$$= -p \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) < 0.$$

Thus, for all values of the buyer's order in the range $X \geq D$, his marginal profit is negative. Consequently, the buyer will not order above end-customer demand.

Evaluating the decision spectrum $X \leq D$, the buyer profit from (22), given $w = p$, turns out to be zero:

$$\Pi_B^{\mathrm{WHP}}(X) = (p - p) \cdot L\left(X, Q^{\mathrm{WHP}}\right) = 0.$$

Because the buyer's profit is zero for any order quantity below end-customer demand, he is indifferent between all values from 0 to D. Assuming that the buyer orders $X^{\mathrm{WHP}} = D$ units and given $w = p$, it follows from the supply chain's and the supplier's profits in (6) and (15) that

$$\Pi_S^{\mathrm{WHP}}\left(Q^{\mathrm{WHP}} \middle| X^{\mathrm{WHP}} = D\right) = p \cdot L(D, Q) - c \cdot Q = \Pi_{\mathrm{SC}}(Q).$$

Thus, the supplier receives the total supply chain profit while the buyer does not generate any profit when ordering D units. Hence, the buyer does not agree on the contract and the business does not take place at all. Consequently, coordination cannot be achieved by the simple wholesale price contract if the two above conditions hold. The buyer only participates in the business if the wholesale price is below the retail price. However, in this case it holds that $c/p < c/w$ and, consequently, $M(X, Q^{\mathrm{WHP}}) > M(D, Q^*)$. As $\partial M(X, Q)/\partial Q < 0$, it follows that the supplier's production quantity is too low to coordinate the supply chain. Only a wholesale price value as large as the retail price incentivizes the supplier to produce the supply chain optimal quantity when the buyer's order equals demand.

(II) *Wholesale price* $w = c/\theta$

However, a low wholesale price might induce the buyer to order larger amounts which compensate the unwillingness of the supplier to inflate the order enough to reach the supply chain optimum. For that reason, another extreme case for the wholesale price is evaluated.

If the supplier sells at her expected production cost to the buyer ($w = c/\theta$), it is obvious that a production quantity larger than the order quantity makes no sense. Thus, case S(I) $Q \leq X$ must be analyzed with the profit function from (13). Setting $w = c/\theta$ yields

$$\Pi_S^{\mathrm{WHP}}(Q) = \left(\frac{c}{\theta} \cdot \theta - c\right) \cdot Q = 0.$$

Because the supplier's profit is zero for all possible production choices, she is indifferent between all values from 0 to X^{WHP}. That being the case, it will be assumed that the supplier produces $Q^{\mathrm{WHP}} = X^{\mathrm{WHP}}$ units. Anticipating this behavior, the buyer maximizes his profit for case B(II) $X \geq D$ in (26)

$$\Pi_B^{\mathrm{WHP}}(X) = p \cdot L(D, Q) - w \cdot L(X, Q)$$

Given $Q^{\mathrm{WHP}} = X^{\mathrm{WHP}}$, it follows that $F_S\left(z_{X,Q}\right) = 1$ and $f_S\left(z_{X,Q}\right) = 0$. Thus, the buyer's profit function transforms to

$$\Pi_B^{\mathrm{WHP}}\left(X^{\mathrm{WHP}} \middle| Q^{\mathrm{WHP}} = X^{\mathrm{WHP}}\right) = p \cdot L(D, Q) - c \cdot Q = \Pi_{\mathrm{SC}}(Q)$$

because according to (5) $w \cdot L(X, Q) = \frac{c}{\theta} \cdot L(X, Q) = \frac{c}{\theta} \cdot Q + \frac{c}{\theta} \cdot (1 \cdot (Q - \theta \cdot Q) + \sigma_{Y(Q)} \cdot 0) = c \cdot Q$ is given.

As $X^{\mathrm{WHP}} = Q^{\mathrm{WHP}}$ and $\Pi_B^{\mathrm{WHP}}(X^{\mathrm{WHP}}|Q^{\mathrm{WHP}} = X^{\mathrm{WHP}}) = \Pi_{\mathrm{SC}}(Q)$, it obviously follows that $X^{\mathrm{WHP}} = Q^*$ and $\Pi_B^{\mathrm{WHP}}(X^{\mathrm{WHP}}) = \Pi_{\mathrm{SC}}(Q^*)$.

Thus, it can be shown that given $w = c/\theta$, coordination of the supply chain could be enabled with the buyer ordering the supply chain optimal production quantity and the supplier producing the exact order quantity. However, as the supplier is left with no profit, her participation constraint is violated and she does not agree on the contract. Thus, coordination of the supply chain is impeded by violating the supplier's participation constraint.

Summarizing, each case violates the participation constraint of one actor in the supply chain ($\Pi_B^{\mathrm{WHP}}(X) = 0$ for $w = p$ and $\Pi_S^{\mathrm{WHP}}(Q|X) = 0$ for $w = c/\theta$) and, thus, terminates the interaction.

4.2 Overproduction risk-sharing contract

Under the overproduction risk-sharing (ORS) contract, the risk of producing too many units (i.e., those units which exceed the order quantity) is shared among the two parties. Thus, the supplier bears less risk and is motivated to respond to the buyer's order with a higher production quantity. Under this contract, the buyer commits to pay for all units produced by the supplier. While he pays the wholesale price w per unit for deliveries up to his actual order volume, quantities that exceed this amount are compensated at a lower price w_0. To exclude situations where the supplier will generate unlimited profits from overproduction the following parameter restrictions are set: $w_0 < c/\theta < w$. As the supplier is able to generate revenue for every produced unit she has an incentive to produce a larger lot compared to the situation under the simple WHP contract. This increase might provide the potential to align the supplier's production decision with the supply chain optimal one.

In this context, two contract variants have to be distinguished depending on the way a possible overproduction is handled by the parties. Under the first variant the buyer just financially compensates the supplier for overproduction without physically receiving deliveries that exceed his order size. This Pull-ORS contract leaves him in a different risk position as when the parties agree that the supplier will deliver the whole production output irrespective of the buyer's order. This variant is denoted as a Push-ORS contract.

4.2.1 *Supplier decision*

The profit to optimize by the supplier is identical for both contract variants. Different from the WHP profit function in (12) it includes the compensation for overproduction and is given by

$$\Pi_S^{\mathrm{ORS}}(Q|X) = w \cdot E[\min(X, Y(Q))] + w_O \cdot E\left[(Y(Q) - X)^+\right] - c \cdot Q \qquad (29)$$

Like in the WHP contract analysis, two cases are analyzed separately, S(I) ($Q \leq X$) and S(II) ($Q \geq X$).

Case S(I)

From case S(I) $(Q \leq X)$ it results that $Y(Q) \leq Q \leq X$ and the supplier's profit transforms to

$$\Pi_S^{\mathrm{ORS}}(Q|X) = w \cdot E[Y(Q)] + w_O \cdot 0 - c \cdot Q = (w \cdot \theta - c) \cdot Q \tag{30}$$

For the first-order derivative it holds that

$$\frac{\mathrm{d}\Pi_S^{\mathrm{ORS}}(Q|X)}{\mathrm{d}Q} = w \cdot \theta - c \begin{cases} > 0 & \text{for } w > c/\theta \\ \leq 0 & \text{else} \end{cases}$$

From that, the optimal input decision under case S(I) is given by

$$Q_{\mathrm{S(I)}}^{\mathrm{ORS}}(X) = \begin{cases} X & \text{for } w > c/\theta \\ 0 & \text{else} \end{cases} \tag{31}$$

Consequently, it has to be evaluated whether case S(II) $(Q \geq X)$ is preferable for the supplier.

Case S(II)

In this case, the supplier profit is given by

$$\begin{aligned} \Pi_S^{\mathrm{ORS}}(Q|X) &= w \cdot E[\min(X, Y(Q))] + w_O \cdot E[Y(Q) - \min(X, Y(Q))] - c \cdot Q \\ &= (w - w_O) \cdot E[\min(X, Y(Q))] + w_O \cdot E[Y(Q)] - c \cdot Q \end{aligned}$$

so that we can formulate

$$\Pi_S^{\mathrm{ORS}}(Q|X) = (w - w_0) \cdot L(X, Q) + w_0 \cdot \mu_{Y(Q)} - c \cdot Q \tag{32}$$

with $L(X, Q)$ from (16). The first-order derivative of the supplier's profit is given by

$$\begin{aligned} \frac{\mathrm{d}\Pi_{\mathrm{S}}^{\mathrm{ORS}}(Q|X)}{\mathrm{d}Q} &= (w - w_0) \cdot \frac{\partial L(X, Q)}{\partial Q} + w_0 \cdot \theta - c \\ &= (w - w_0) \cdot M(X, Q) + w_0 \cdot \theta - c \end{aligned} \tag{33}$$

with $\partial L(X, Q)/\partial Q$ from (17). The supplier's production quantity under case S(II), $Q_{\mathrm{S(II)}}^{\mathrm{ORS}}$, results from the first-order condition $\mathrm{d}\Pi_S^{\mathrm{ORS}}(Q|X)/\mathrm{d}Q \overset{!}{=} 0$ and is implicitly given from:

$$\frac{c - w_0 \cdot \theta}{w - w_0} = M\left(X, Q_{\mathrm{S(II)}}^{\mathrm{ORS}}\right). \tag{34}$$

Thus, the supplier's production decision under an ORS contract can be formulated as

$$Q^{\mathrm{ORS}}(X) = \begin{cases} Q_{S(II)}^{\mathrm{ORS}} & \text{if } w > c/\theta \\ 0 & \text{else} \end{cases}. \tag{35}$$

Note that for $w_O = 0$ the optimal decision is identical to that under a WHP contract.

The supplier's profit is concave as the second-order derivative is negative:

$$\frac{\mathrm{d}^2 \Pi_S^{\mathrm{ORS}}(Q|X)}{\mathrm{d}Q^2} = (w - w_0) \cdot \frac{\partial M(X,Q)}{\partial Q}$$
$$= -f_S(z_{X,Q}) \cdot \frac{(w - w_0) \cdot \theta^2}{4}$$
$$\cdot \frac{\left(X + \mu_{Y(Q)} + \sigma_{Y(Q)}\right) \cdot \left(X + \mu_{Y(Q)} - \sigma_{Y(Q)}\right)}{\sigma_{Y(Q)} \cdot \mu_{Y(Q)}^2} < 0.$$

Since $M(X,Q)$ in (34) is a constant like for the WHP contract, the first-order derivative $\mathrm{d}Q^{\mathrm{ORS}}(X)/\mathrm{d}X$ is identical to that in (20).

4.2.2 Buyer decision

The buyer's profit function depends on the specific type of ORS contract that is applied. Under a *Pull-ORS type* (exclusion of over-delivery) the buyer maximizes a profit which compared to the WHP contract is reduced by the supplier's compensation for overproduced items

$$\Pi_B^{\mathrm{ORS}}(X) = p \cdot E[\min(D, X, Y(Q))] - w \cdot E[\min(X, Y(Q))] - w_0 \cdot E\left[(Y(Q) - X)^+\right]. \tag{36}$$

As for the supplier, the buyer analysis treats two separate cases.

Case B(I)

Under case B(I) ($X \leq D$), the buyer's profit is given by

$$\Pi_B^{\mathrm{ORS}}(X) = (p - w) \cdot E[\min(X, Y(Q))] - w_0 \cdot E\left[(Y(Q) - X)^+\right]$$
$$= (p - w + w_0) \cdot E[\min(X, Y(Q))] - w_0 \cdot E[Y(Q)]$$

which delivers

$$\Pi_B^{\mathrm{ORS}}(X) = (p - w + w_0) \cdot L(X,Q) - w_0 \cdot \mu_{Y(Q)} \tag{37}$$

The total first-order derivative of (37) is given by

$$\frac{\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)}{\mathrm{d}X} = (p - w + w_0) \cdot \left(1 - F_S(z_{X,Q})\right) + ((p - w + w_0) \cdot M(X,Q) - w_0 \cdot \theta) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X} \tag{38}$$

with $M(X,Q)$ from (17) and $\mathrm{d}Q(X)/\mathrm{d}X$ from (20). Depending on whether the first-order derivative is positive or negative, the order quantity under case B(I), $X_{B(I)}^{\mathrm{ORS}}$, ranges from zero up to demand D.

Case B(II)

For case B(II) ($X \geq D$) the buyer maximizes the following profit

$$\Pi_B^{\mathrm{ORS}}(X) = p \cdot E[\min(D, Y(Q))] - (w - w_0) \cdot E[\min(X, Y(Q))] - w_0 \cdot E[Y(Q)]$$

that equals

$$\Pi_B^{\mathrm{ORS}}(X) = p \cdot L(D,Q) - (w - w_0) \cdot L(X,Q) - w_0 \cdot \mu_{Y(Q)} \tag{39}$$

with $L(D,Q)$ from (5) and $L(X,Q)$ from (16). The profit maximizing order quantity for case B(II), $X_{B(II)}^{\mathrm{ORS}}$, results from the first-order derivative below

$$\frac{\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)}{\mathrm{d}X} = -(w - w_0) \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + (p \cdot M(D,Q) - (w - w_0) \cdot M(X,Q) - w_0 \cdot \theta) \cdot \frac{dQ(X)}{\mathrm{d}X} \tag{40}$$

with $M(D,Q)$ and $M(X,Q)$ from (7) and (17), respectively, by setting $\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)/\mathrm{d}X \stackrel{!}{=} 0$.

4.2.3 Interaction of buyer and supplier

Under the extended contract with two parameters w and w_0 it has to be analyzed whether there exists a combination of contract parameters which guarantees that the total supply chain profit is maximized while both, supplier and buyer, accept the contract. Coordination is achieved if the optimality conditions of supply chain and supplier under an ORS contract are identical. They are given from (8) and (34), respectively:

$$\frac{c}{p} = M(D,Q^*) \text{ and } \frac{c - w_0 \cdot \theta}{w - w_0} = M\left(X, Q^{\mathrm{ORS}}\right).$$

This condition is fulfilled if (i) the buyer orders at demand level, i.e., if $X^{\mathrm{ORS}} = D$ and (ii) $M(D,Q^*) = M(X,Q^{\mathrm{ORS}})$ holds, i.e., if the following condition for the contract parameters is satisfied

$$c \cdot (w - w_0) = p \cdot (c - w_0 \cdot \theta) \tag{41}$$

which ensures that $c/p = (c - w_0 \cdot \theta)/(w - w_0)$. This condition also implies that $p = (w - w_0) \cdot c/(c - w_0 \cdot \theta) > w - w_0$.

For this parameter setting the supplier's marginal profit under case S(II) in (33) turns out to be

$$\frac{\mathrm{d}\Pi_S^{\mathrm{ORS}}\left(Q^{\mathrm{ORS}} = Q^* \middle| X^{\mathrm{ORS}} = D\right)}{\mathrm{d}Q} = (w - w_0) \cdot \frac{(c - w_0 \cdot \theta)}{(w - w_0)} + w_0 \cdot \theta - c = 0.$$

The supplier's marginal profit being zero, shows that the supplier actually chooses the respective quantity. As the buyer anticipates this behavior, it can be evaluated which order decision maximizes the buyer's profit. Under case B(II) $(X \geq D)$, for $Q^{\mathrm{ORS}} = Q^*$ the buyer's marginal profit from (40) transforms to

$$\begin{aligned}\frac{\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)}{\mathrm{d}X} &= -(w-w_0)\cdot\left[1-F_S(z_{X,Q})\right]\\ &\quad+\left(p\cdot\frac{c}{p}-(w-w_0)\cdot\left(\frac{c-w_0\cdot\theta}{(w-w_0)}\right)-w_0\cdot\theta\right)\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}\\ &= -(w-w_0)\cdot\left[1-F_S(z_{X,Q})\right]+(c-c)\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}\\ &= -(w-w_0)\cdot\left[1-F_S(z_{X,Q})\right]<0\end{aligned}.$$

Due to the first-order derivative being negative, the buyer will not order above demand. Assuming an order quantity of $X^{\mathrm{ORS}}=D$ and the coordinating parameter setting from (41), the buyer maximizes the profit under case B(I) $(X\leq D)$ in (37) according to

$$\Pi_B^{\mathrm{ORS}}\left(X^{\mathrm{ORS}}=D\right)=(p-w+w_0)\cdot L(D,Q^*)-w_0\cdot\mu^*_{Y(Q)}.$$

Rearranging the above profit yields:

$$\begin{aligned}\Pi_B^{\mathrm{ORS}}\left(X^{\mathrm{ORS}}=D\right) &= p\cdot L(D,Q^*)-c\cdot Q^*+c\cdot Q^*-(w-w_0)\cdot L(D,Q^*)-w_0\cdot\theta\cdot Q^*\\ &= \Pi^*_{\mathrm{SC}}-(w-w_0)\cdot L(D,Q^*)+(c-w_0\cdot\theta)\cdot Q^*\\ &= \Pi^*_{\mathrm{SC}}-(w-w_0)\cdot L(D,Q^*)+\frac{c}{p}\cdot(w+w_0)\cdot Q^*=\Pi^*_{\mathrm{SC}}-(w-w_0)\cdot\frac{\Pi^*_{\mathrm{SC}}}{p}\end{aligned}$$

$$\Pi_B^{\mathrm{ORS}}\left(X^{\mathrm{ORS}}=D\right)=\Pi^*_{\mathrm{SC}}\cdot\left(1-\frac{w-w_0}{p}\right). \tag{42}$$

Due to (41) it holds that $p>w-w_0$ and thus, $\Pi_B^{\mathrm{ORS}}(X^{\mathrm{ORS}}=D)>0$. Utilizing the first-order condition of the above profit, the optimal order quantity is determined. The relation in (42) allows us to conclude that $\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)/\mathrm{d}X>0$ since $\mathrm{d}\Pi^*_{\mathrm{SC}}(X)/\mathrm{d}X>0$ (with $\Pi^*_{\mathrm{SC}}(X)=\Pi^*_{\mathrm{SC}}$ for $D=X$) and thus, $X^{\mathrm{ORS}}=D$.

So, both conditions for coordination are fulfilled which proves that the *Pull-ORS* contract can enable supply chain coordination, because the buyer incentivizes the supplier to produce the supply chain optimal amount by ordering at demand level if the contract parameters are fixed appropriately, i.e., according to (41).

If the actors agree on a *Push-ORS* contract the situation changes. In case all produced items are physically delivered, the buyer's sales are not restricted by his own order and his profit turns out to be identical for the cases B(I) and B(II), i.e., for $X\leq D$ and $X\geq D$, and is given from (39):

$$\Pi_B^{\mathrm{ORS}}(X)=p\cdot L(D,Q)-(w-w_0)\cdot L(X,Q)-w_0\cdot\mu_{Y(Q)}.$$

From the previous analysis of the interaction between supplier and buyer, it is given that coordination requests $X^{\mathrm{ORS}}=D$ and $c\cdot(w-w_0)=p\cdot(c-w_0\cdot\theta)$. These conditions result in the following marginal profit for the buyer:

$$\begin{aligned}\frac{\mathrm{d}\Pi_B^{\mathrm{ORS}}(X)}{\mathrm{d}X} &= -(w-w_0)\cdot\left(1-F_S\left(z_{X,Q}\right)\right)+\left(p\cdot\frac{c}{p}-(w-w_0)\cdot\frac{c-w_0\cdot\theta}{w-w_0}-w_0\cdot\theta\right)\cdot\frac{\mathrm{d}Q(X)}{\mathrm{d}X}\\ &= -(w-w_0)\cdot\left(1-F_S\left(z_{X,Q}\right)\right)<0\end{aligned}.$$

As the buyer's marginal profit is negative (given $w_0 < w$), it is no option for the buyer to order at demand level. Through the design of the contract, orders below demand may be optimal. As the delivered quantity can exceed the order or even end-customer demand, the buyer can still meet demand by 'under-ordering'. Assuming the buyer orders below demand, there may be combinations of w and w_0 which incentivize the supplier to produce the supply chain optimal quantity (obviously, a larger wholesale price or a higher compensation for overstock is necessary). However, higher prices are less profitable for the buyer who would further reduce his order quantity. This downward trend continues until nothing is ordered at all. Thus, the *Push-ORS* contract cannot coordinate the supply chain.

4.3 Penalty contract

If a *penalty* (PEN) contract is applied the supplier will bear a higher risk than under a simple WHP contract since she is punished for under-delivery. The supplier is penalized by the buyer (in the amount of π) for each unit ordered that cannot be delivered because of insufficient production yield. Given the potential penalty the supplier has an incentive to produce more than under the simple WHP contract which might be sufficient to achieve coordination of the supply chain.

4.3.1 Supplier decision

Under the PEN contract, the profit to optimize by the supplier includes the revenue from product delivery as well as a penalty for under-delivery and is given by

$$\Pi_S^{\text{PEN}}(Q|X) = w \cdot E[\min(X, Y(Q))] - \pi \cdot E\left[(X - Y(Q))^+\right] - c \cdot Q. \tag{43}$$

In the following, the two cases S(I) $(Q \leq X)$ and S(II) $(Q \geq X)$ are, again, analyzed separately.

Case S(I)

Given case S(I) $(Q \leq X)$ the supplier's profit simplifies to

$$\begin{aligned} \Pi_S^{\text{PEN}}(Q|X) &= w \cdot E[Y(Q)] - \pi \cdot (X - E[Y(Q)]) - c \cdot Q \\ &= ((w + \pi) \cdot \theta - c) \cdot Q - \pi \cdot X \end{aligned} \tag{44}$$

From the first-order derivative of (44) which is given by

$$\frac{\mathrm{d}\Pi_S^{\text{PEN}}(Q|X)}{\mathrm{d}Q} = (w + \pi) \cdot \theta - c$$

it follows that the supplier produces either zero or the ordered amount depending on the parameter constellation as formulated below

$$\frac{\mathrm{d}\Pi_S^{\text{PEN}}(Q|X)}{\mathrm{d}Q} \begin{cases} > 0 & \text{for } w + \pi > \dfrac{c + \pi}{\theta} \\ \leq 0 & \text{else} \end{cases}.$$

Note that if $Q = X$, then $\Pi_S^{\text{PEN}}(Q|X) = ((w + \pi) \cdot \theta - c - \pi) \cdot X$ which constitutes the parameter condition above. Finally, the production quantity under case S(I), $Q_{\text{S(I)}}^{\text{PEN}}$, is formulated as follows

$$Q_{S(I)}^{PEN}(X) = \begin{cases} X & \text{for } w + \pi > \dfrac{c+\pi}{\theta} \\ 0 & \text{else} \end{cases}. \tag{45}$$

Case S(II)

Assuming that $w + \pi > (c + \pi)/\theta$ holds, case S(II) $(Q \geq X)$ has to be evaluated. The profit generated by the supplier is according to (43)

$$\Pi_S^{PEN}(Q|X) = w \cdot E[\min(X, Y(Q))] - \pi \cdot E[X - \min(X, Y(Q))] - c \cdot Q$$

and can be expressed as

$$\Pi_S^{PEN}(Q|X) = (w + \pi) \cdot L(X, Q) - \pi \cdot X - c \cdot Q. \tag{46}$$

Taking the first-order derivative yields

$$\frac{\mathrm{d}\Pi_S^{PEN}(Q|X)}{\mathrm{d}Q} = (w + \pi) \cdot \frac{\partial L(X, Q)}{\partial Q} - c = (w + \pi) \cdot M(X, Q) - c \tag{47}$$

with $\partial L(X, Q)/\partial Q$ from (17). Hence, from $\mathrm{d}\Pi_S^{PEN}(Q|X)/\mathrm{d}Q \overset{!}{=} 0$ the optimal production input under case S(II), $Q_{S(II)}^{PEN}$, satisfies the following equation

$$\frac{c}{w + \pi} = M\left(X, Q_{S(II)}^{PEN}\right) \tag{48}$$

Hence, the supplier's production policy under a PEN contract is the following

$$Q^{PEN}(X) = \begin{cases} Q_{S(II)}^{PEN} & \text{for } w + \pi > \dfrac{c+\pi}{\theta} \\ 0 & \text{else} \end{cases}. \tag{49}$$

Note that for $\pi = 0$ the optimal decision is identical to that under a WHP contract.

The supplier's profit is concave as the second-order derivative is negative:

$$\begin{aligned} \frac{\mathrm{d}^2\Pi_S^{PEN}(Q|X)}{\mathrm{d}Q^2} &= (w + \pi) \cdot \frac{\partial M(X, Q)}{\partial Q} \\ &= -f_S\left(z_{X,Q}\right) \cdot \frac{(w + \pi) \cdot \theta^2}{4} \\ &\quad \cdot \frac{\left(X + \mu_{Y(Q)} + \sigma_{Y(Q)}\right) \cdot \left(X + \mu_{Y(Q)} - \sigma_{Y(Q)}\right)}{\sigma_{Y(Q)} \cdot \mu_{Y(Q)}^2} < 0. \end{aligned}$$

Since $M(X, Q)$ in (48) is a constant like for the WHP contract, the first-order derivative $\mathrm{d}Q^{PEN}(X)/\mathrm{d}X$ is identical to that in (20).

4.3.2 *Buyer decision*

The buyer under a PEN contract is compensated for missing units by the penalty rate. The profit the buyer generates is the following

$$\Pi_B^{\text{PEN}}(X) = p \cdot E[\min(D, X, Y(Q))] - w \cdot E[\min(X, Y(Q))] + \pi \cdot E\left[(X - Y(Q))^+\right].$$

The two cases B(I) $(X \leq D)$ and B(II) $(X \geq D)$ are evaluated in the next section.

Case B(I)

The buyer's profit in case B(I) $(X \leq D)$ transforms to

$$\begin{aligned}\Pi_B^{\text{PEN}}(X) &= (p - w) \cdot E[\min(X, Y(Q))] + \pi \cdot E\left[(X - Y(Q))^+\right]\\ &= (p - w - \pi) \cdot E[\min(X, Y(Q))] + \pi \cdot X\end{aligned}$$

$$\Pi_B^{\text{PEN}}(X) = (p - w - \pi) \cdot L(X, Q) + \pi \cdot X \tag{50}$$

with $L(X, Q)$ from (16). Taking the first-order derivative yields the expression below

$$\frac{\mathrm{d}\Pi_B^{\text{PEN}}(X)}{\mathrm{d}X} = (p - w - \pi) \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + \pi + (p - w - \pi) \cdot M(X, Q) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X} \tag{51}$$

with $M(X, Q)$ from (17) and $\mathrm{d}Q(X)/\mathrm{d}X$ from (20). The optimal order quantity under case B(I), $X_{\text{B(I)}}^{\text{PEN}}$, then results from $\mathrm{d}\Pi_B^{\text{PEN}}(X)/\mathrm{d}X \overset{!}{=} 0$. However, also the case $X \geq D$ has to be analyzed.

Case B(II)

Under case B(II), i.e., $X \geq D$, the buyer maximizes the subsequent profit $\Pi_B^{\text{PEN}}(X) = p \cdot E[\min(D, Y(Q))] - (w + \pi) \cdot E[\min(X, Y(Q))] + \pi \cdot X$ that equals

$$\Pi_B^{\text{PEN}}(X) = p \cdot L(D, Q) - (w + \pi) \cdot L(X, Q) + \pi \cdot X \tag{52}$$

with $L(D, Q)$ from (5) and $L(X, Q)$ from (16). The buyer's optimal decision under case B(II), $X_{\text{B(II)}}^{\text{PEN}}$, is derived from exploiting the first-order condition $\mathrm{d}\Pi_B^{\text{PEN}}(X)/\mathrm{d}X \overset{!}{=} 0$ concerning the derivative below

$$\begin{aligned}\frac{\mathrm{d}\Pi_B^{\text{PEN}}(X)}{\mathrm{d}X} = &-(w + \pi) \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + \pi + (p \cdot M(D, Q) - (w + \pi) \cdot M(X, Q))\\ &\cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X}\end{aligned} \tag{53}$$

with $M(D, Q)$ from (7), $M(X, Q)$ from (17) and $\mathrm{d}Q(X)/\mathrm{d}X$ from (20).

4.3.3 Interaction of buyer and supplier

As under the ORS contract, it has to be analyzed whether there exists a combination of contract parameters which guarantees that total supply chain profit is maximized while both, supplier and buyer, accept the contract. To coordinate the supply chain, the optimality conditions of supply chain and supplier under a PEN contract have to be identical. They are given from (8) and (48), respectively:

$$\frac{c}{p} = M(D, Q^*)$$

and

$$\frac{c}{w + \pi} = M\left(X, Q^{\text{PEN}}\right).$$

This condition is fulfilled if the buyer orders at demand level, i.e., if $X^{\text{PEN}} = D$ and if $M(D, Q^*) = M(X, Q^{\text{PEN}})$, i.e., if the following condition for the contract parameters is satisfied

$$p = w + \pi \tag{54}$$

which ensures that $c/p = c/(w + \pi)$. Given the parameter condition, the supplier's marginal profit in (47) turns out to be zero:

$$\frac{\mathrm{d}\Pi_S^{\text{PEN}}(Q|X)}{\mathrm{d}Q} = (w + \pi) \cdot \frac{c}{w + \pi} - c = 0.$$

As the supplier's marginal profit is zero, she actually chooses the corresponding input quantity. Because the buyer anticipates this behavior, it can be evaluated which order decision maximizes his profit. Under case B(II) ($X \geq D$), the buyer's marginal profit from (53) in combination with the parameter condition in (54), transforms to

$$\begin{aligned}\frac{\mathrm{d}\Pi_B^{\text{PEN}}(X)}{\mathrm{d}X} = & -(w + \pi) \cdot \left(1 - F_S\left(z_{X,Q}\right)\right) + \pi \\ & + \left((w + \pi) \cdot \frac{c}{w + \pi} - (w + \pi) \cdot \frac{c}{w + \pi}\right) \cdot \frac{\mathrm{d}Q(X)}{\mathrm{d}X}\end{aligned}$$

and yields

$$\frac{\mathrm{d}\Pi_B^{\text{PEN}}(X)}{\mathrm{d}X} = -w + (w + \pi) \cdot F_S\left(z_{X,Q}\right). \tag{55}$$

For proving that $\mathrm{d}\Pi_B^{\text{PEN}}(X)/\mathrm{d}X < 0$, it will be shown that the penalty π must not be too large. Thus, the determination of the penalty needs particular analysis. Under coordination (given $p = w + \pi$ and $X^{\text{PEN}} = D$ which leads to $Q^{\text{PEN}} = Q^*$), and using the supply chain profit from (6), the supplier's and the buyer's profits from (46) and (52) can be expressed as follows

$$\begin{aligned}\Pi_S^{\text{PEN}}\left(Q^{\text{PEN}} \middle| X^{\text{PEN}} = D\right) &= (w + \pi) \cdot L\left(D, Q^{\text{PEN}}\right) - \pi \cdot D - c \cdot Q^{\text{PEN}} \\ &= p \cdot L(D, Q^*) - c \cdot Q^* - \pi \cdot D = \Pi_{\text{SC}}(Q^*) - \pi \cdot D\end{aligned}$$

and

$$\Pi_B^{\text{PEN}}\left(X^{\text{PEN}} = D\right) = \pi \cdot D.$$

Consequently, for the supplier's participation constraint to hold, i.e., to generate a non-negative profit, the maximum penalty π^+ that results in $\Pi_S^{\text{PEN}}(Q^{\text{PEN}}|X^{\text{PEN}} = D.) = 0$, is given by

$$\pi^+ = \frac{\Pi_{\text{SC}}(Q^*)}{D} \tag{56}$$

From $\Pi_{SC}(Q^*) = p \cdot \left(1 - F_S\left(z^*_{D,Q}\right)\right) \cdot D - \left(p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c\right) \cdot Q^*$ in (10) we get:

$$\pi < \pi^+ = p \cdot \left(1 - F_S\left(z^*_{D,Q}\right)\right) - \left(p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c\right) \cdot \frac{Q^*}{D}.$$

Given the coordinating parameter constellation $p = w + \pi$, the restriction $\pi < \pi^+$ transforms to

$$\pi < (w + \pi) \cdot \left(1 - F_S\left(z^*_{D,Q}\right)\right) - \left(p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c\right) \cdot \frac{Q^*}{D}.$$

From that we further get

$$-w + (w + \pi) \cdot F_S\left(z^*_{D,Q}\right) < -\left(p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c\right) \cdot \frac{Q^*}{D}. \tag{57}$$

Under case B(II), from (55), the optimal buyer decision of $X^{PEN} = D$ is only given if

$$\frac{d\Pi_B^{PEN}(X)}{dX} = -w + (w + \pi) \cdot F_S\left(z_{X,Q}\right) < 0.$$

According to (57) this holds if $p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c > 0$.

From (7) and (8) we know that

$$F_S\left(z^*_{D,Q}\right) = \frac{c}{p \cdot \theta} + \frac{\sigma^*_{Y(Q)}}{2 \cdot \mu^*_{Y(Q)}} \cdot f_S\left(z^*_{D,Q}\right)$$

so that $p \cdot \theta \cdot F_S\left(z^*_{D,Q}\right) - c = p \cdot \theta \cdot \frac{\sigma^*_{Y(Q)}}{2 \cdot \mu^*_{Y(Q)}} \cdot f_S\left(z^*_{D,Q}\right) > 0$.

Thus, if the participation constraint for the supplier is fulfilled and if the penalty π is restricted to be lower that π^+, the buyer's optimal order quantity will be $X^{PEN} = D$ in case B(II). Since for $X \leq D$ the first-order derivative in (53) reduces to $d\Pi_B^{PEN}(X)/dX = \pi > 0$ the contract coordinating parameter condition $p = w + \pi$ also initiates $X^{PEN} = D$ in case B(I). Thus, analogously to the ORS contract, the PEN contract can enable supply chain coordination because the buyer incentivizes the supplier to produce the supply chain optimal amount by ordering at demand level while the contract parameters are fixed appropriately, i.e., under $p = w + \pi$.

5 Conclusion and outlook

The analyses in this paper are the first that address the problem of coordination through contracts in supply chains with binomially distributed production yield. They reveal several interesting insights for a buyer–supplier chain with deterministic end-customer demand. The simple WHP contract fails to coordinate, while more sophisticated contracts with reward or penalty scheme enable coordinated behavior in the supply chain without violating the actors' participation constraints. However, the ORS contract's ability to coordinate a supply chain depends on the variant that is applied. If a *Pull*-type contract (without the delivery of excess units) is used, coordination can be achieved. However, if physical delivery of overstock is allowed (*Push* variant), the contract loses its coordination power. For the PEN contract, however, it can be shown that the design enables SC coordination and, depending on the parameter setting (including a maximum penalty restriction), guarantees an arbitrary profit split.

A comparison with the results from Inderfurth and Clemens (2014) obtained for stochastically proportional yields reveals that all contract designs retain their ability or disability to trigger coordination. For the coordinating contract types, *Pull*-ORS and PEN, it furthermore turns out that coordination is always coupled with a buyer's order at demand level. It is also interesting to see that the contract parameter setting which is necessary to coordinate the supply chain under both contract types, i.e., (w, w_0) in (41) and (w, π) in (54), is exactly the same as in the case of stochastically proportional yield. So it becomes evident that the general coordination properties of the studied contracts, including the ability of profit split, do not differ between the different yield types although under binomial yield, different from stochastically proportional yield, the level of the yield uncertainty is critically dependent on the size of the production batch. This property, however, will in first line affect the size of the production and order decision.

Regarding the production quantity, it is found in this paper that demand is inflated to some extent to cope with yield losses. The respective inflation factor, however, is not a constant multiplier of demand like in the case of stochastically proportional yield (see Inderfurth and Clemens 2014). Instead, depending on the cost, price and yield data this inflation factor might increase or decrease with increasing demand level and approaches the reciprocal of the expected yield rate when demand tends to become very large. This is due to the characteristic of binomial yields to monotonically decrease the output risk as the production input level rises up to a level where this risk almost vanishes. The consequences are twofold. First, under comparable parameter settings and identical demand the production level under binomial yield is lower and the expected supply chain profit is higher than in the case of stochastically proportional yield. Second, in high-demand environments the coordination deficit of the simple WHP contract becomes negligible because the yield risk almost disappears in case of binomial yield so that the production decisions in the centralized and decentralized supply chain setting tend to coincide. This is completely different from what is valid under stochastically proportional yield.

The contract analysis for the case of binomial production yield in this paper also permits to study the effects of yield misspecification in the sense that it is assumed that the yield is stochastically proportional, but the real underlying model is binomial. A respective numerical study has been carried out for both settings, the centralized and decentralized one (see " Effects of yield misspecification if real yield is binomial" and "Effects of yield misspecification if real yield is stochastically proportional" in Appendices). In this study the production and order decisions under the wrong yield assumption are inserted in the profit function with correct yield specification with yield parameters that are identical for both yield models. In the centralized case it turns out that a major profit loss of more than 30 % can emerge from such a misspecification, especially if the profitability in terms of price/cost ratio is very small as can be verified in "Effects of yield misspecification if real yield is binomial" in Appendix. In the case of decentralized decision making under a WHP contract, however, the profit loss for the whole supply chain is in general smaller. In some specific cases the supply chain can even profit from yield misspecification since the wrong buyer's order and supplier's reaction can improve the total supply chain performance. "Effects of yield misspecification if real yield is stochastically proportional" in Appendix reveals that the same qualitative outcome (with different quantitative results) is found in the case of a reverse misspecification, i.e., if binomial yield is assumed but the real yield is stochastically proportional. The lesson that can be learnt from this specific investigation is that it is very important to specify the yield type correctly. It would be highly interesting to find out if one can distinguish data settings where it really matters to use the true yield model. Such a study, however, is beyond the scope of this paper and will be a matter of future research.

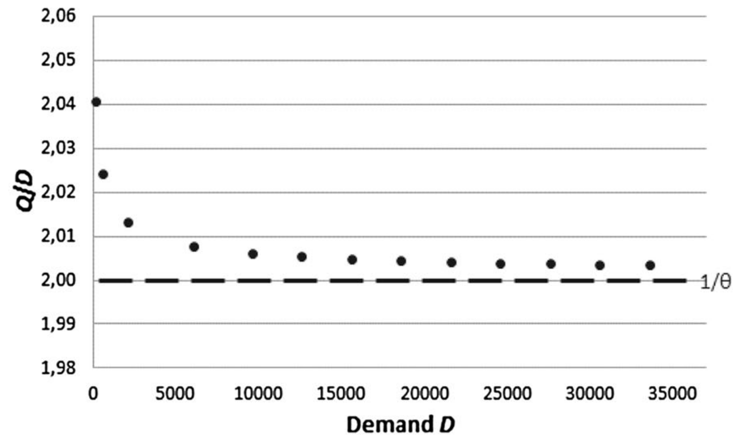
Additionally, further research should focus on extending the supply chain to an emergency option for procuring extra units in case of under-delivery. This option was introduced by Inderfurth and Clemens (2014) and it was shown to coordinate the supply chain by applying the WHP contract. This, however, only holds if the supplier, and not the buyer, is able to utilize the emergency source. In the current setting, this option might reveal a similar performance. Besides, the setting can also be adjusted with respect to supply chain structure. An important aspect in this context is the extension from a serial to a converging supply chain. Another interesting extension of the current work would lie in a contract analysis for an environment where demand is also random. From research in the case of stochastically proportional yield (see Yan and Liu 2009) we know that the simple contracts considered in this paper cannot guarantee coordination while more complex ones might do so. It is an open question, however, if these results also hold under binomially distributed yields.

Concentrating on further types of yield uncertainty, the all-or-nothing type of yield realization, also known as disruption risk (see Xia et al. 2011), has hardly received any attention in literature so far. The same holds for additional yield types mentioned in Yano and Lee (1995), like interrupted geometric yield or yield uncertainty from random capacity. Furthermore, it would be a challenging task to study how contracts can be used for supply chain coordination in planning environments with multiple productions runs that are addressed in Grosfeld-Nir and Gerchak (2004).

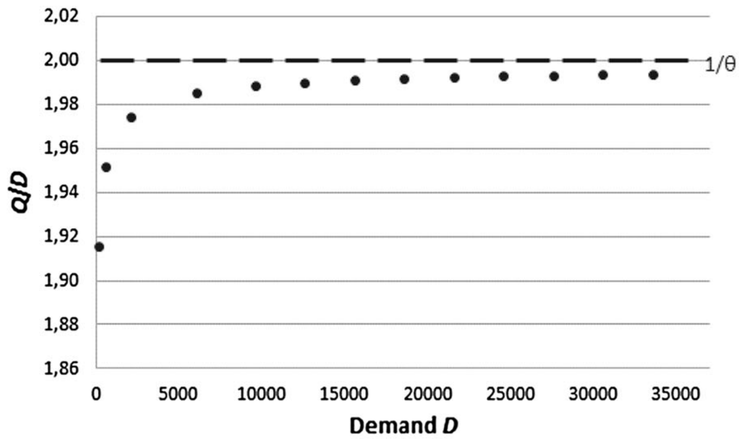
Appendix

Examples for the development of the production/demand ratio

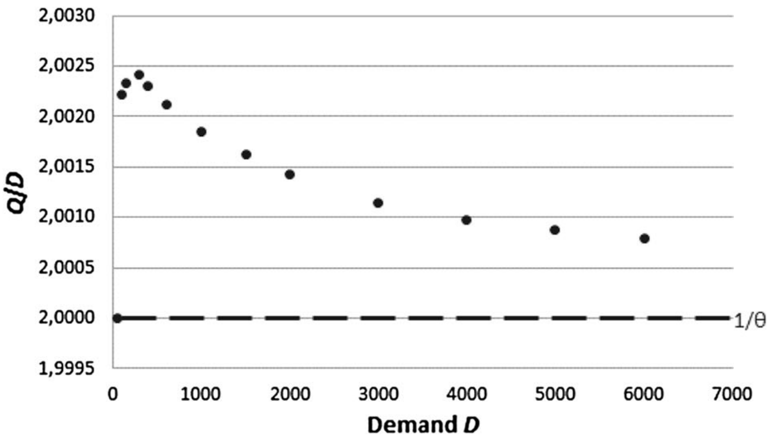
Figure 2 illustrates three exemplary curves for the Q/D-ratio with increasing demand.



(a) Data: $c = 1$, $p = 6$



(b) Data: $c = 1$, $p = 2.5$



(c) Data: $c = 1$, $p = 4.17$

Fig. 2 Three exemplary developments for production input/demand ratio for 50 % success probability which approaches $1/\theta$

It is evident from the different curves that there is no monotony in the Q/D-ratio. Yet, the results in (a) and (b) are comparable with typical newsvendor settings where the critical ratio (here it is given by c/p) determines whether optimal production quantities are below or above expected demand (which corresponds to

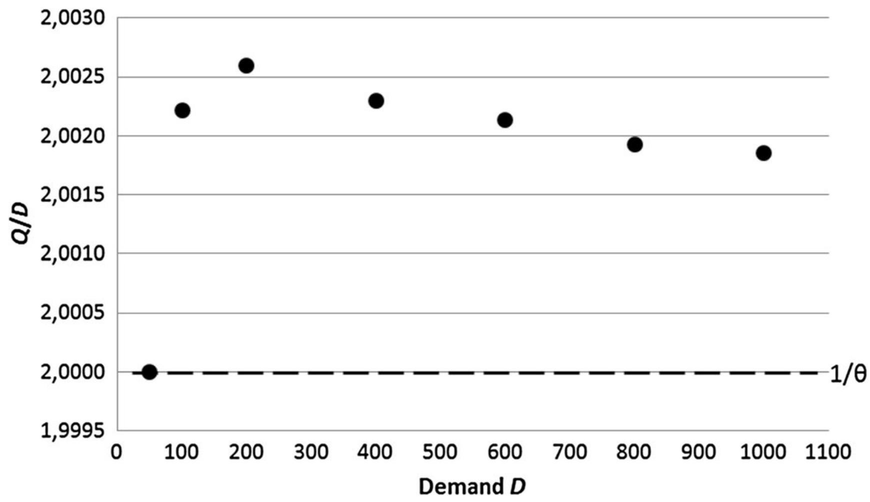


Fig. 3 Extraction from Fig. 2 part (c)

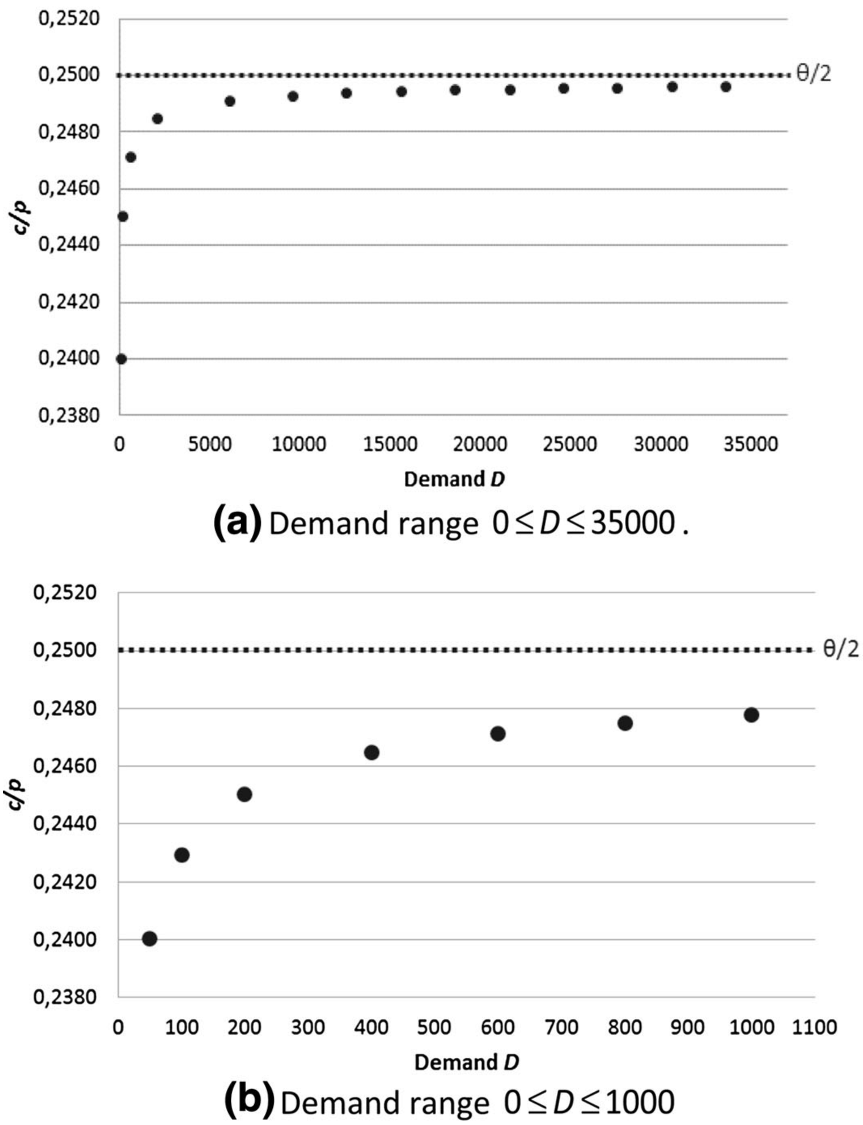


Fig. 4 Critical parameter ratio (c/p) which guarantees a Q/D ratio of $1/\theta$

production yield in our setting). The major difference is that, in addition to prices and costs, also demand has an influence on the production decision as the production risk decreases with increasing quantity. A high margin [as in (a)] causes Q/D ratios above $1/\theta$ while low margins [compare (b)] lead to production inputs below the expected yield. Yet, the shape of the curve in (c) is quite interesting. The changes in Q/D are minor with increasing demand, however, at one point the curve intersects with $1/\theta$ (which is at $D = 50$). For illustrative purpose, the segment $0 \leq D \leq 1000$ from curve (c) is extracted in Fig. 3.

The intersection with $1/\theta$ raises the question whether there exist parameter combinations which always guarantee an inflation of demand in the amount of $1/\theta$. Figure 4 part (a) answers this question by illustrating the c/p ratio which results in $Q/D = 1/\theta$ for increasing demand.

Part (b) of the above figure extracts the range $0 \leq D \leq 1000$ from part (a). Comparing this illustration with Fig. 3, the point $Q/D = 1/\theta$ at $D = 50$ corresponds to the starting point of the curve in Fig. 4b which is at $c/p = 1/4.17 = 0.24$.

Effects of yield misspecification if real yield is binomial

For presenting numerical examples we set the parameters as follows: $c = 1$, $p = 14$ and $D = 100$. The binomially distributed yield is approximated by the normal distribution with mean and standard deviation from (1) and (2). For $Q \geq D = 100$ this approximation is feasible for $0.06 \leq \theta \leq 0.94$ because for these values the condition $Q \cdot \theta \cdot (1 - \theta) > 5$ is satisfied. In the following Tables 1, 2, 3 and 4, miscalculated decision variables and the respective profits are indicated by the superscript *mis*.

Table 1 Supply chain decisions and profit deviations (in %) for changing retails prices under centralized decision making for 50 % success probability

p	Q^{mis}	Q^*	Π_{SC}^{mis}	Π_{SC}^*	$\Delta\Pi_{SC}$ (%)
2	100	100	0	0	0.00
3	122	194	61	92	33.73
4	141	200	141	189	25.06
5	158	203	237	286	17.14
6	173	205	346	384	9.95
7	187	208	463	483	4.05
8	200	209	577	582	0.72
9	212	211	680	681	0.02
10	224	212	775	780	0.65
11	235	213	865	879	1.56
12	245	214	955	978	2.36
13	255	214	1045	1077	3.00
14	265	215	1135	1177	3.52

Table 2 Supply chain decisions and profit deviations (in %) for changing wholesale prices under decentralized decision making for 50 % success probability

w	Q^{mis}	X^{mis}	Q_S^{WHP}	X^{WHP}	Π_{SC}^{mis}	Π_{SC}^{WHP}	$\Delta\Pi_{SC}^{WHP}$ (%)
2	265	265	215	215	1135	1177	3.52
3	220	179	211	109	1176	1176	0.00
4	196	138	207	104	1148	1173	2.11
5	180	114	205	101	1077	1170	7.97
6	173	100	205	100	1039	1171	1.30
7	187	100	208	100	1114	1173	5.07
8	200	100	209	100	1161	1175	1.20
9	212	100	211	100	1176	1175	−0.07
10	224	100	212	100	1174	1176	0.19
11	235	100	213	100	1165	1176	0.97
12	245	100	214	100	1155	1177	1.84
13	255	100	214	100	1145	1177	2.70
14	265	100	215	100	1135	1177	3.52

Effects of yield misspecification if real yield is stochastically proportional

Table 3 Supply chain decisions and profit deviations (in %) for changing retail prices under centralized decision making for a mean yield rate of 0.5

p	Q^{mis}	Q^*	Π_{SC}^{mis}	Π_{SC}^*	$\Delta\Pi_{SC}$ (%)
2	100	100	0	0	0.00
3	194	122	29	55	47.68
4	200	141	100	117	14.43
5	203	158	174	184	5.42
6	205	173	249	254	1.99
7	208	187	324	326	0.64
8	209	200	400	400	0.09
9	211	212	476	476	0.00
10	212	224	552	553	0.12
11	213	235	629	631	0.35
12	214	245	706	710	0.65
13	214	255	782	790	0.97
14	215	265	859	871	1.31

Table 4 Supply chain decisions and profit deviations (in %) for changing wholesale prices under decentralized decision making for a mean yield rate of 0.5

w	Q^{mis}	X^{mis}	Q_S^{WHP}	X^{WHP}	$\Pi_{\text{SC}}^{\text{mis}}$	$\Pi_{\text{SC}}^{\text{WHP}}$	$\Delta\Pi_{\text{SC}}^{\text{WHP}}$ (%)
2	215	215	265	265	859	871	1.31
3	211	109	220	179	857	862	0.51
4	207	104	196	138	855	847	−1.00
5	205	101	180	114	853	831	−2.70
6	205	100	173	100	854	823	−3.79
7	207	100	187	100	855	839	−1.95
8	209	100	200	100	856	850	−0.72
9	210	100	212	100	857	858	0.12
10	211	100	224	100	858	863	0.67
11	212	100	235	100	858	867	1.03
12	213	100	245	100	859	869	1.24
13	214	100	255	100	859	870	1.32
14	215	100	265	100	859	871	1.31

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4. Decision behavior in supply chains with random production yields

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Josephine Clemens

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Decision behavior in supply chains with random production yields

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Abstract

Dealing with supply risks is one of the challenges of decision makers in supply chains as producing and sourcing become more and more complex. Theoretical research on different types of supply uncertainty as well as their management is well covered. Behavioral aspects in this context, however, have not received much attention so far. In this paper, we present an experimental study which aims at investigating how subjects make decisions of ordering and producing in the presence of random production yields at a supplier, i.e. production output is a random fraction of production input. Subjects were confronted with the situation of either the buyer or the supplier in a simple two-tier supply chain with deterministic demand and had to make the respective quantity decisions. Results show that buyers have a good understanding of the situation and are likely to follow a probabilistic choice rule. In addition to that, hedging against supply risks drives their behavior of over-ordering. Suppliers on the other hand start off with moderate production decisions but improve over time which indicates learning effects. Furthermore, the study shows that additional sharing of information on yield rates is no cure for inefficient behavior of the buyer.

Keywords: Behavioral operations management, supply chain interaction, random yield, supply risk

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1. Introduction and motivation

The issue of random production yields has been discussed in literature variously (see e.g. Yano/Lee (1995)) as it is of high practical relevance. The cultivation of agricultural products, the transformation of chemicals into pharmaceuticals or the manufacturing of highly sophisticated semiconductors are examples for random yield processes in that the same production input results in different and unknown production outputs. The presence of such uncertainty increases the complexity of decision making which is less an issue from an analytical point of view than from a behavioral perspective.

Nowadays, the topic gains more practical relevance as supply risks increase steadily because production processes and procurement strategies becoming more and more complex. Tang (2006) reviews numerous aspects of supply risks, their origin, and how to handle them. A practical example for our special supply risk of yield uncertainty is provided by Kazaz (2004) who addresses olive oil production in Turkey which is highly vulnerable to weather conditions and infestation. The production of vaccinations and other pharmaceuticals is another example for processes which underlie risks of uncertain outcome (Chick et al. (2007)).

The intention of this experimental study is to capture the specific allocation of risks in a supply chain which is characterized by production yield uncertainty. Considering a two-level supply chain with one buyer (or retailer) and one supplier (or manufacturer) the risk is directly present at the supplier stage. Depending on production input and yield realization, the supplier faces risks of understocking as well as overstocking which result in lost revenues or unprofitable production efforts. Nevertheless, the yield risk also impacts the performance of the buyer. As the buyer needs to fulfill end customer demand (which is known) by procuring units from the unreliable supplier the risk spreads out through the supply chain in downstream direction. Assuming that production output cannot be corrected by another production run, rework, or external procurement, the buyer faces a risk of underdelivery. This can have a direct impact on his profit performance as underdelivery can result in unsatisfied end customer demand. Additionally, and comparable to the supplier's situation, a possible overstock incurs cost but cannot be converted into revenues.

The purpose of this paper is to shed light on how the supply chain handles the supply risk by conducting laboratory experiment with human subjects as decision makers. On the one hand, we investigate how supplier and buyer separately react to the risk and how that affects their and the total supply chain's performance. For that reason, they were each confronted with an automated opponent. On the other hand, the paper tries to answer the question on how the supply risk is propagated through the supply chain by interaction between buyer and supplier. Thus, experiments were implemented in which human suppliers and buyers interacted with each other.

The rest of the paper is organized as follows. The underlying theoretical model as well as related literature are presented in §2. Subsequently, the experimental design and the hypotheses are introduced in §3 and §4, respectively. In §5 we present and discuss the results of the experiments. And finally, concluding remarks and an outlook to future research is presented in §6.

2. Analytical background and related literature

2.1 Model

The underlying single-period model is a two-level supply chain with a single buyer procuring from a single supplier. The buyer has to fill deterministic end customer demand *D* at a retail price *p* by ordering an amount *X* from the supplier. The supplier has to produce items at a per unit cost *c* in order to fill the buyer's order. Her production decision is denoted by *Q* and she receives the wholesale price *w* per unit delivered (simple wholesale price contract). However, the underlying production process is subject to some risks which result in uncertain production yields. It is assumed that production yields are stochastically proportional, i.e. production output is a random fraction *z* of production input (with mean μ_z). This assumption is reasonable as this yield type describes a situation where the whole production lot is exposed to risks which may affect the output to a smaller or greater extend. As a consequence, a portion of the lot may turn out unusable. This is the case in agriculture, e.g., where weather or vermin can cause (parts of) the harvest to be destroyed.

In case production output is below order quantity, the supplier cannot fulfill the buyer's order. Consequently, the buyer may not be able to fill end customer demand in full amount. If production output exceeds the order quantity, excess units are not shipped to the buyer but are disposed off at zero cost. In case the buyer ordered and got delivered more than demand, those excess units are also of no value. In this single period game, all information on cost and price parameters as well as on demand and yield distribution is common knowledge. The course of actions and decisions is depicted in Figure 1.

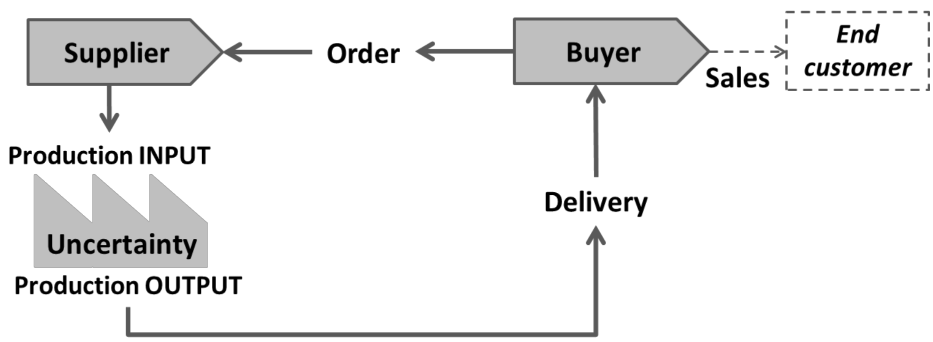

Figure 1: Sequence of events in the supply chain

The profit functions of the buyer $\Pi_B(X)$ and the supplier $\Pi_S(Q|X)$ are given by

$\Pi_B(X) = p \cdot E\left[\min(z \cdot Q, X, D)\right] - w \cdot E\left[\min(z \cdot Q, X)\right]$ and

$\Pi_S(Q|X) = w \cdot E\left[\min(z \cdot Q, X)\right] - c \cdot Q$

It is assumed that the production yield rate *z* is uniformly distributed between 0 and 1 with a mean of 0.5. From the first order condition of the buyer's and the supplier's expected profits, respectively, the closed form solutions of the optimal order quantity and production input are given by $X = D \cdot \sqrt{(c \cdot p) / \left(w \cdot \left(\sqrt{2 \cdot c \cdot w} - c\right)\right)}$ and $Q(X) = X \cdot \sqrt{w/(2 \cdot c)}$. The optimal expected profits transform to $\Pi_B = \left[p - \sqrt{p \cdot \left(\sqrt{8 \cdot c \cdot w} - 2 \cdot c\right)}\right] \cdot D$ and $\Pi_S = \left[w - \sqrt{2 \cdot c \cdot w}\right] \cdot X$ (see Inderfurth and Clemens (2014) for details on all formulas' derivations). The underlying analysis always assumes profitability of the business, i.e. that (expected) per unit cost is no larger than per unit price ($c / \mu_z \leq w \leq p$). Depending on the parameter setting, it can be optimal for the buyer to order above demand level in order to account for the supply risk. Thus, in such settings both actors face the risk of overstocking. If the buyer orders exactly the demand (ordering below demand is never optimal) he faces only the risk of understocking.

We note that the simple wholesale price contract is not efficient. The double marginalization effect hinders the contract to achieve coordination (see Inderfurth and Clemens (2014)). However, in the experimental study, profits and profit losses of the actors always refer to the maximum profits under the wholesale price contract (individual optimization) and not to the supply chain optimum (joint optimization).

2.2 Related literature

The theoretical side of supply uncertainty in production and ordering decisions within supply chains is one stream of literature relevant to our research. The most fundamental review of research on random yields in production systems, a special type of supply risk which is focal in our research, is provided by Yano and Lee (1995). To name a selection, Gerchak et al. (1988) and Henig and Gerchak (1990) analyze optimal production policies in periodic review systems while Gerchak et al. (1994), Gurnani et al. (2000) and Pan and So (2010) go further by considering random yields in assembly systems. He and Zhang (2008, 2010), Keren (2009), Wang (2009) and Xu (2010) as well as Inderfurth and Clemens (2014) and Clemens and Inderfurth (2015) extend the research to interaction in a two-tier supply chain where yields at the first stage of the supply chain are random.

Another stream of literature concerns behavioral economics in supply chains with uncertainty. The analysis of behavioral aspects in random demand supply chains (newsvendor problem) is well advanced. Schweitzer and Cachon (2000) found subjects to order suboptimal but were able to explain their behavior with simple heuristics. Their research has been extended notably ever since. Relevant to our research are studies by Benzion et al. (2007), Bolton and Katok (2008) as well as Bostian et al. (2008) who investigate the role of learning over time in newsvendor type experiments. Another adjacent field of studies includes the impact of information sharing on decision making in this setting (see Bolton et al. (2012)). Both aspects connect with the supplier behavior we observe in our experiments.

Regarding buyer behavior, two aspects are found to be main drivers for our observations, namely random or probabilistic choices and social preferences. Probabilistic choices as a form of bounded rational behavior (Luce (1959) and McKelvey and Palfrey (1995)) indicate that choices which lead to only small profit losses are made with higher probability than decisions which incur greater mismatches with expected profit. See Lim and Ho (2007), Ho and Zhang (2008), Su (2008), Kremer et al. (2010), Chen and Zhao (2012), Wu and Chen (2014), and Pavlov et al. (2016) for bounded rationality models in a supply chain context.

Social preferences such as fairness concerns on the other side explain behavior which is suboptimal but leads to more even profit allocations between actors (Fehr and Schmidt (1999) and Bolton and Ockenfels (2000)). Related to our research are approaches in a supply chain context as in Loch and Wu (2008), Katok and Pavlov (2013), Katok et al. (2014), and Hartwig et al. (2015).

The combination of those two streams, random yield in production systems and behavioral economics in supply chains under uncertainty, form the frame for our current research. First approaches to this field were made by Gurnani et al. (2014) who study how subjects place orders in supply chains where one supply source underlies two types of risk, namely disruption risk and yield uncertainty. They find that bounded rational behavior can explain the observed sourcing decisions by subjects. However, they model a supply chain without interaction where the buyer always has the option to procure from a reliable supplier in addition to the uncertain source and find that subjects tend to diversify in placing orders. Goldschmidt (2014) as well as Goldschmidt et al. (2014) come close to the approach by Gurnani et al. (2014) but focus solely on an all-or-nothing risk for the buyer. In their setting, a disruption in supply is very rare but has substantial impact on the performance of the buyer. In experiments, they find that buyers move from single-sourcing to dual-sourcing and back to single-sourcing in the aftermath of a disruption.

Craig et al. (2016) conduct field experiments in the apparel industry to analyze buyer behavior in response to performance increases of an unreliable supplier (in terms of fill rate). They find significant order increases when suppliers become more reliable.

All approaches, however, leave out the decision made by the unreliable supplier and consider it given at all times. Furthermore, the dominant risk considered in the approaches is the disruption in supply which leads to a total loss in delivery and not just a fraction. Also, interaction between buyer and supplier in the supply chain is not addressed by any of the aforementioned and thus, no insight is given into the decision making in complex random yield supply chains. By considering both actors' decisions, ordering and producing, we reveal insights into supply chain members' behavior independently but also in interaction with each other and the effects on the supply chain as a whole.

3. Experimental implementation

The experimental setting aims at investigating the model described above and analyzes how the present risk is perceived and handled in the supply chain. As both stages have to account for the existing uncertainty in the supply chain it is worthwhile to investigate the actors' behavior separately but also in interaction with each other.

3.1 Experimental design

In total, three different experiments were conducted. In the first experiment, the so-called baseline buyer game (BUYER), the part of the supplier was automated (i.e. a computer chose the profit maximizing input quantity given the incoming order) while the buyer was played by a subject. The subject was confronted with the buyer's situation from the above described supply chain and had to make an order decision. The supplier's decision is the *best response* to the buyer's order. After the yield rate has materialized, production output was calculated and a corresponding delivery was made to the buyer. The delivery was used to fulfill end customer demand as far as possible.

The counterpart to that experiment was the baseline supplier game (SUPPLIER) with the reverse situation. The buyer's order was automated with the objective of maximizing profits. The supplier was played by a subject who received the order and had to decide on a production input quantity. The course of events is as described above.

The treatment conducted in a third experiment was to eliminate all automated decisions and let both parts be played by subjects, i.e. the subjects interacted in the supply chain (INTERACT game). Subjects were informed that they were matched with the same partner in every round (for the instructions handed out to the subjects see Appendix A.4). An overview of the experiments is given in Figure 2.

	Experiments
Supplier automated	**BUYER game**
Buyer automated	**SUPPLIER game**
Suppplier and buyer as subjects	**INTERACT game**

Figure 2: Overview of experiments

In the experiments, data was as follows: $c = 1$, $w = 6$, $p = 25$, and $D = 100$. The prices were chosen according to practical observations. In commodity goods industries, e.g., it is common that buying firms earn a multiple of the prices they pay to manufacturers for their goods (Voigt (2012)). Comparing profits in the supply chain, the supplier usually receives only 20% – 30 % of the total supply chain profit while the buyer gains the major portion.

The yield rate is uniformly distributed between 0 and 1 with mean 0.5. Furthermore, yield rates in consecutive rounds in this single-period game are independent and identically distributed.

The optimal order quantity of the buyer is $X = 130$, the optimal response of the supplier is $\sqrt{3}$ times the incoming order, i.e. if the buyer orders 130, the supplier's optimal production input is $Q = 225$. The maximum expected profits of buyer and supplier are $\Pi_B = 1.390$ and $\Pi_S = 330$, respectively.

In all experiments, feedback was given after each period, i.e. after all actors' decisions in one round have been made. For the buyer, feedback was given on his ordering decision, on delivery from the supplier, sales, and generated profit. The supplier received information on the incoming order, her decision on production input, the materialized yield rate and output as well as the quantity delivered to the buyer and her profit of the round.

3.2 Experimental protocol

The experiments were conducted in the Magdeburg Experimental Laboratory of Economic Research (MaxLab) with subjects recruited using the tool ORSEE (Greiner (2004)). Altogether, 78 subjects participated in the experiments with between subject design, i.e. each subject participated only once. The BUYER game and the SUPPLIER game were played by 20 subjects each, representing 20 supply chains. In the INTERACT game, 19 supply chains were generated, i.e. 38 subjects were present to play the game (19 buyers and 19 suppliers). The game was implemented using the software tool zTree (Fischbacher (2007)). In all sessions, the subjects arrived at the Lab and were handed out instructions which were read together with the experimenter. Afterwards, subjects were randomly assigned to computers and the role of either supplier or buyer. The experiments consisted of 30 one-shot games, i.e. the subjects played 30 rounds of the game. In order to maintain comparability between the experiments, a string of 30 random yield rates was generated in advance and implemented in each game.[1] All participants were paid by performance.

4. Hypotheses

It is hypothesized that theory holds and that actors are rational profit maximizers. The Hypotheses below specify the general assumption.

H1. In BUYER and INTERACT, buyers order an amount of 130 throughout the experiment.

H2. In SUPPLIER, suppliers always produce 225 units or, more generally, $\sqrt{3}$ times the order quantity.

H3. In INTERACT, suppliers produce $\sqrt{3}$ times the incoming order in every period of the experiment.

5. Results

The following chapter summarizes the results of our experiments. We start by analyzing the effects of our treatment variations on supply chain performance (Section 5.1). We then continue by analyzing the buyers' orders (Section 5.2.) which are the input for the suppliers' production decision (Section 5.3). Section 5.4 analyzes supply chain behavior when yield information is shared between the supplier and the buyer.

5.1 Overall supply chain results

In terms of expected supply chain profits (sum of buyer and supplier expected profits), the benchmark for rational and expected profit maximizing supply chain parties is 1789[2]. We observe that the performances in all treatments (measured by mean profits) are significantly lower than the benchmark of 1789 (Wilcoxon, p=0.00 for BUYER and INTERACT, p=0.02 for SUPPLIER, two-sided), ranging from 1752 in BUYER over 1735 in SUPPLIER to 1634 in INTERACT.[3] Boxplots for supply chain profits of all games are illustrated in Figure 3.

[1] For each period's yield rate, see Table 5 in the Appendix.

[2] Note that the expected maximum profits for buyer, supplier, and supply chain differ from the theoretical benchmarks. This results from the very limited scenario of 30 rounds with 30 randomly drawn yield rates. This stream of yield rates generates higher than benchmark profits. However, they would level towards the theoretical values when extending the duration of the game to more rounds.

[3] See Table 2 and Table 3 for mean values and standard deviations.

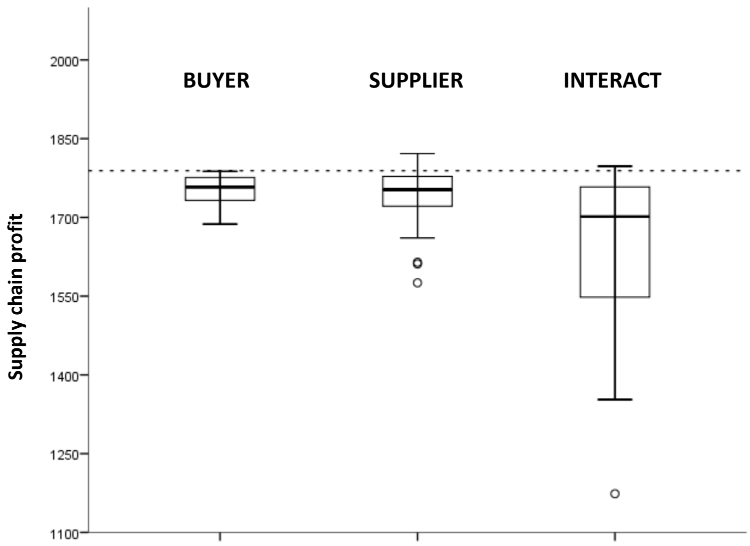

Figure 3: Boxplots for supply chain profits per game with benchmark at 1789[4]

An interesting observation is that mean profits in INTERACT are considerably lower than in the two separate games. The following Table 1 summarizes the profit losses by actor and game. We observe that buyers perform worse in interaction than in the isolated game while suppliers perform better (in terms of mean deviations from optimal profits[5]). Hence, buyer behavior seems to trigger the lower performance in INTERACT.

	Buyer profit loss	Supplier profit loss	Supply chain profit loss
BUYER	-31 (-2,2%)	[-6]	-37 (-2,1%)
SUPPLIER	[-39]	-15 (-4,1%)	-54 (-3%)
INTERACT	-144 (-10,1%)	-10 (-2,8%)	-155 (-8,7%)

Table 1: Mean deviations from benchmark profits per game[6]

The situation is somewhat different when comparisons between games are made. Comparing BUYER with SUPPLIER, total supply chain profits do not deviate significantly from each other, i.e. even though profits are suboptimal, subjects on average do not behave worse in one game than in another (MWU, p=0.892, two-sided). However, the interaction effect is considerable. Profits are significantly lower in INTERACT than in BUYER (p=0.023) or SUPPLIER (p=0.043) (MWU, two-sided).

[4] Outliers are indicated by circles and are not included in the determination of the boxplot. Outliers are defined as values which are between 1.5 and 3 box lengths from a hinge of the box.

[5] Throughout the analyses, supplier profit losses are measured in terms of deviation from best response (to the order arriving from the buyer, whether it is optimal or not) while all other values are deviations from theoretically predicted profits (i.e. all actors behave optimally).

[6] The numbers in square brackets are profit losses caused by suboptimal behavior of the counterpart alone as the decision maker is automated by a computer.

The reasons for the observed differences in buyer and supplier behavior are discussed in the following sections 5.2 and 5.3.

5.2 Buyer results

We first consider the buyers' ordering behavior when the supplier reacts optimally (BUYER game, Section 5.2.1) and when interacting with another human subject (Section 5.2.2). We then analyze the effects of human interaction by comparing the BUYER treatment with the INTERACT treatment in Section 5.2.3 and discuss yield chasing effects in Section 5.2.4.

5.2.1 BUYER game

We observe that buyers' orders are not significantly different from the benchmark of 130 units in every round (Wilcoxon, p=0.247). Table 2 summarizes the mean deviations with standard deviations (in brackets) from optimal ordering as well as the corresponding profits for BUYER. [7]

		Optimum	Observed
Treatment: BUYER	Order	130	127 (20)
	Deviation from optimum	-	-3 (20)
	Buyer profit	1426	1395 (47)
	Deviation from optimum	-	-31 (47)**
	Supply Chain profit	1789	1752 (30)
	Deviation from optimum	-	-37 (30)***
Treatment: INTERACT	Order	130	139 (36)
	Deviation from optimum	-	9 (36)
	Buyer profit	1426	1282 (112)
	Deviation from optimum	-	-144 (112)***
	Supply Chain profit	1789	1634 (173)
	Order	130	139 (36)
[*** (p<0.00); ** (p<0.05); * (p<0.1)]			

Table 2: Means and standard deviations of optimum vs. observed orders and profits per treatment

While orders are on average not significantly different from 130, we observe a substantial degree of variance. Thus, hypotheses H1 (subjects order always 130) can be rejected.

Figure 4 visualizes the observation by showing mean deviations from the optimal order per subject (ID). Having a closer look at the deviation from predicted orders, 11 out of 20 subjects (55%) order significantly below and 6 out of 20 (30%) significantly above 130 (sign-test, p<0.05, two-tailed). The remaining 15% of subjects order not significantly different from optimum.

[7] Note that mean values per subject (over all 30 rounds) were used in the analysis because one subject's decisions are not independent between rounds.

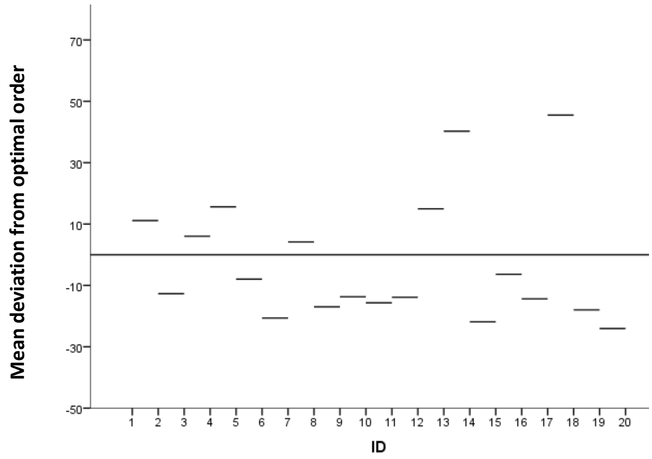

Figure 4: Mean deviations from optimal order per subject (ID)

As mean orders do not show systematic patterns, it is worthwhile having a closer look at decisions over time. Figure 5 shows how mean orders develop over time with the thick black line indicating the optimal order of 130.

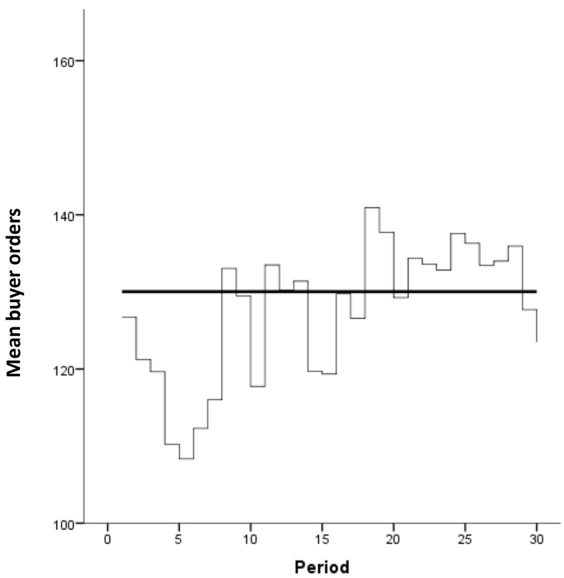

Figure 5: Development of orders over time in BUYER

Orders come quite close to the optimum after a "warm-up" in the first 7 periods. They are significantly below optimum in the first 10 periods by -11 units on average (Wilcoxon, p=0.000, two-tailed), but approach optimum in the last 10 periods (on average deviation is +3 units) (Wilcoxon, p=0.636, two-tailed). Mean deviation over all periods is -2.6 (sign-test, p<0.00, two-sided).

As tests for patterns in behavior do not reveal much insight, it is reasonable to imply that subjects follow a trial-and-error-pattern to the best of their understanding. The histograms in Figure 6 illustrate the deviation of buyer profits from optimal profits in BUYER and INTERACT which may support the hypothesis that buyers make probabilistic choices.

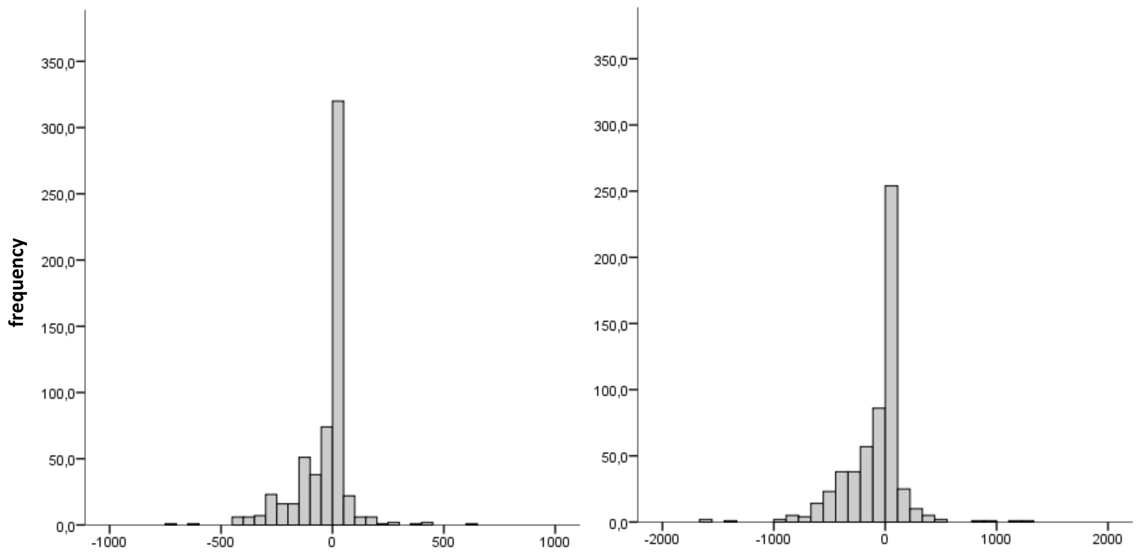

Figure 6: Histograms for deviation from optimal profit in BUYER and INTERACT

The histograms show that order decisions which have no or only marginal impact on the subject's profit are made with the highest frequency, i.e. decisions which lead to only small profit losses are made with higher probability than those which incur higher losses. This indicates that actors are not making optimal but still "good" decisions when translated into profits. The presumption is convincing that actors cannot calculate the optimal order quantity but still have a good understanding of the situation and try to minimize the mistake they make when placing an order to the best of their ability. Having a closer look on the data reveals that nearly 47% of all orders deviate between 1 and 20 units from the optimal order. The resulting profit loss (given the best response of the automated supplier) is -22 (-1,5%). 35% of all orders deviate between 21 and 40 units from the optimum with a resulting profit loss of -63 (-4,4%). Thus, the majority of orders are close to optimum and only small profit losses result from this suboptimal behavior. Testing the data with respective probabilistic choice models which identify parameters for the occurrence of random choices is left for future research.

5.2.2 INTERACT ordering

In INTERACT mean orders, again, alternate around the optimum of 130. We observe a non-significant mean deviation from the optimum of +9 (sign-test, p=0.295, two-sided). Having a closer look, 9 subjects order significantly above optimum, 6 significantly below, and 2 are not significantly different from the optimum. Figure 7 visualizes the observation per subject.

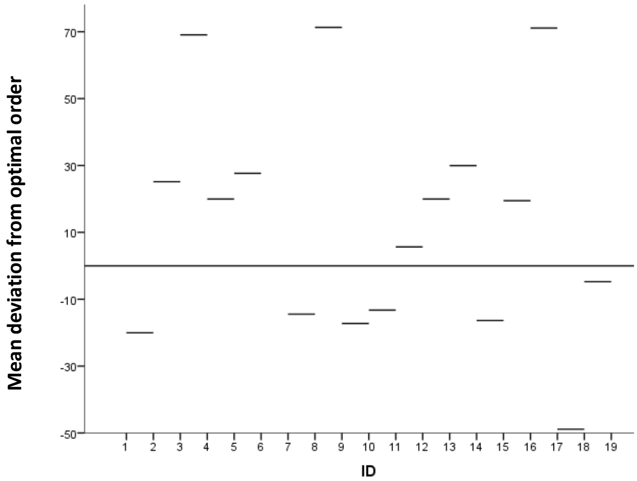

Figure 7: Mean deviations from optimal order by subject (ID) in INTERACT

Considering repetition as an influential factor, orders also show no significant pattern (see Figure 8). Splitting the total game span into three equal intervals, orders are not significantly above 130 in any of the three (e.g. Wilcoxon, p=0.295 in periods 1-10 and p=0.212 in periods 21-30, two-tailed). Furthermore, testing early vs. late decisions, no significant effect is observable between the first 10 and the last 10 rounds of the game (Wilcoxon, p=0.906, two-sided).

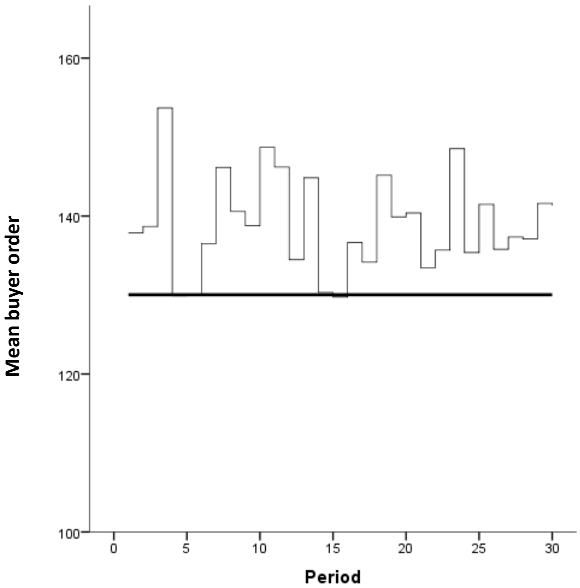

Figure 8: Development of mean orders over time in INTERACT[8]

It can be assumed, just as in BUYER, that subjects make probabilistic choices when ordering. A comparable graph to Figure 6 could be drawn which shows that "good" decisions are made with higher probability and profit losses due to suboptimal behavior are not severe.

[8] Note that buyer orders are not significantly above optimum even though it could be predicted from the figure.

5.2.3 Treatment comparisons

Ordering decisions show no systematic pattern stand-alone for the games BUYER and INTERACT. However, if orders between those two games are analyzed, interesting insights are observed. Buyers in interaction order significantly more than in case of a computerized counterpart (MWU, p=0.00). This may be explained by two considerations, namely social preferences and hedging against coordination risks.

First, by using the decision support tool provided (profit calculator) subjects can easily detect that profit allocation is highly unequal in the game. Given the structure of the interaction approximately 20% of profits are generated by the supplier but 80% by the buyer.[9] This may have an impact on the buyer's decision because by ordering more, a larger portion of the total profit can be awarded to the supplier. Such social or fairness preferences may trigger the buyer's behavior. Analyzing this aspect, it is revealed that mean supplier profits are closer to optimum in INTERACT than in the baseline SUPPLIER game where orders are automated and always amount to 130 (mean deviation in supplier profit is -15 in SUPPLIER and -10 in INTERACT; MWU, p=0.866, two-sided). However, this observation may be triggered by either improved supplier decisions or by higher orders as discussed above.

Second, the uncertainty about how much the supplier will produce may encourage the buyer to safeguard against potential stock outs by ordering a higher amount and thus, setting incentives for the supplier to produce a larger lot. Testing the data shows that on average overstocks are indeed higher in INTERACT than in BUYER (21 vs. 18 units) but the differences are not significant (MWU, p=0.978, two-sided). The issue of mismatching demand will be discussed in more detail in the next chapter.

5.2.4 Yield chasing

As mentioned above, the buyer's stocking behavior is worth further investigation. In order to dig for reasons behind this behavior linear regression models were run on the data. For that matter, it was tested whether outcomes in one period impact the decision in the next period. The regression model includes the previous period's amounts of missed demand[10] $\left(Demand - Delivery\right)^+$ and overstock $\left(Delivery - Demand\right)^+$ at the buyer site as well as fixed effects for periods and subjects into the analysis.[11]

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[9] A numerical example for this relation is provided in Table 7 in the Appendix.

[10] Missed demand is a more reasonable indicator than underdelivery ($\left(Order - Delivery\right)^+$) because it has a direct negative impact on profits whereas in case of underdelivery end customer demand may still be met.

[11] For the regression model and all results, see Table 8 and Table 9 in the appendix.

It is discovered that both quantities, overstock and missed demand, have significant impacts on the decision.[12] Interestingly, the direction of adjustment is counterintuitive. In BUYER, orders decrease after a stock out in the previous period and increase when an overstock occurred before. The observed behavior can be explained by a phenomenon called "Gambler's fallacy"[13] which states that actors make mistakes when estimating probabilities of uncertain events. The irrational assumption is that if delivery was high, it will be low in the next period with a higher probability (and vice versa). Additionally, buyers tend to adjust orders stronger after a stock-out than after an overstock (in numbers, the adjustment to one unit of understock is -0.604 while an overstock of one units leads to an order increase of 0.184). Thus, "having too much" is obviously considered worse than "having too little".

Oddly, in INTERACT the impact of both, overstock and missed demand, on the order is negative. This effect may result from the specific situation the buyer finds himself in. In comparison to the BUYER game where the supplier's decision is automated the buyer faces the additional uncertainty of a real decision maker in INTERACT. Knowing about the supplier's uncertain production process in addition to not knowing what the subject supplier will chose as a production input may cause high insecurity for the buyer.

5.3 Supplier results

After having analyzed the buyer situation, we now consider the suppliers' input decisions, first, with automated buyers who order always 130 (SUPPLIER game, Section 5.3.1) and second, when interacting with a human buyer (Section 5.3.2). We then analyze the effects of human interaction by comparing the SUPPLIER game with the INTERACT treatment in Section 5.3.3 and discuss yield chasing effects in Section 5.3.4.

5.3.1 SUPPLIER game

The analysis of the baseline game SUPPLIER with an automated buyer (always ordering 130 units) reveals that subjects do not choose production quantities according to theory. Rather, the choices of production input are on average below the predicted ones (or the "best response" of $\sqrt{3}$ times the order from the buyer). Thus, hypotheses H2 (In SUPPLIER, suppliers always produce 225 units or, more generally, $\sqrt{3}$ times the order quantity) can be rejected. Mean inputs are 213 but they are not significantly below the optimum of 225 (Wilcoxon, p=0.232, two-sided). On a per subject level, analyses show that 8 subjects produce significantly below, 6 significantly above best response, and 6

[12] Note that yield rates are *i.i.d.* while observations may draw another picture.
[13] See Tversky and Kahneman (1971, 1974) for details on the phenomenon.

not significantly different from it. Figure 9 a) provides this rather inconclusive picture of mean deviations from the optimal input per subject in the game.

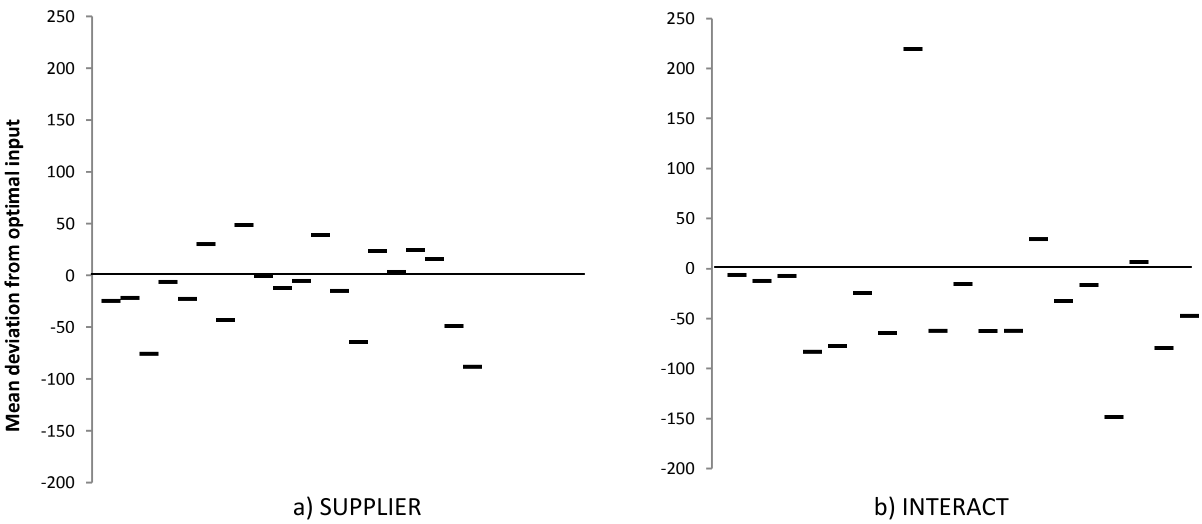

Figure 9: Mean deviations from optimal input per subject (ID) in a) SUPPLIER and b) INTERACT

As a consequence of suboptimal input choices, supplier and total supply chain profits are significantly below optimum (Wilcoxon, p<0.1, two-sided). The first section in Table 3, summarizes means and standard deviations for input choices and corresponding profits in SUPPLIER.[14]

		Optimum/Best response	Observed
Treatment: SUPPLIER	Input	225	213 (38)
	Deviation from optimum	-	-12 (38)
	Supplier profit	362	347 (15)
	Deviation from optimum	-	-15 (15)***
	Supply Chain profit	1789	1735 (67)
	Deviation from optimum	-	-54 (67)**
Treatment: INTERACT	Input	241	212 (104)
	Deviation from optimum	-	-29 (73)**
	Supplier profit	362	352 (86)
	Deviation from optimum	-	-10 (86)***
	Supply Chain profit	1789	1634 (173)
	Deviation from optimum	-	-155 (173)***
[*** (p<0.00); ** (p<0.05); * (p<0.1)]			

Table 3: Means and standard deviations of optimum vs. observed production decisions and profits

[14]Mean values per subject (over all 30 rounds) were used in the analysis because one subject's decisions are not independent between rounds.

5.3.2 INTERACT input

When dealing with a human counterpart, the results are more pronounced. In INTERACT, mean input quantities are 212 which is significantly lower than best response of 241 given the incoming orders from the human buyers (Wilcoxon, p=0.007, two-sided).[15] Furthermore, variation is higher in INTERACT than in SUPPLIER. Consequently, hypothesis H3 can be rejected.

We observe 14 out of 20 (70%) subjects ordering on average significantly below best response, 3 out of 20 (15%) significantly above best response, and only for 2 subjects the mean deviation is not significantly different from zero (see Figure 9 b)).

Given that incoming orders may differ from round to round due to the subject buyer, optimal production decisions are not constant. Figure 10 shows the relation of subject inputs versus best response inputs in INTERACT with the 45° line indicating a 1:1 relation. The majority of realizations is to the left of the line which means that inputs are too low compared to what would have been optimal given the incoming order.[16] Explaining the root-causes for this behavior is left for future research.

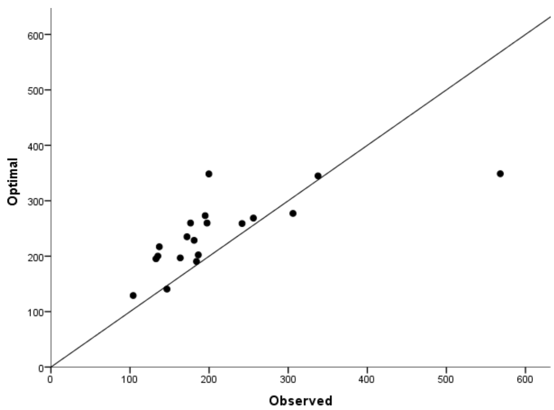

Figure 10: Scatter plots of mean subject inputs vs. optimal inputs in INTERACT

Analyzing decision behavior over time, it can be observed that there exists a tendency to higher inputs in later periods, and partial overshooting in the last periods. Mean decisions move closer to the optimum in later periods which may indicate some learning effects. The graphs below illustrate mean inputs over time and how they approach optimal inputs and even overshoot them (mainly) in later periods for both games.

[15]Mean values per subject (over all 30 rounds) were used in the analysis because one subject's decisions are not independent between rounds.
[16] A similar plot can be generated for SUPPLIER but as best response is always 225, the resulting graph is not as illustrative.

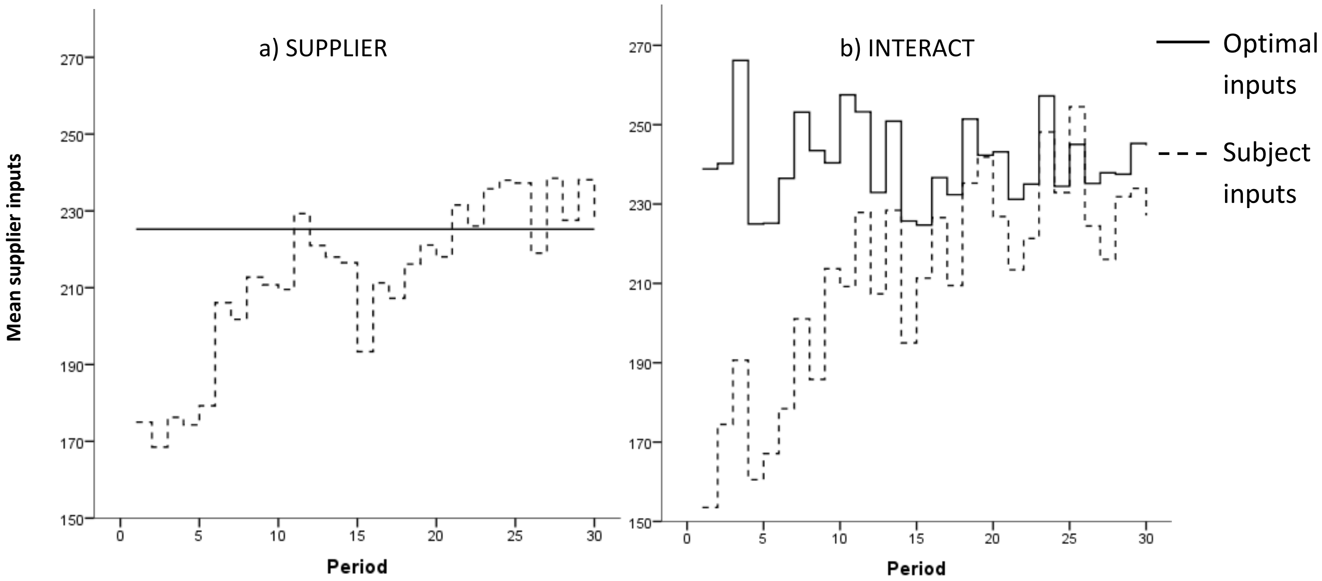

Figure 11: Mean inputs in a) SUPPLIER and b) INTERACT over time

Testing the data reveals that in the first 10 periods mean deviation from best response inputs is -34 units in SUPPLIER and -59 units in INTERACT which is significantly lower than zero (for both: Wilcoxon, p=0.00, two-sided). In the last 10 periods, however, it is +7 units in SUPPLIER and -10 in INTERACT but only deviations in INTERACT are still significantly different from zero (Wilcoxon, p=0.000, and for SUPPLIER, p=0.407, both two-sided).

5.3.3 Treatment comparisons

Figure 12 shows how mean deviations from best response input develop over time for both games.

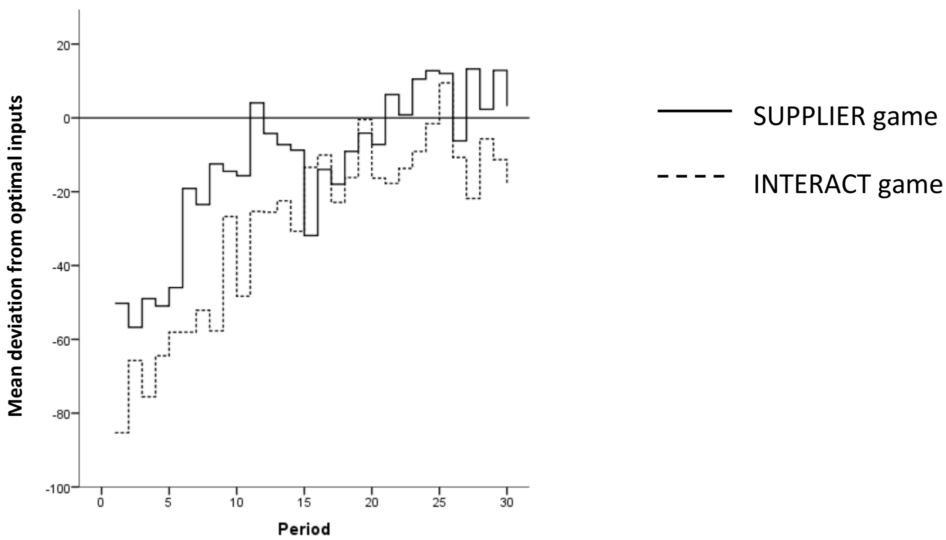

Figure 12: Mean deviations from optimal inputs over time

Thus, the decisions improve over time as the gaps between subject and optimal choices (illustrated by horizontal line at value of zero) diminish. Nevertheless, inputs especially for SUPPLIER game overshoot the optimal value in the last periods.

The gap between subject and optimal input choices is significantly larger in INTERACT than in SUPPLIER (MWU, p=0.065, two-sided; see Table 3 for mean values) with significantly higher variation. That means subjects behave quite differently when facing a computerized or a subject buyer which indicates that the supplier is influenced by variations in incoming orders (even though mean orders are not significantly larger than 130, variances are significantly different from zero (Levene test, p=0.000, two-sided, see Table 2). Interestingly, when comparing early vs. late decisions in both games, only decisions in the last 10 periods differ significantly between games (MWU, p=0.049, two-sided). More specifically, in each interval deviations from best response are higher in INTERACT. However, while subjects produce always below optimum in INTERACT, in SUPPLIER they overshoot the optimum in the last 10 periods which results a significant deviation between games in this interval.[17]

5.3.4 Yield chasing

In optimum, the best response to incoming orders is to produce $\sqrt{3} = 1.73$ times the order. In order to gain more insight into the supplier decision making, linear regressions with subjects and yield rates as fixed effects were ran on the data.[18] The effect of adjustment to changing orders in INTERACT is 1.724 (highly significant) which is close to the optimal multiplier.[19] Yet, subjects take further factors into consideration when making input decisions which results in suboptimal input choices. The results highlight that there is a significant effect of mismatching the ordered amount, just as observed for the buyers. In short, subjects incorporate the amount of underdelivery $\left[Order - Output\right]^+$ as well as overstock $\left[Output - Delivery\right]^+$ of the previous period into their decision making process. As for the buyer's order choices, the direction of adjustment is counterintuitive. A shortage leads to a reduction in input in the next period while an overstock increases the next period's input. Again, the observation can be explained by "Gambler's fallacy". Moreover, we observe that an underage leads to a significantly lower downward adjustment of the input in the next period than an overage which implies that "having too much" is considered worse than "having too little". The results for INTERACT are similar.

[17] See Table 10 and Table 11 in the appendix for all MWU-test results for the supplier decision.
[18] The model and the full set of results are summarized in the appendix, Table 12 and Table 13.
[19] Note: There is no order adjustment effect in SUPPLIER since the order size is constant over all periods in this game.

Analyzing early versus late decisions, regressions over ‘early’ data (rounds 1 to 15) and ‘late’ data (rounds 16 to 30) show that the Gambler’s fallacy described above is observable for the first 15 periods in both games with a slightly higher adjustment to overstock than understock in INTERACT. Yet, the results change for the last 15 periods and the phenomenon is not observed any longer. Instead, adjustments are always negative, regardless of whether there was a stock out or an overstock in the past period. This may be explained by the before mentioned overshooting later in the game, i.e. subjects started correcting their choices towards lower inputs. However, results are not significant any longer.

5.4 Results on information sharing

5.4.1 BUYER-INFO game and INTERACT-INFO game

Given the results from the aforementioned experiments, the question arises how deficits in decision making can be eliminated. One option is to reduce the level of information lack the buyer is facing. So far, the buyer was informed after each round about the delivery quantity from the supplier (and the resulting sales quantity and profit). Yet, the buyer gains no information on how the delivery quantity emerges. More specifically, he does not know whether underdeliveries result from low production quantities by the supplier (decision) or from low yield rates (random event) or both. In order to account for that issue, additional experiments with information sharing were conducted (BUYER-INFO and INTERACT-INFO). In these experiments, ceteris paribus, buyers received additional feedback after each round on the supplier’s decision as well as on the realized yield rate of the specific round. Furthermore, production output (input * yield rate) was shown which determined the delivery quantity to the buyer.

In BUYER-INFO, buyers saw the optimal response to their order while in INTERACT-INFO they received feedback on subject’s response to their order. Thus, uncertainties about supplier behavior in INTERACT was reduced as buyers were able to “learn” to some extent about the counterpart‘s pattern of decision making.

The table below summarizes the descriptive statistics of mean orders, production inputs, profits, and the respective deviations from optimum (with standard deviations in brackets).

		Optimum	Observed
BUYER-INFO	Order	130	133 (25)
	Deviation from optimum	-	3 (25)
	Buyer profit	1426	1356 (122)
	Deviation from optimum	-	-71 (122)**
	Supply Chain profit	1789	1719 (125)
	Deviation from optimum	-	-70 (125)***
INTERACT-INFO	Order	130	132 (27)
	Deviation from optimum	-	2 (27)
	Buyer profit	1426	1337 (80)
	Deviation from optimum	-	-89 (80)***
	Input	229	211 (52)
	Deviation from optimum	-	-18 (32)**
	Supplier profit	362	344 (65)
	Deviation from optimum	-	-18 (65)
	Supply Chain profit	1789	1682 (109)
	Deviation from optimum	-	-107 (109)***
[*** (p<0.00); ** (p<0.05); * (p<0.1)]			

Table 4: Means and standard deviations of optimum vs. observed ordering decisions and profits in INFO treatments

Using Wilcoxon tests (two-sided), we observe that mean orders in BUYER-INFO are on average, but not significantly, larger than optimal when yield information is available (p=0.940). As a result, however, profits are significantly lower for the buyer (p=0.004) and the supply chain as a whole (p=0.001).

Compared to the setting without information sharing, in INTERACT-INFO mean order quantities decrease from 139 to 132 but the deviations from optimum are not significant (p=0.970). Suppliers, though, react by significantly too low input quantities (p=0.040) which deteriorates the performance of the whole supply chain and leaves the buyer with significantly lower profits than optimum (p=0.000) while the supplier's loss is not significant (p=0.204). Comparable to the situation without information sharing, the supplier "learns" over time and increases her production quantities over the course of the game. The rise in inputs between the first and the last third is statistically significant (Wilcoxon, p=0.001, two-sided) but with a mean of +19 units highly overshoots the optimum towards the end.

5.4.2 Treatment comparisons: with and without information sharing

When comparing situations with and without information sharing on yield information and supplier decision, we do not find any significant effects. Orders increase from BUYER to BUYER-INFO while buyer and supply chain profits decrease due to this adjustment in order. None of these effects are statistically significant (MWU, two-sided). In INTERACT, the results are reverse. Orders decrease when information is shared and suppliers react by slightly lowered input quantities. While the buyer and the supply chain benefit from this situation in term of profits, the supplier loses. The notion that buyers may follow social preferences and award higher profits to the supplier by ordering more than optimal seems to vanish when information on the supplier decision is available. This finding may serve as an indicator that subject's behavior in BUYER is rather hedging against coordination risks than pursuing social preferences. Again, using MWU tests, no significant effects exist between the games. Counterintuitively, information sharing in this special case does not seem to be the "cure" for stemming performance deficits.

6. Concluding discussion

The risks of supply uncertainty in today's production systems is unavoidable which makes it necessary to learn about how to handle them and achieve efficient management of supply processes. Theoretical effort has been made widely but approaches to analyze behavioral aspects in decision making under supply uncertainty is lacking. Thus, our research provides insight into this specific area.

In our setting, decision makers form a supply chain with one buyer and one supplier where the supply side underlies production yield uncertainty while all information on parameters is common knowledge. In separate games, buyers and suppliers individually decide on their respective quantity, namely orders and production input while the counterpart is automated. In an additional game, all decisions are made by subjects, i.e. everyone interacts with another human decision maker.

The results show the following:

Buyers' orders alternate around the optimum of 130 but are not significantly different from it in both cases, a) with an automated supplier and b) when interacting with a subject supplier. However, profits are significantly lower than expected. The analysis suggests that buyers have a sound understanding of the situation but face bounded rationality, i.e. they lack the ability to calculate optimal quantities. They seem to follow a probabilistic choice rule when making order decisions which lead to suboptimal but still good results in terms of profits. When interacting with a human supplier, buyers were observed to order higher amounts which leave the supplier with an increased share of total supply chain profit. A reasonably explanation are fairness preferences towards the other supply chain party. However, further analyses of the data reveals that hedging against coordination risks is the more likely driver for this behavior as higher orders lead to higher production inputs which in turn enhances the buyer's chances of matching end customer demand. Regressions on order choices furthermore reveal that buyers follow the so-called "Gambler's fallacy", i.e. they expect deliveries to be high when they were low in the previous period and thus, reduce their order quantity. Moreover, the adjustments follow a pattern which suggests that "having too much" is considered worse than "having too little".

Suppliers on the other hand also alternate their input quantities around the optimum. For both games, inputs on average are too low, but results are only significant for the case of interaction with a human counterpart. In terms of profits, suppliers in all scenarios are worse off. However, their decisions improve in the course of the experiment. The analyses show that suppliers learn over time and their performance increases towards the end of the game, i.e. while input quantities are significantly too low in the first third, they move towards the optimum in the last third of the game. Supplier inputs with a computerized buyer are not significantly different from optimum any longer in the last periods. In interaction with a human buyer, input choices also approach the optimum but are still significantly below optimum in the last 10 periods of the experiment. As observed for the buyer, suppliers also follow the "Gambler's fallacy" when making production decisions in adjacent periods. Again, they seem to learn over time as the effect vanished in the last periods of the experiment.

Our analysis also revealed that information sharing is no cure for the inefficiencies in our supply chain. When provided with yield information of the previous round, buyers adjusted their behavior but not in an effective, i.e. significant way. Total supply chain performance even deteriorates in the single buyer game. In the interaction case, supply chain profits increase but not significantly and, furthermore, suppliers are worse off than they have been already. This highlights the notion that buyers are driven rather by hedging against risks than by fairness preferences when ordering above optimum.

Two of our main findings are also detected by Gurnani et al. (2014) who model a supply chain with supplier disruption risk in order to investigate whether decision makers diversify their orders between reliable and unreliable suppliers. They find that subjects behave boundedly rational, i.e. they have a sound understanding of the situation and are likely to make good choices, however, not the best ones. Furthermore, they reveal learning effects of subjects over time. They state that decision makers may use simple heuristics at the beginning of the experiment but improve their decisions throughout the course of the game.

Summarizing our findings, it seems reasonable that the observed obstacles can be reduced, especially for the supplier, by appropriate training on the tasks. While sharing yield information seems to be non-effective, other decision support tools, i.e. advanced calculators (and training on them) may help improving the actors' decisions.

Our research leaves room for further analyses and extensions. We do not answer the question of how to model observed ordering decisions by buyers using quantal response equilibria (QRE). The data shows patterns which can be explained by probabilistic choices but a thorough modelling and prediction of parameters with QRE are open to future research. Moreover, an in-depth examination of the reasoning behind supplier's too low production pattern with an overshooting towards the end of the experiment is left for further analysis.

The introduced work can also be extended in various ways. One option is to alter the supply chain setting by allowing overstock at the supplier to be shipped to the buyer, at a reduced costs. Under such a Push-variant of the wholesale price contract (see Inderfurth and Clemens (2014) for details on the contract) risk sharing in the supply chain is promoted which can reveal interesting insights into the subjects' behavior. Furthermore, we initially concluded from the data that buyers may follow fairness preferences, but found evidence that their behavior was triggered by other factors. However, this does not mean that social preferences are irrelevant in their decision making process. Appropriately designed experiments may help reveal whether other-regarding preferences still play a role in this supply chain setting.

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Appendix

A.1 General information

Table 5: Realized yield rates per period

Period	Yield rate
1	0,87
2	0,66
3	0,95
4	0,65
5	0,66
6	0,30
7	0,21
8	0,45
9	0,88
10	0,21
11	0,83
12	0,92
13	0,97
14	0,99
15	0,15
16	0,74
17	0,04
18	0,28
19	0,79
20	0,13
21	0,51
22	0,44
23	0,49
24	0,04
25	0,85
26	1,00
27	0,54
28	0,54
29	0,65
30	0,05

Table 6: Parameter values and optimal decisions and profits

	Notation	Value
Production cost	c	1
Wholesale price	w	6
Retail price	p	25
Demand	D	100
Yield rate	z	$\sim U[0{,}1], \mu_z = 0{,}5$
Optimal order	X	130
Optimal production input	Q	225 ($= \sqrt{3} \cdot X$)
Optimal buyer profit	Π_B	1.390
Optimal supplier profit	Π_S	330

A.2 Buyer results

Table 7: Profit allocation between buyer and supplier depending on order quantity[20]

Order	Buyer profit	Supplier profit (given best response)	Supplier's profit share
0	0	0	*0%*
10	135	25	*16%*
20	270	51	*16%*
30	405	76	*16%*
40	541	101	*16%*
50	676	127	*16%*
60	811	152	*16%*
70	946	178	*16%*
80	1.081	203	*16%*
90	1.216	228	*16%*
100	1.352	254	*16%*
110	1.374	279	*17%*
120	1.386	304	*18%*
130	1.390	330	19%
140	1.387	355	*20%*
150	1.379	380	*22%*
160	1.366	406	*23%*
170	1.350	431	*24%*
180	1.331	456	*26%*
190	1.309	482	*27%*
200	1.286	507	*28%*
210	1.260	533	*30%*
220	1.233	558	*31%*
230	1.205	583	*33%*
240	1.175	609	*34%*
250	1.144	634	*36%*

[20] The numerical example is conducted using the data from the experiment (demand = 100, mean yield rate = 0.5, unit production cost = 1, unit wholesale price = 6, and unit retail price = 25). Profit maximizing responses by the supplier are assumed.

Table 8: Linear regression model for order decisions (buyer)

$$q_t = \alpha + \beta_1 \cdot y_{t-1}^U + \beta_2 \cdot y_{t-1}^O + \sum_{i=1}^{n} \lambda_i^S \cdot D_i^S + \sum_{t=1}^{30} \lambda_t^P \cdot D_t^P + \varepsilon_t$$

Symbol	Description
t	time index
i	subject index
q_t	Order in $t = 2,\dots,30$
y_t^U	Understock: $\left[Demand_t - Delivery_t\right]^+$, $t = 2,\dots,30$
y_t^O	Overstock: $\left[Delivery_t - Demand_t\right]^+$, $t = 2,\dots,30$
D_i^S	$\left.\begin{cases} 1 & \text{if decision relates to subject } i \\ 0 & \text{else} \end{cases}\right\}, i = 1,\dots,n$
D_i^P	$\left.\begin{cases} 1 & \text{if period } t \\ 0 & \text{else} \end{cases}\right\}, i = 1,\dots,n$
ε_t	error term

Table 9: Linear regression results on buyer orders

	BUYER						INTERACT					
	All periods		1st half		2nd half		All periods		1st half		2nd half	
	coeff.	p	coeff.	p	coeff.	p	coeff.	p	coeff.	p	coeff.	p
Constant	141.191	.000	124.130	.000	150.217	.000	127.311	.000	111.426	.000	163.630	.000
Understock t-1	-.604	.000	-.537	.019	-.171	.444	-.376	.000	-.560	.000	-.008	.948
Overstock t-1	.184	.000	.129	.041	.095	.180	-.160	.001	-.176	.020	-.202	.002

A.3 Supplier results

Table 10: MWU-test results for input decisions in SUPPLIER and INTERACT

	Deviation BR
Mann-Whitney-U-Test	124,000
Wilcoxon-W	314,000
U	-1,854
Asymp. Sig. (2-seitig)	,064
Exakte Sig. [2*(1-seitige Sig.)]	,065[b]

Table 11: MWU-test results for input decisions in SUPPLIER and INTERACT over time

Interval		Deviation BR
1	Mann-Whitney-U-Test	143,000
	Wilcoxon-W	333,000
	U	-1,321
	Asymp. Sig. (2-seitig)	,187
	Exakte Sig. [2*(1-seitige Sig.)]	,194[b]
2	Mann-Whitney-U-Test	140,000
	Wilcoxon-W	330,000
	U	-1,405
	Asymp. Sig. (2-seitig)	,160
	Exakte Sig. [2*(1-seitige Sig.)]	,166[b]
3	Mann-Whitney-U-Test	120,000
	Wilcoxon-W	310,000
	U	-1,967
	Asymp. Sig. (2-seitig)	,049
	Exakte Sig. [2*(1-seitige Sig.)]	,050[b]

a. Gruppierungsvariable: TREAT

b. Nicht für Bindungen korrigiert.

Table 12: Linear regression model for production input decisions (supplier)

$$Y_t = \alpha + \beta \cdot q_t + \gamma_1 \cdot x_{t-1}^U + \gamma_2 \cdot x_{t-1}^O + \sum_{i=1}^{n} \lambda_i^S \cdot D_i^S + \sum_{t=1}^{30} \lambda_t^P \cdot D_t^P + \varepsilon_t$$

Symbol	Description
t	time index
i	subject index
Y_t	Input in $t = 2,...,30$
q_t	Order in $t = 2,...,30$
x_t^U	Underdelivery: $[Order_t - Output_t]^+$, $t = 2,...,30$
x_t^O	Over stock: $[Output_t - Order_t]^+$, $t = 2,...,30$
D_i^S	$\left.\begin{cases} 1 & \text{if decision relates to subject } i \\ 0 & \text{else} \end{cases}\right\}, i = 1,...,n$
D_i^P	$\left.\begin{cases} 1 & \text{if period } t \\ 0 & \text{else} \end{cases}\right\}, i = 1,...,n$
ε_t	error term

Table 13: Linear regression results on production inputs

	SUPPLIER						INTERACT					
	All periods		1st half		2nd half		All periods		1st half		2nd half	
	coeff.	p	coeff.	p	coeff.	p	coeff.	p	coeff.	p	coeff.	p
Constant	265.645	.000	126.956	.000	278.314	.000	191.660	.000	-109.92	.000	-55.98	.001
Order	-	-	-	-	-	-	1.724	.000	1.565	.000	1.590	.000
Delivery gap t-1	-1.262	.000	-.819	.000	-.419	.049	-1.032	.000	-.517	.002	-.139	.212
Overstock t-1	.380	.000	.358	.000	-.015	.883	.371	.000	.659	.000	-.076	.120

A.4 Instructions

Please read the following instructions carefully and contact us, if you have any questions about the content. If you have questions during the experiment, please raise your hand.

Initial situation

You are one of two members in a two-member supply chain consisting of one supplier and one buyer as depicted in the figure below:

At the beginning of the experiment you will be randomly assigned to either the role of the supplier or to the role of the buyer. This allocation will remain valid throughout the experi-ment. Furthermore, a second player will be assigned to your supply chain who will play the other member. This allocation will also remain unchanged throughout the experiment..

Course of events and decisions

There are 30 rounds to play and each round is independent from the previous one.

The buyer of the supply chain can sell exactly 100 units per round to an external end customer. The selling price to the end customer is fixed at 25 ECU per unit. In order to sell units to the end customers, they have to be ordered from the supplier. The order quantity can amount from 0 to 400. The wholesale price paid by the buyer to the supplier for each delivered unit is exactly 6 ECU. Each round, the buyer places exactly one order to the supplier.

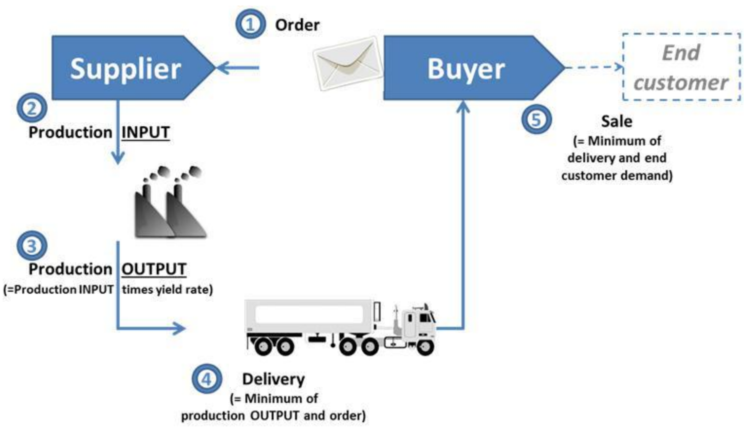
The supplier receives the information on the order quantity from the buyer. The supplier has to produce the ordered units in order to supply them to the buyer. For this purpose the supplier chooses a production lot size in every round (also named production input). Each input unit incurs production costs of 1 ECU. But the supplier's production process is not reliable, so the production yield (also named production output) is unknown in the beginning. The yield can vary between zero and the production input, which means that the yield rate (production output as fraction of production input) of production lies between 0% and 100%. The computer randomly generates a new yield rate in every round. All realizations between 0% and 100% occur with the same probability (uniform distribution) and the mean of the yield rate is 50%.

After the production process (and materialized production output) the order will be delivered as far as possible. If production output is less than order the required order quantity cannot be delivered in full amount and the supplier loses 6 ECU per unit of underdelivery. Thus, also for the buyer also it is uncertain which quantity will be delivered. However, if the production output is higher than the order excess units cannot be delivered to the buyer and no revenue can be generated for these units.

When the supplier delivers less than 100 units to the buyer the buyer is not able to fully supply total end customer demand. The buyer will lose 25 ECU in revenue per unit of missed end consumer demand. In case more than 100 units are ordered and delivered, only 100 units can be sold to the end consumer. The buyer incurs a per unit cost of 6 ECU (the wholesale price) for every excess unit, but is no revenues are generated from these units.

To provide an incentive to the supplier for increasing her production input, it could be useful if the order quantity is higher than the end customer demand. All excess units (both for the supplier and the buyer) are not available for sale in the next round.

The sequence of events and decisions is shown in the following figure:



1. The buyer makes an order decision to the supplier.

2. The supplier decides on a production input (given the buyer's order).

3. The (uncertain) production process takes place and the production output is realized with the following properties: $\text{Production OUTPUT} = \text{yield rate} \cdot \text{production INPUT}$
   such that $0 \leq \text{production OUTPUT} \leq \text{production INPUT}$

4. The delivery quantity to the buyer is calculated as follows: $\text{Delivery} = \text{Minimum}\{\text{Production OUTPUT, Order}\}$

5. The sales volume to the end customer is realized as follows: $\text{Sale} = \text{Minimum}\{\text{Delivery, Demand}\}$

Your task:

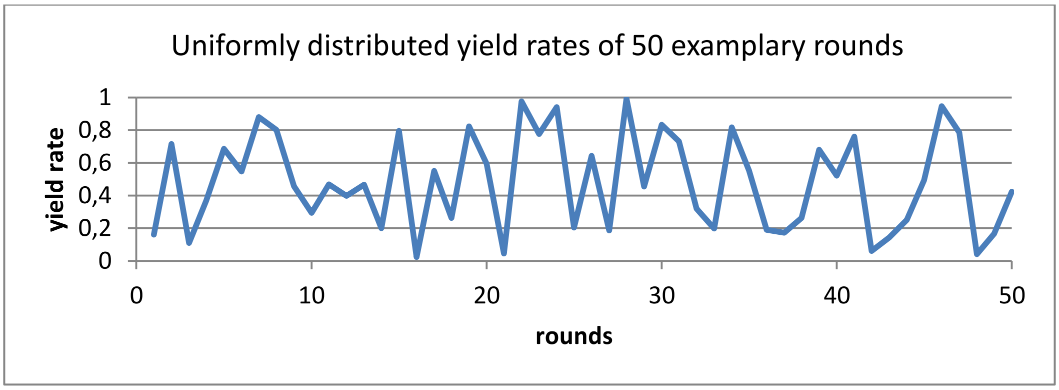
As buyer: decide on the order quantity!

As supplier: decide on the production input!

Summary of parameters

The end consumer demand amounts to 100 units per round. The buyer generates 25 ECU out of the sale to the end consumer. He pays 6 ECU to the supplier for each delivered unit. He incurs no other costs. Each unit of the production input costs 1 ECU for the supplier even if that unit will not be transformed into production output.

The yield rate is **uniformly distributed between 0 and 1 (i.e. btw. 0% and 100%)** with a mean of 0,5 (i.e. 50%). A uniform distribution means that all values between 0 and 1 occur with the same probability. A new random yield rate will be generated in every round. The realization of the yield rate of the past 50 rounds is exemplary shown in the figure below. Please note there is no correlation between the yield rates below and the ones drawn throughout the experiment.



Calculation of profits

The profits per rounds for the supplier and the buyer are calculated as follows:

$$\text{Profit supplier} = 6 \text{ Talers} \cdot \text{delivery} - 1 \text{ Taler} \cdot \text{production input}$$
$$\text{Profit buyer} = 25 \text{ Talers} \cdot \text{sale} - 6 \text{ Talers} \cdot \text{delivery}$$

For decision support you will be given a calculator which, given arbitrary entries for decisions (order and production input) and chance (yield rate between 0 and 1), returns the corresponding values for the production output, the order quantity, the sales volume and the profit for both actors. The decision support tool works for any combination of decisions (order quantity and production input) and random values (between 0 and 1).

Example

The buyer is ordering 110 units (given that the end consumer demand is 100 units). The supplier chooses a production input of 120. The value of the yield rate is 0,75 (i.e. 75%). Hence, production output is 90 units.

Decisions

$$\text{Order} = 110$$
$$\text{Production input} = 120$$

Chance

Yield rate = 0.75 (i.e. 75%)

Calculations

Production output $= 0.75 \cdot 120$ Input units (production input)
$= 90$ units
Delivery $=$ Minimum of 90 units (production output)
and 110 units (order)
$= 90$ units
Supplier profit $= 6$ Talers $\cdot$ 90 units (delivery)
$- 1$ Taler $\cdot$ 120 Input units (production input)
$= 420$ Talers

Sale $=$ Minimum of 90 units (delivery)
and 100 units (end customer demand)
$= 90$ units
Buyer profit $= 25$ Talers $\cdot$ 90 units (sale)
$- 6$ Talers $\cdot$ 90 units (delivery)
$= 1{,}710$ Talers

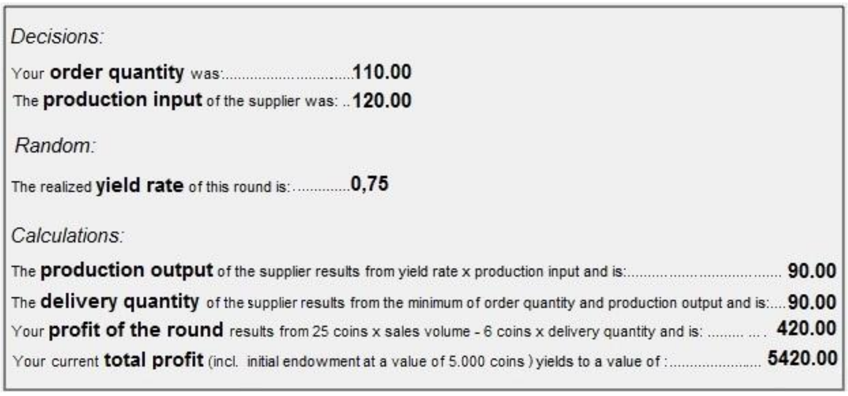
Initial endowment

The available initial endowment of 5000 ECU will be used if you incur losses. The experiment will be terminated if you lose all of your profits including the initial endowment during the experiment. The sum of all rounds' profits (positive or negative) is your total profit after finishing the last round.

Feedback

After you and the second member in your supply chain have made the respective decisions in a round each of you will be given the following feedback screen (depending on the role you are assigned to) (here, the first round of the example introduced above is illustrated):

Supplier:



Buyer:

Decisions:

Your **order quantity** was: **110.00**

Calculations:

The **delivery quantity** of the supplier is: **90.00**

Your **sales volume** resluts from the minimum of the delivery quantity and the end customer demand and is:....... **90.00**

Your **profit of the round** results from 25 coins x sales volume - 6 coins x delivery quantity and is: **1710.00**

Your current **total profit** (incl. initial endowment at a value of 5.000 coins) yields to a value of :......................**6710.00**

Number of rounds and payoff

There are 30 rounds to play. The game starts over again in every round and you have to decide on an order quantity or a production input, respectively. Your role as buyer or supplier remains valid throughout the experiment.

You will get a payoff after the last round. Your payoff (in €) is calculated from the sum of the profit from all rounds plus initial endowment and is divided by 2.500, i.e. at the end 100 experimental ECU are equal to a value of 4 Cent. Additionally to this payment you will receive a payment of 3€ which is independent of you performance in the experiment.. At the end of the experiment you will get be paid in cash. Please wait until your name is called.

Please give a hand sign, if you have additional questions. Please leave the instructions at your place after the experiment has finished.

Good luck!

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