Comparative Analysis of Signals Restoration by Different Kinds of Approximation

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***Abstract*—The comparative analysis of continuous signals restoration by different kinds of approximation is performed. The software product, allowing to define optimal method of different original signals restoration by Lagrange polynomial, Kotelnikov interpolation series, linear and cubic splines, Haar wavelet and Kotelnikov-Shannon wavelet based on criterion of minimum value of mean-square deviation is proposed. Practical recommendations on the selection of approximation function for different class of signals are obtained.**

***Keywords:* restoration, signal, approximation, Kotelnikov series, spline functions, wavelet functions, cubic spline, deviation, software product, mean-square deviation.**

I. INTRODUCTION

Significant attention is paid to the signals restoration in telecommunications. It is connected with the fact that the substantial part of analog signals requires the restoration at different stages of analog-to-digital conversion. Furthermore, to use a single approach to signals restoration is not always possible. Indeed, original signals can differ in forms, make hops or “splashes” on the amplitude. The traditional method of signal restoration is the application of Kotelnikov series. However, the application of Kotelnikov series as the approximation function is effective in the infinite interval of time, while in the real systems it is necessary to examine a signal in a certain limited interval. In this case, there appears an error in signals restoration by means of Kotelnikov series. Therefore, to examine other kinds of approximation is reasonable. The following: Lagrange exponential polynomial, spline function, wavelet function and many others are examined as the approximation functions [1]-[5]. In certain cases [1]-[3] the application of spline functions and wavelet functions allows to reach the best results of continuous signal restoration in the finite time interval, rather than with the application of Kotelnikov series.

The signals restoration by means of Kotelnikov interpolation series, cubic spline and wavelet functions were examined by the authors in works [1]-[5]. In works [1]-[3] the signals of linear frequency modulation, the random signals, as well as the signals, which are characterized by the amplitude steepness were investigated. Restoring such signals by Kotelnikov series and cubic splines, the estimations of restoration error are obtained and it is shown that under specific conditions the application of cubic splines is more reasonable. In works [4][5] it is found that for some signals the application of wavelet functions as approximation ones is more preferencial in comparison with the use of Kotelnikov series and cubic splines. The task to select the optimum method of signals restoration, depending on their original kind, emerges.

In this work let us consider the restoration of different signal kinds by means of the above mentioned methods, that will allow us to give recommendation on the application of one or another restoration method depending on the examined signal.

The objective of this work is the comparison of the signals restoration methods with the use of different approximation functions for their more precise restoration.

To achieve the set objective let us find errors of original signal restoration by means of different approximation functions. As the criterion we will use a minimum value of mean-square deviation between the original signal and the approximation function. We will consider Kotelnikov interpolation series, Lagrange polynomial, spline function and wavelet function as the approximation functions. For this purpose let us develop the program realization, which will allow to investigate different methods of signals restoration. Based on the developed program, let us carry out investigation of different signals restoration and develop practical recommendations on the selection of the approximating device.

II. CONTINUOUS SIGNALS RESTORATION BY KOTELNIKOV SERIES AND SPLINE FUNCTIONS

Let us consider the continuous signal $f(x)$ in the interval $0 \le x \le 1$, that is given by the function of the form [2][3]:

$$f(x) = 2\frac{\sin(16\pi(x-\frac{1}{2}))}{16\pi(x-\frac{1}{2})} - 3\frac{\sin(8\pi(x-\frac{1}{2}))}{8\pi(x-\frac{1}{2})}, \qquad (1)$$

It is considered that $f(x)$ is limited on the spectrum $F_{\max} = 8$ kHz, then the sample spacing $\tau = \frac{1}{2F_{\max}}$.

Let us consider samples of the original signal $f(x)$ as interpolation points. Let be given function values as $f_k = f(x_k)$, $k = \overline{0, N-1}$, in the interval [0;1] in mesh points Δ: $0 = x_0 < x_1 < ... < x_N = 1$. The given original signal $f(x)$ belongs to the class $|f(x)| < \frac{A}{|x^2|}$, where $f(x)$ is original signal, A is a certain constant. To restore the signals of such class, Kotelnikov interpolation series and cubic spline were examined in works [2][3].

Fig.1 illustrates the obtained results of given above signals restoration, examined in the works [2][3], particularly: diagram of the given signal $f(x)$ (diagram 1), values of function $f(x)$ in the mesh points (diagram 2), the diagram of the restored signal by Kotelnikov series (diagram 3), the diagram of the restored signal by the cubic spline (diagram 4).

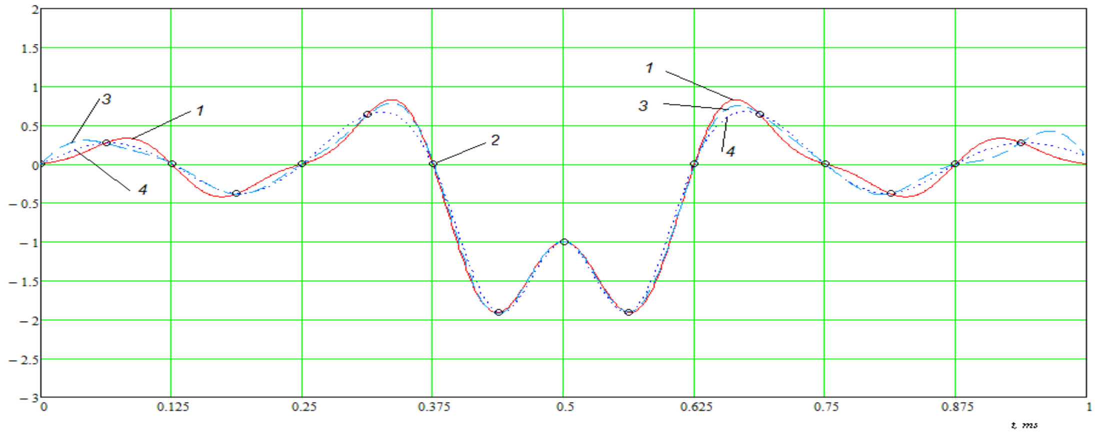


Fig. 1 – Original continuous signal $f(x)$ and its restoration by Kotelnikov series and c-spline.

The obtained results [2][3] show that in the intervals of interpolation “with amplitude smooth change”, where the form of the signal continuously changes without “amplitude steepness” the best results are given by spline approximation, and in the intervals with “amplitude steepness” – by Kotelnikov interpolation series. As it is shown in [2], the approximation error by application of Kotelnikov series and cubic spline can reach 6.2% and 9.2% in the first and the second cases respectively due to the fact, that with the uniform sampling the sample points can not coincide with the amplitude maximums or the minimums. It is found that the use of cubic spline as the approximation function during continuous signals restoration is reasonable for the signals, which do not have bumping parameters changes. At the same time, it is much more difficult to select the approximation function for the signals, which possess the amplitude hops and “rapid” oscillations.

Therefore it would be reasonable to consider another restoration method, for which this approximation error would be less. In our opinion, such method is wavelet transform device.

III. Continuous Signals Restoration by Wavelet Functions

Recently special development was given to the wavelet analysis device in information theory, theory of coding, theory of signals and images. To the number of the major advantages of the wavelet functions application as the approximation function during the signals restoration can be referred the following [6][7]:

– the signals restoration with the low information losses, in other words, distortion in the restored signal, appearing as a result of quantization, while using the wavelet functions can be made comparatively small and thus to elliminate the noise interferences;

– the ability of wavelet functions to reveal the local special features of the investigated signal, in particular for the pulse and digital signals and the images (point of discontinuity, peaks, sharp drops and self-similarity), which miss other methods of signals restoration, allows to reproduce the most important characteristics of the original signal;

– the use of wavelet functions during the signals restoration allows to obtain information about the nature of the parameters change of the investigated function, that is particular important in examining the non-stationary signals, for example, consisting of different components, which act in different time intervals, the modulated signals, etc., which nowadays find much wider application than stationary or quasi-stationary signals, as well as processes and systems generating them;

– the possibility of the independent analysis of the function on the different scales of its change, because, each frequency signal component is additionally described by the scaling function, that allows to determine the time interval, where a certain signal frequency characteristic changed.

Wavelet transform allows to perform the signals restoration both in the temporary parameters with the basic wavelet function $\psi(x)$, and in the frequency parameters with the scaling function $\varphi(x)$ [6]:

$$\psi(x) = \sqrt{2}\sum_k g_k \varphi(2x-k)\,, \quad \varphi(x) = \sqrt{2}\sum_k h_k \varphi(2x-k)\,, \qquad (2)$$

where
$h_k = \sqrt{2}\int \varphi(x)\overline{\varphi}(2x-k)dx$, $g_k = (-1)^k h_{(N-1)-k-1}$ – expansion coefficient, k, N – natural numbers.

All the wavelets are obtained from the basic wavelet with the scaled transformation $1/2^k$ and shifts $j/2^k$ [6]:

$$\psi_{j,k} = 2^{j/2}\psi(2^j x - k)\,,\ \varphi_{j,k} = 2^{j/2}\varphi(2^j x - k)\,. \qquad (3)$$

Then, according to [6], any function $f(x)$ from Hilbertian space $L^2(R)$ can be expressed in the series of the form:

$$f(x) = \sum_k s_{j_n,k}\varphi_{j_n,k}(x) + \sum_{j \ge j_n, k} d_{j,k}\psi_{j,k}(x)\,, \qquad (4)$$

where
$f(x)$ – temporary function of the original signal;
$\varphi_{j,k}(x)$ – scaling function (father wavelet);
$\psi_{j,k}(x)$ – basic wavelet function (mother wavelet);
$s_{j,k}$ and $d_{j,k}$ – wavelet coefficients;
j_n – scaling level;
k, j, j_n – natural numbers.

Wavelet coefficients $s_{j,k}$ and $d_{j,k}$ for the expression (4) are calculated as follows [6]:

$$s_{j-1,k} = \sum_m h_m s_{j,2k+m}\,,\ d_{j-1,k} = \sum_m g_m s_{j,2k+m}\,, \qquad (5)$$

where
h_m, g_m – approximation and expansion coefficients.

On the most detailed level of scaling $j = j_{max}$ there left only s coefficients and the signal is given by the scaling function [6]:

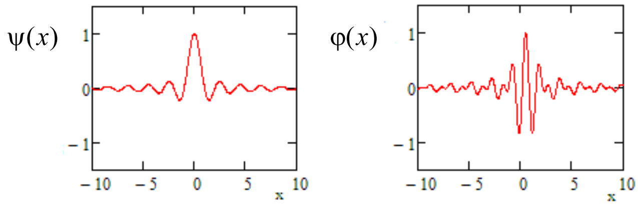
$$f(x) = \sum_k s_{j_{\max},k}\varphi_{j_{\max},k}(x)\,. \qquad (6)$$

In the case of signal restoration on the most detailed level, i.e. by using the formula (6), coefficients $s_{j_{\max}}$ are calculated as follows [6]:

$$s_{j\max,n} = f(n\tau)/2^{\frac{j\max}{2}}\,. \qquad (7)$$

where
τ - sampling interval.

Let us consider the restoration of the continuous signal $f(x)$ of the form (1) by wavelet function. To restore the signal let us use Kotelnikov-Shannon wavelet for which basic wavelet function $\psi(x)$ and scaling function $\varphi(x)$ take the following form (Fig. 2, a) and (Fig. 2, b) relatively:



a) b)

Fig. 2 The basic wavelet function $\psi(x)$ a) and scaling function $\varphi(x)$ b) for Kotelnikov-Shannon wavelet

For Kotelnikov-Shannon wavelet, according to [7], analytic note for its basic function $\psi(x)$ and scaling function $\varphi(x)$ t akes the form:

$$\psi(x) = 2\frac{\sin 2\pi(x-1/2) - \sin \pi(x-1/2)}{\pi(x-1/2)}\,,$$
$$\varphi(x) = \begin{cases} \sin \pi x/\pi x, & x \ne 0, \\ 1, & x = 0, \end{cases}\,. \qquad (8)$$

Mother wavelet $\psi(x)$ and scaling function $\varphi(x)$ are connected by the relations (2), besides, for Kotelnikov-Shannon wavelet [7]:

$$h_k = \begin{cases} 1/\sqrt{2}, & k = 0, \\ (-1)^{(k-1)/2}\sqrt{2}/\pi k, & k - \text{odd}, \\ 0, \text{ for the rest } k \end{cases} \qquad (9)$$
$$g_k = (-1)^k h_{N-1-k}\,,\ k = \overline{0, N-1}\,,$$

where
N – natural number, which defines the coefficients h_k and g_k number.

The necessary condition of the signal restoration by the wavelet function is its definition by samples number that is equal to $N = 2^j$, where $j \ge 1$ defines the maximum possible number of scaling levels [6]. In our case $j = j_{\max} = 4$, $N = 2^4 = 16$.

Lets us calculate the h_k and g_k expansion by the formula (9).

The obtained values are given in the table 1.

TABLE I
VALUES OF h_k AND g_k COEFFICIENTS

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
h_k	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{\pi}$	0	$-\frac{\sqrt{2}}{3\pi}$	0	$\frac{\sqrt{2}}{5\pi}$	0	$-\frac{\sqrt{2}}{7\pi}$	0	$\frac{\sqrt{2}}{9\pi}$	0	$-\frac{\sqrt{2}}{11\pi}$	0	$\frac{\sqrt{2}}{13\pi}$	0	$-\frac{\sqrt{2}}{15\pi}$
g_k	h_{15}	0	h_{13}	0	h_{11}	0	h_9	0	h_7	0	h_5	0	h_3	0	h_1	$-h_0$

Table 2 shows the number values of $s_{j,k}$ and $d_{j,k}$ coefficients, obtained according to (5), besides, $s_{0,k}$ coefficients are values of function $f_k = f(x_k)$ in the interpolation points.

Lets us perform $f(x)$ signal restoration by the formula (4) [6], using Kotelnikov-Shannon wavelet, for which $\psi(x)$ and $\varphi(x)$ are defined by the formula (8) [7].

TABLE II
VALUES OF $S_{J,K}$ AND $d_{J,K}$ COEFFICIENTS

	k	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
j=1	$s_{j,k}$	0,217	0	0	-0,478	-1,714	0	0	0,123
	$d_{j,k}$	-0,048	0,068	-0,113	0,338	0,161	-0,113	0,068	-0,048
j=2					k	0	1	2	3
					$s_{j,k}$	0,134	-0,026	-1,065	-0,095
					$d_{j,k}$	0,082	0,318	-0,443	-0,082
j=3							k	0	1
							$s_{j,k}$	0,097	-0,796
							$d_{j,k}$	0,103	-0,191
j=4								k	0
								$s_{j,k}$	-0,29
								$d_{j,k}$	0,062

The results of the original signal $f(x)$ restoration by Kotelnikov-Shannon wavelet [4][5] are obtained in the Mathcad 15 environment and are shown on the Fig. 3. As it is shown on the Fig. 3 the original signal $f(x)$ restoration by Kotelnikov-Shannon wavelet allowed to obtain the result that corresponds to the kind of the original signal [4][5]. It is allows to make the conclusions that it is the most reasonable to use wavelet functions as the approximation function during the signals restoration, which are characterized by the presence of rapid oscillations.

It is possible intuitively to see that the signals with the abrupt changes must be analyzed rather by the wavelet functions than by the classical approach on the basis of Kotelnikov series, that is used as the basic $\sin x$ function.

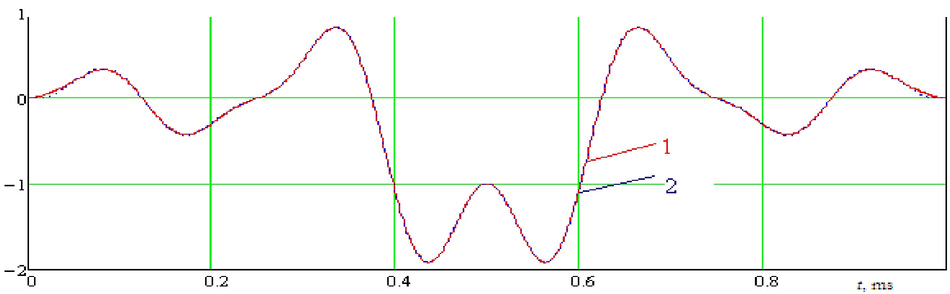


Fig. 3. Restoration of the original signal $f(x)$ by Kotelnikov-Shannon wavelet.

Let us compare the results obtained during the signal $f(x)$ restoration, that is characterized by the amplitude higher steepness, using Kotelnikov series, cubic spline and Kotelnikov-Shannon wavelet.

IV. Analysis of the Results of Continuous Signals Restoration by Different Approximation Functions

To compare the results of the original $f(x)$ signal restoration by Kotelnikov-Shannon wavelet [4][5] let us consider the results obtained earlier in works [2][3], the approximation of the same signal by Kotelnikov series and cubic spline.

Fig. 4 illustrates [4][5] the signal restoration of the form (1) in the interval $[0;1]$ by means of different approximation functions, and Fig. 5 – in the interval $[0,625;0,75]$: the diagram of the given signal $f(x)$ (diagram 1), the approximation of the original signal $f(x)$ by Kotelnikov series (diagram 2), the approximation of the original signal $f(x)$ by cubic spline (diagram 3), the approximation of the original signal $f(x)$ by Kotelnikov-Shannon wavelet (diagram 4).

As we can see from the Fig. 4 and 5, the proposed method of continuous signals restoration, which are characterized by the amplitude abrupt changes, based on wavelet functions allows to obtain the best results of restoration (curve 4), rather than restoration methods by Kotelnikov series (curve 2) and by cubic spline (curve 3) [4][5].

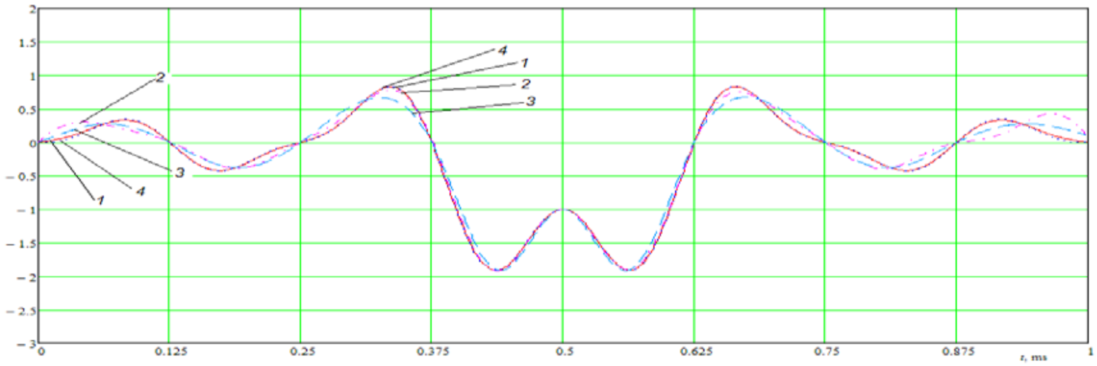


Fig. 4. Restoration of the original signal $f(x)$ by Kotelnikov series, cubic spline and Kotelnikov-Shannon wavelet in the interval $[0;1]$.

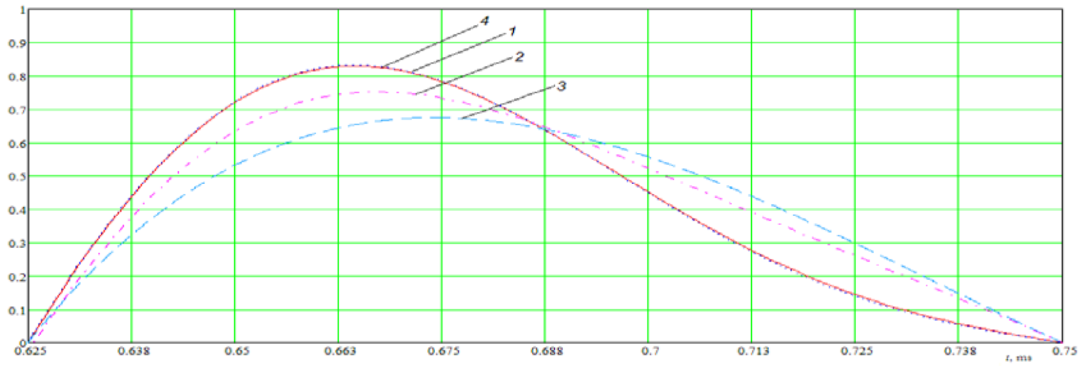


Fig. 5. Restoration of the original signal $f(x)$ by Kotelnikov series, cubic spline and Kotelnikov-Shannon wavelet in the interval $[0{,}625;0{,}75]$.

To estimate the fact how precisely one or another method (Kotelnikov series, cubic spline or Kotelnikov-Shannon wavelet) restores the original signal, let us define the value of deviation ε_n of the restored signal values form the original signal.

The results are given on the Fug.6: diagram ε_n with the approximation of the original signal $f(x)$ by Kotelnikov series (curve 1), diagram ε_n with the approximation of the original signal $f(x)$ by cubic spline (curve 2), diagram ε_n with the approximation of the original signal $f(x)$ by Kotelnikov-Shannon wavelet (curve 3).

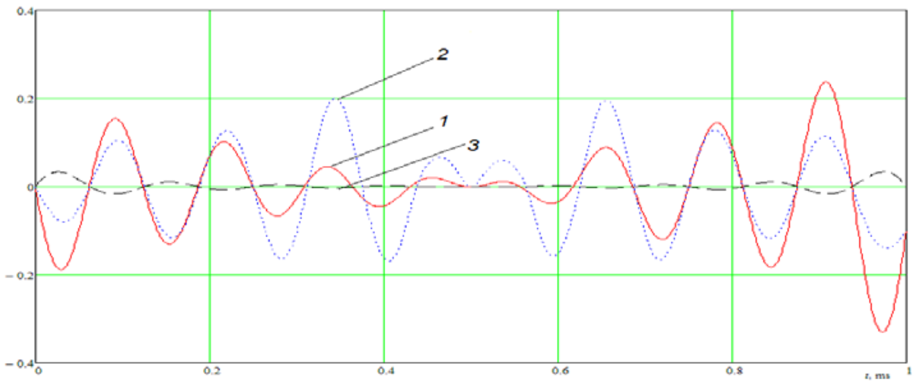


Fig. 6. Deviation of the restored signal from the original one.

Similarly to [2], let us define the mean-square deviation (MSD) of the restored and original signals difference. The results of the MSD values of the restored and original signals are given in table 3. As we can see from Fig. 4-6, the application of Kotelnikov series for continuous signals restoration, which are characterized of the higher steepness of the amplitude and a sharp function increase, leads to the global signal smoothing and therefore local peaks at the moment of abrupt changes in the signal amplitude are restored badly.

The analogous approximation results are given by the application of cubic spline. An approximation error in this case reaches 6% [2]. However, the wavelet transform application does not prevent the restoration of these peaks due to the property of scalability.

Fig. 7 illustrates the dependence of mean-square deviation on the number of scaling level in the signal $f(x)$ restoration by Kotelnikov-Shannon wavelet.

TABLE III
DEVIATION VALUES ε_n AND MSD σ_n OF THE RESTORED SIGNAL BY APPROXIMATION FUNCTIONS (KOTELNIKOV SERIES, CUBIC SPLINE AND KOTELNIKOV-SHANNON WAVELET)

№	Interval	Number limits of the interval	ε_n Kotelnikov series	ε_n cubic spline	ε_n Kotel'nikov-Shannon wavelet	σ_n Kotelnikov series	σ_n cubic spline	σ_n Kotel'nikov-Shannon wavelet
1	$[t_0, t_1]$	[0; 0,063]	0,119	0,05	0,021	0,059	0,025	0,01
2	$[t_1, t_2]$	[0,063; 0,125]	0,098	0,066	-0,011	0,048	0,032	0,0051
3	$[t_2, t_3]$	[0,125; 0,188]	0,083	0,074	0,0065	0,041	0,036	0,0032
4	$[t_3, t_4]$	[0,188; 0,25]	0,066	0,082	0,0044	0,031	0,038	0,0021
5	$[t_4, t_5]$	[0,25; 0,313]	0,039	0,103	0,0029	0,023	0,051	0,0014
6	$[t_5, t_6]$	[0,313; 0,375]	0,023	0,128	0,0019	0,02	0,061	0,0092
7	$[t_6, t_7]$	[0,375; 0,438]	0,027	0,106	0,0011	0,017	0,053	0,00055
8	$[t_7, t_8]$	[0,438; 0,5]	0,012	0,041	0,00035	0,0072	0,022	0,00022
9	$[t_8, t_9]$	[0,5; 0,563]	0,0048	0,036	0,00035	0,0049	0,02	0,00022
10	$[t_9, t_{10}]$	[0,563; 0,625]	0,022	0,099	0,0011	0,015	0,049	0,00053
11	$[t_{10}, t_{11}]$	[0,625; 0,688]	0,059	0,121	0,0018	0,026	0,062	0,00095
12	$[t_{11}, t_{12}]$	[0,68; 0,75]	0,079	0,107	0,0029	0,035	0,05	0,0014
13	$[t_{12}, t_{13}]$	[0,75; 0,813]	0,091	0,081	0,0043	0,046	0,04	0,0021
14	$[t_{13}, t_{14}]$	[0,813; 0,875]	0,117	0,076	0,0012	0,056	0,036	0,0031
15	$[t_{14}, t_{15}]$	[0,875; 0,938]	0,149	0,072	-0,01	0,075	0,037	0,0054
16	$[t_{15}, t_{16}]$	[0,938; 1]	0,225	0,102	0,02	0,092	0,038	0,0065

Thus, we can state, that the smallest restoration error by the wavelet functions can be obtained with the maximum scaling level, and an increase of the maximum scaling level is possible with the samples number increase.

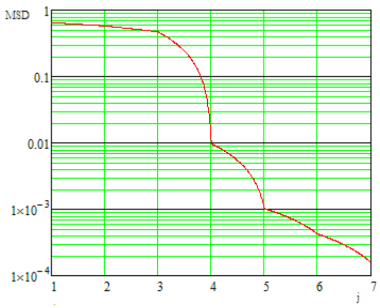


Fig. 7. Dependence of the MSD on the number of scaling levels.

One of the tasks be solved with the use of wavelet functions to restore the continuous signals, is what samples number is necessary to take so that the original and restored signals deviation would be minimum with the set computational volume.

V. SOFTWARE REALIZATION OF THE SIGNALS RESTORATION BY DIFFERENT APPROXIMATION FUNCTIONS

To compare the methods of signals restoration by means of different approximation functions (Kotelnikov series, Lagrange polynomial interpolation, linear spline functions, cubic spline functions, as well as wavelet functions) the software realization "*Signal Restoration*", which realizes the mentioned methods of the signals restoration is developed [5]. Software product is executed by means of framework Q_t with using the C++ programming language. The control-flow chart is given in Fig. 8.

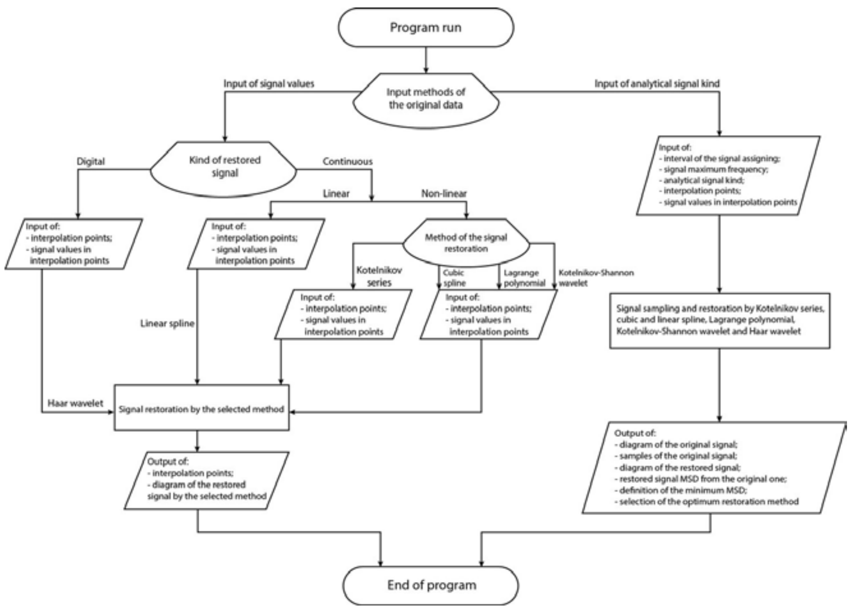


Fig. 8. Algorithm diagram of the «*Signal Restoration*» program.

Functionally, the given product allows to perform the signals restoration of different classes (both continuous and discrete) by all indicated methods, as well as to obtain the estimation of restoration error according to the MSD criterion of the restored signal from the original one. The best method of signal restoration based on the obtained MSD is chosen.

The algorithm of the given software product consists in the following:

1. Input of the original data about the signal (to set a signal by means of the samples or in the analytical form).
2. Selection of the signal restoration method (by means of Kotelnikov series, Lagrange polynomial interpolation, linear spline functions, cubic spline functions, Haar wavelet or Kotelnikov-Shannon wavelet).
3. The signal restoration by the selected method.
4. MSD definition for each restoration method.
5. Conclusion of the obtained results.

Linear frequency-modulated signal (LFM signal), LFM signal with the linearly changing amplitude (modified LFM signal), frequency-modulated signal (FM signal) and the signal, which is characterized by the amplitude higher steepness and sharp function increase were considered for the analysis.

In particular, Fig. 9 illustrates the results of frequency-modulated signal restoration by means of Kotelnikov series, Lagrange polynomial interpolation, linear spline functions, cubic spline functions, Haar wavelet and Kotelnikov-Shannon wavelet [5].

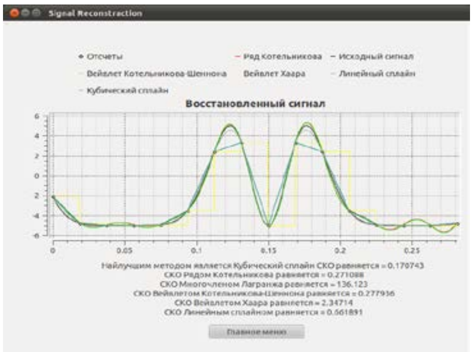


Fig. 9. Restoration of FM signal.

By means of the software product the following results and recommendations regarding the use of the specific approximation functions depending on the original signal kind, i.e., its class, on the MSD criterion are obtained [5]:

– continuous FM signal is reasonable to restore by means of cubic spline, for which MSD is approximately equal to 0,171;

– during the LFM signal restoration, it turned out that it is reasonable to use Kotelnikov-Shannon wavelet, in this case MSD is approximately equal to 0,169;

– in case of the restoration of the LFM signal with the linearly changing amplitude, it is recommended to use Kotelnikov series as the approximation function (MSD is approximately equal to 0,125);

– for the continuous signals, which are characterized by the presence of a large number of oscillations and sharp function increase, it is recommended to use wavelet function based on Kotelnikov-Shannon wavelet;

– as for Haar wavelet, it is necessary to use only for the signals restoration of the "squared pulse" type.

VI. CONCLUSION

1. As a result of carried out analysis of the continuous signals restoration on their samples by Kotelnikov series, cubic spline and Kotelnikov-Shannon wavelet it is shown, that the first two restoration methods do not consider such special features of signal as the amplitude abrupt change, which leads to the restored signal values deviation from its original value. The use of Kotelnikov-Shannon wavelet allows to decrease this deviation. This implies, that the mathematical device of wavelet analysis can be used for the signals restoration, which are characterized by the presence of rapid oscillations.

2. The estimations of the restoration error of different signal classes by the functions, pointed out above, are obtained, which allowed to speak about the advantage of a certain method of the signals restoration depending on its kind, i.e., the class of signals.

3. The program, which allows to find the restoration errors of different signals based on the MSD minimum value criterion by means of Lagrange polynomial, Kotelnikov interpolation series, linear and cubic splines, Haar wavelet and Kotelnikov-Shannon wavelet is developed.

4. The practical recommendations regarding the selection of the approximation function based on the mean-square functions deviation criterion for different signal kinds are obtained. The results of work with the software realization "*Signal Restoration*" allow to carry out the substantiated selection of the optimum restoration method of a certain original signal.

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