Application of Fuzzy Variables for Systems of Management Decision Support

Leonid Mylnikov
Perm National Research Polytechnic University - Electrotechnical Department
Komsomolsky Ave, 29, 614990, Perm, Russia
E-mail: leonid@pstu.ru

***Abstract*—The following article describes an approach covering the variety of opinions and uncertainties of estimates within the chosen technique of decision support. Mathematical operations used for assessment of options are traced to operations of working with functions that are used for assessment of possible options of decision-making. Approach proposed could be used within any technique of decision support based on elementary mathematical operations. In this article the above-mentioned approach is described under analytical hierarchy process.**

Keywords: **fuzzy set, management, decision-making, algorithm, analysis, prediction, membership function, set, analytical hierarchy process.**

I. INTRODUCTION

Throughout the whole life man, social community or organizational system have to face numerous choices and to make a variety of decisions. Decision-making based on the given choices is inseparable from their activities. As any product of human activity, decision-making mechanism has a dual nature: subjective and objective [3]. It was noticed that the subjective component by no means always leads to the best result. Thus, in order to enlarge the objective component of choice of this or that scenario a variety of decision support methods were developed.

All decision support methods could be divided into 2 big groups: 1) methods handling accurate practical data (OLAP, Data Mining etc) and presenting it in a convenient form for decision-making person; 2) methods handling expert analysis data or Q-data (SWOT-analysis, analytical hierarchy process, economic techniques etc).

Methods of the second type are commonly used in economics and organizational system management. In these particular systems the objective expert estimate is the most challenging part. So the problem of accuracy improvement of such methods' estimates is crucial.

The only way to improve methods' accuracy is to improve estimates. As experts are not able to give different estimates, it makes sense whether to use the theory of probability. to assess the level of certainty of experts in their estimates and thus neutralize experts' errors or to use additional experts for estimates' precision. However methods in an explicit form are unable to cover several expert opinions. For those purposes the approaches based on average values given by an expert group or interval value estimates are used [4][11]. They reduce accuracy and bring additional simplifications to methods which are already not always precise enough.

II. TECHNIQUE OF FUZZY VARIABLES APPLICATION

Description of complex estimates (estimates including the level of certainty or estimates of experts' group) is possible through functions (even in the format of a table or matrix). These functions could be considered as membership functions $\mu_A(x)$ from fuzzy-set theory (fuzzy logic). Application of fuzzy-set theory in comparison with other theories has lots of advantages as here all elementary mathematical operations could be defined. Moreover, other theories are based on the probabilities and could be narrowed to fuzzy-set theory [1][5][10].

Another advantage of fuzzy-set theory is that lots of data collected through opinions on this or that problem could not be presented as one figure but as some verbal estimates (e.g. bad, good, adult, child, prospective etc). These estimates in practice are considered as linguistic variables that are as well described by membership functions [7]. Though, if considering several opinions, even such estimates could vary making the general estimate quite blurry.

Accordingly, the general scheme of decision-making methods within fuzzy-set theory could be presented as following: (Fig. 1 [9]).

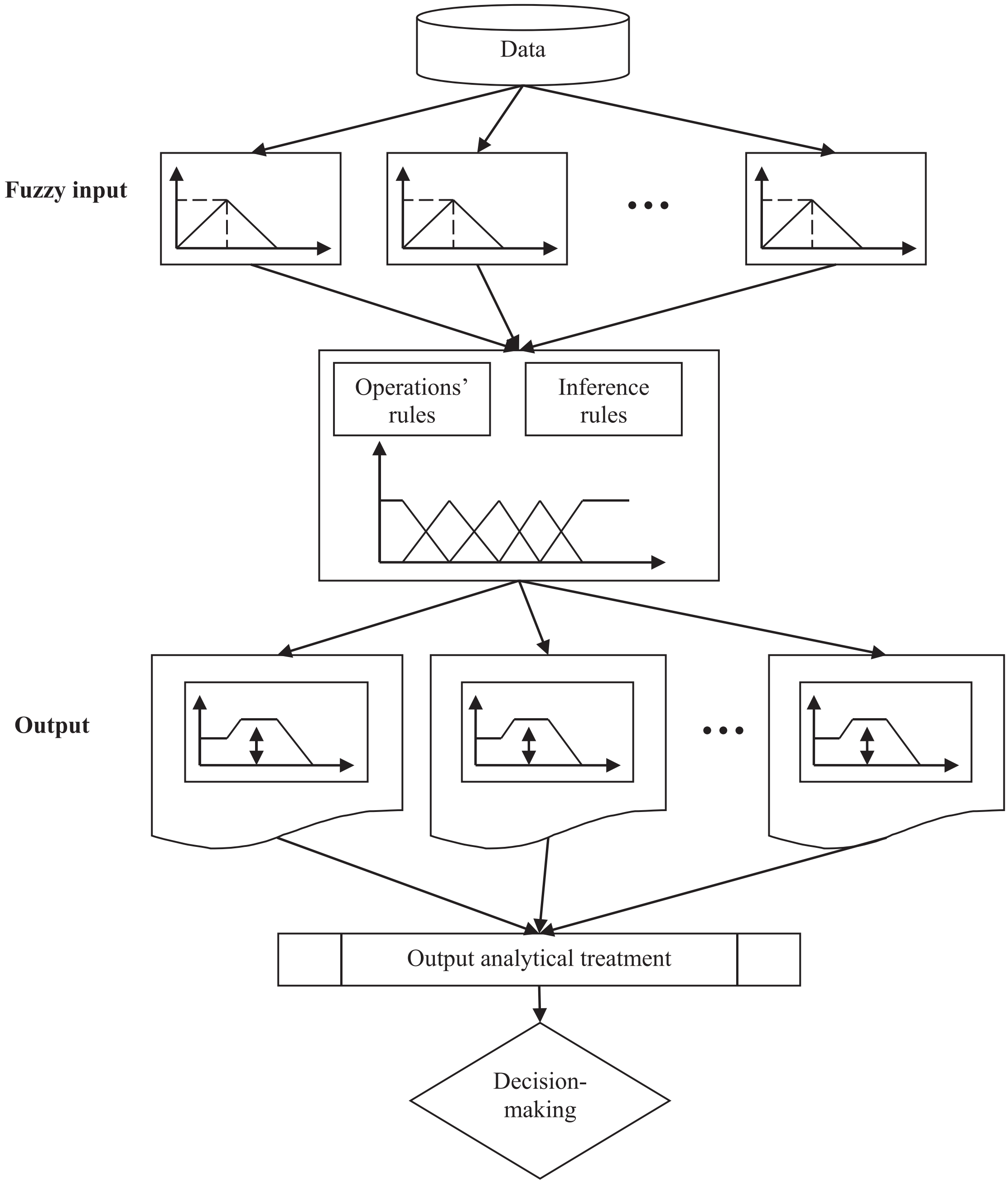

Fig. 1. Method application scheme.

The first stage is the transfer of numerical estimates into fuzzy form (fuzzification). Here the Gaussian function could be used. The calculation is based on the following formula:

$$\mu(x) = e^{-\left(\frac{x-c}{\sigma}\right)^2}$$

where

$\mu(x)$ - the level of membership to fuzzy set;

σ- set as level of expert certainty (from 99 % to 1%, that equals values of σ 0.01 and 0.99 respectively);

x - range of variation of expert estimates, varies from minimum possible to maximum possible total value of the indicator (on Fig. 2 value variation is from 0 to 2);

c - value of expert estimate or weighting factor.

Another way of transferring numbers into fuzzy form is to define fuzzy set as a triangular fuzzy number [2].

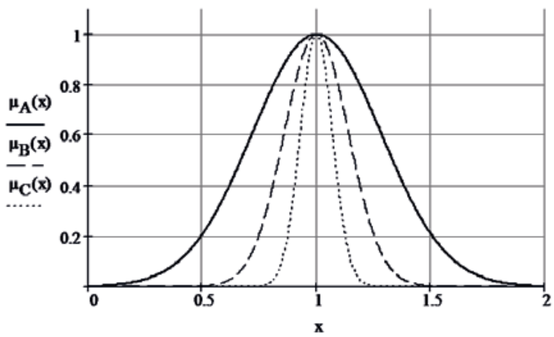

Fig. 2. Graphs of Gaussian function membership with different level of expert certainty.

The majority of techniques and methods needs only 4 basic operations as addition, subtraction, multiplication and division to make a decision. These operations are defined for fuzzy sets and described in literature both for fuzzy sets based on Euler formula and for triangular fuzzy numbers (including operations between variables in fuzzy and explicit form). It is important to notice that performing operations with fuzzy numbers, for instance in triangular shape, gives as a result a number different from a "triangular" one. Therefore some assumptions could be introduced only at the stage of transfer to fuzzy form.

Some arithmetical operations have a number of realization techniques for fuzzy sets. Each technique has its own advantages and should be chosen in accordance with the established task (for example, sum of Zadeh gives the highest accuracy in comparison to other methods). However, despite the difference of realization techniques, the performance of operations does not bring any assumptions or errors into initial data which makes final values true.

Some methods include steps based on various functional connections or conditions. At present such type of theory for fuzzy sets is elaborated and called "Fuzzy systems of logical inference" [2].

III. Fuzziness of Decision Support Methods

Let's analyze operations with fuzzy variables with consideration of several opinions by the classic example of selecting a school for a child based on analytical hierarchy process. Let us assume that the parents choose one of 3 available options. First, they need to define criteria for estimation of different options. Criteria should cover, if possible, all areas of the problem. If decomposition received is not sufficient to compare the importance of criteria, further decomposition should be done. Let's assume the received scheme looks as shown on Fig.3:

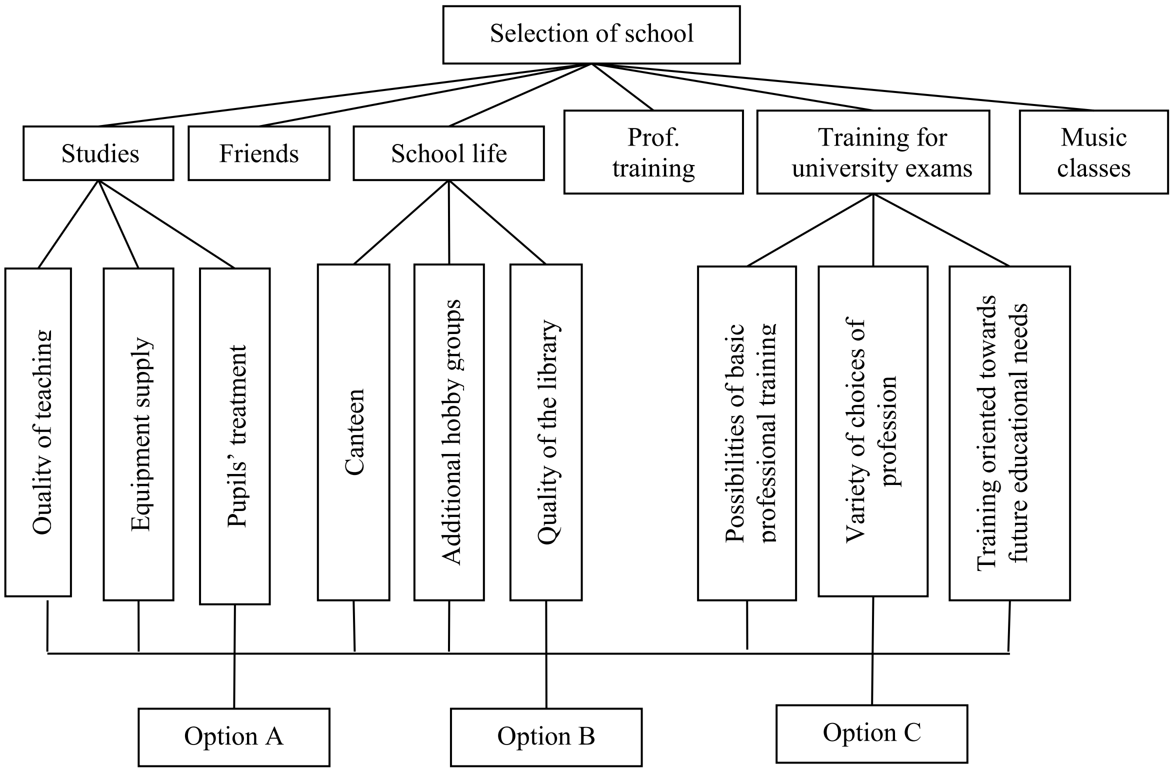

Fig. 3. Hierarchy of Selection of school based on analytical hierarchy process[6].

Decomposition of criteria Friends, Prof. training and Music Classes are skipped for short. After the following scheme is developed it is necessary to do double estimates. If parents have different opinions (the importance of criteria of upper level in relation to a target) then double estimates could be done by different parents.

For example, one of the parents compiled the following matrix:

Selection of school	Studies	Friends	School life	Prof. training	Training for university exams	Music classes
Studies	1	5:1	1	6:1	3:1	8:1
Friends	1:5	1	1:5	1:5	1:3	3:1
School life	1	5:1	1	4:1	2:1	6:1
Prof. training	1:6	5:1	1:4	1	4:1	5:1
Training for university exams	1:3	3:1	1:2	1:4	1	4:1
Music classes	1:8	1:3	1:6	1:5	1:4	1

The other parent's matrix looks as following:

Selection of schools	Studies	Friends	School life	Prof. training	Training for university exams	Music classes
Studies	1	4:1	1	6:1	3:1	8:1
Friends	1:4	1	1:5	1:5	1:3	3:1
School life	1	5:1	1	5:1	2:1	4:1
Prof. training	1:6	5:1	1:5	1	6:1	5:1
Training for university exams	1:3	3:1	1:2	1:4	1	4:1
Music classes	1:8	1:3	1:4	1:5	1:4	1

Based on the criterion *Studies* parents made the following matrixes:

Studies	Quality of teaching	Equipment supply	Pupils' treatment
Quality of teaching	1	2:1	1:3
Equipment supply	1:2	1	1:5
Pupils' treatment	3:1	5:1	1

Studies	Quality of teaching	Equipment supply	Pupils' treatment
Quality of teaching	1	2:1	1:2
Equipment supply	1:2	1	1:3
Pupils' treatment	2:1	3:1	1

Similar matrixes were compiled for all other criteria of second level by both parents.

Next step is dual comparison of options. We assess the advantage of one option over the other in relation to criteria of third level. It is much easier than to do similar assessment in relation to the problem on the whole. For example, for criteria *Quality of teaching* there were generated the following matrixes:

Quality of teaching	School A	School B	School C
School A	1	3:1	5:1
School B	1:3	1	1:2
School C	1:5	2:1	1

Quality of teaching	School A	School B	School C
School A	1	2:1	2:1
School B	1:2	1	1:2
School C	1:2	2:1	1

After dual comparison is finished we need to consolidate matrixes so we could apply analytical hierarchy process. In order to do that, numerical estimates (dual comparisons from the matrixes) shall be transferred into fuzzy form and summed up. Consolidation of dual estimates transferred into fuzzy shape could be considered as membership function of fuzzy set. The particularity of the membership curve resulting from summarizing is that depending on the range of expert estimates it would have several peaks in different places. Estimates with big variation range (e.g. issues where expert opinions are very different) should have smaller weight. Thus it is necessary to test how true are the expert estimates and modify membership functions accordingly. The test of truth is defined for two fuzzy sets thus by reference to transitivity rule if task has more than two estimates then it is possible to carry out consequently operations with couples of fuzzy variables with further aggregation of obtained results [9] (Fig. 4).

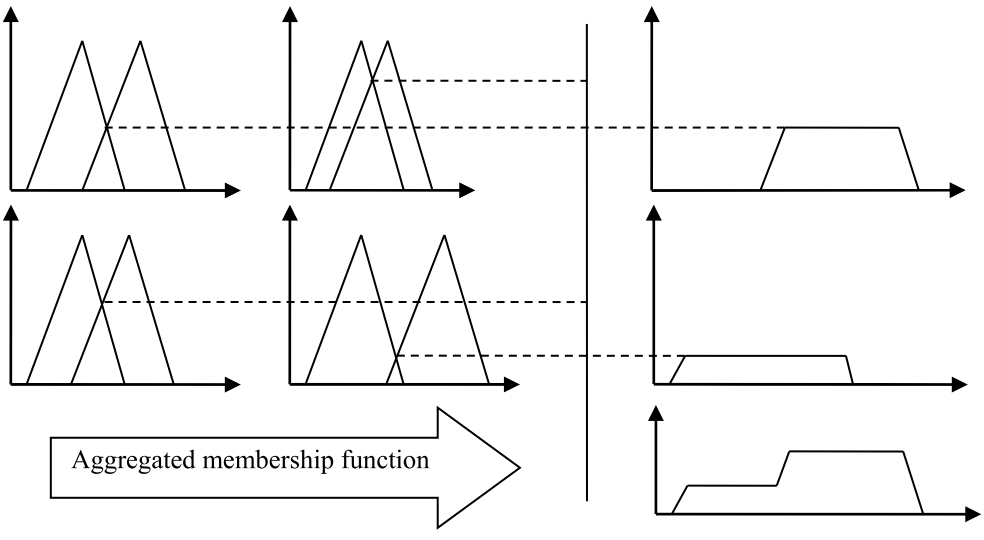

Fig. 4.Test of truth and aggregation of estimates presented as fuzzy variables.

Now it is necessary to calculate eigen vectors (priority vectors) for the obtained matrixes of priority assessment. Calculation could be done by several methods however all of them are restricted by above-mentioned operations and those defined by fuzzy sets. As a result the quantity of vectors will correspond to the quantity of hierarchy levels in the task.

Therefore each criterion will have its vector (priority vector), let us call it V_{ij} where i is a number of hierarchy level and j is a number of level criterion. Each vector has the dimension equal to the quantity of elements under given criterion. For instance, if there are 3 elements of the next level under criterion, then vector's dimension is 3. In addition to elements connected with given criterion, at the next level there are elements that are not connected with it. We may consider that all similar elements have a priority in relation to the given vector that equals 0. Thus it is possible to expand priority vectors for all criteria by adding 0, for elements that are not connected with it. For example, the priority vector $V_{11} = (a, b)$ after expansion will look as $V_{11} = (0, a, b, 0)$. As a result all priority vectors of elements of one level will have identical dimension. The following step requires compilation of matrix of priorities for each hierarchy level, let's call it P_i. Matrix is compiled from expanded priority vectors located in different columns. i.e.

$$P_i = (V_{i0}|V_{i1}| \cdots |V_{ik}| \cdots).$$

After matrixes of priorities are developed it is possible to calculate a priority vector for alternative options. If the quantity hierarchy layers equals N , then vector is calculated under the formula [8]:

$$p = P_{N-1} * P_{N-2} * \cdots * P_0.$$

Studies	0.36
Friends	0.05
School life	0.29
Prof. training	0.16
Training for university exams	0.1
Music classes	0.03

a)

Quality of teaching	0.23
Equipment supply	0.12
Pupils' treatment	0.65

b)

School A	0.66
School B	0.15
School C	0.2

c)

Fig. 5.Developed matrixes of
a– first level;
b–second level;
c– third level.

The results after calculation of priorities based on explicit numbers (for descriptive reasons) for the first matrix enable us to draw the following conclusion: the most significant criteria in school selection are *Studies, School life and the Prof. training*.

In a matrix of second level the most important role for parents plays Pupils' treatment, then goes Level of teaching, and the last is Equipment supply.

Matrix of the third level gives the following conclusion: school A, by criterion Studies, considerably surpasses other schools.

In certain cases, when there is no one-valued occurrence of one fuzzy set into the other or when they are equal, the interpreting of the received results is not as simple, as in scalar form. The complexity lies in the necessity to compare membership functions received after the calculation for each option. And these functions might have quite elaborate configuration. In scalar form such comparisons are elementary. Therefore for comparison these numbers could be converted into habitual scalar form by methods based on calculations of the centre of gravity, the centre of maxima etc.

IV. CONCLUSION

Based on the above reasoning we can draw the following conclusion: transfer of expert estimates into fuzzy form allows to reduce errors by the human factor since instead of the average value, the set of expert estimates will be considered and also the level of expert certainty. The absence of assumptions and simplifications during the decision-making allows to say that described approach does not bring additional errors. Therefore, transfer into fuzzy form allows to use well proved methods in new conditions.

REFERENCES

[1] I. Z. Batyrshin, A.O. Nedosekin, A.A. Stetsko, V. B. Tarasov, A.V. Jazenin, and N.G. Jarushkina, “Theory and practice of fuzzy hybrid systems,” Ed. by N.G.Jarushkina. M: Phismatlit, 2007. (rus).

[2] S. L. Blumin, I.A. Shujkova, P.V. Saraev, and I.V. Cherpakov, “Fuzzy logic: algebraic bases and appendices,” Lipetsk: LEGI, 2002.- 113 p. (rus)

[3] V. N. Burkov, N.A. Korgin, and D.A. Novikov, “Introduction into the theory of organizational systems management,” Moscow: Printing house "Librokom", 2008.-264 p. (rus)

[4] K. Sugihara, Y. Maeda, and H. Tanaka, “Interval Evaluation by AHP with Rough Set Concept,”. Berlin / Heidelberg : Springer , 1999.

[5] V. V. Kruglov, M. I. Dli, and R. Y. Golunov “Fuzzy logic and artificial neural networks,” Training manual.. M: Publishing house of physical and mathematical literature, 2001. – 224 p. (rus)

[6] L. Mylnikov and A. Maksimov, “Use social networks to make decisions,” 3rd International Conference on Applied Social Science (Taipei, Taiwan, 15-16 January 2013), - pp. 137-139.

[7] F. Novikiv, “Discrete mathematics for programmers,” S-Pt: Piter, 2004. (rus)

[8] T. Saaty, “Decision-making: hierarchy analysis method,”. Moscow: "Radio and communication", 1993-278 p. (rus)

[9] T, Arslan, “A hybrid model of fuzzy and AHP for handling public assessments on transportation projects/ Transportation,” N36, 2009, p. 97-112.

[10] L. A. Zadeh, “Fuzzy sets,” Information and Control, vol.8, N 3, 1965, pp.338-353.

[11] Z. Wei-ying, L. Yan, J. Zhuo-Shang,and D. Lin, “Multi-objectives fuzzy optimization model for ship form demonstration based on information entropy,” Journal of Marine Science and Application, Harbin Engineering University, 2006.