Modelling the Generalized Multi-objective Vehicle Routing Problem Based on Costs

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Abstract: The following article addresses a complex combinatorial optimization and integer-programming problem, referred to as the vehicle routing problem, which is typically related to the field of transportation logistics. The aim of the research is to combine a set of objective functions, number of common generalizations and extensions of the problem, arising in distributed services or goods supply. For this purpose, literature on the subject has been analysed, leading to the mathematical modelling method being applied. At the current moment such complicated variants of the problem present high importance for research because of both practical applications and high complexity. The paper proposes a new generalized multi-objective vehicle routing problem with multiple depots and heterogeneous vehicles fleet with regard to various factors affecting costs. The problem statement is presented as a mixed integer linear program. Objectives scalarization approach is proposed in order to reduce decision-maker participation. Shortcomings of the single-criterion formulation and negative effects of replacing the criteria with constraints are shown. The results provide initial data for solving a large number of transportation problems that are reduced to the vehicle routing problem. In particular, the application of the ant colony optimization as a method for solving the problem is discussed.

1 INTRODUCTION

Transportation affects many stages of production and distribution systems and represents an important component of the final product cost [1]. Route planning largely determines the effectiveness of transportation and is often reduced to one of the vehicle routing problems (VRP).

VRP is a combinatorial optimization and integer programming problem, which calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers. In general, the objective is to minimize the overall transportation cost. It is one of the most attractive topics in operation research, logistics, and supply chain management [2, 3]. This interest in VRP is motivated by both its practical relevance and its considerable difficulty.

Indeed, a large number of real-world applications have widely shown that the use of software optimization and automated procedures for solving the VRP yields substantial savings in the global transportation costs [4]. However, the successful application of optimization techniques requires a mathematical model of the problem under consideration.

For today, there are many variants of VRP and its formulations, differing mainly by various additional restrictions. These variants have extended the applicability in real-life cases, but they are often based on models that do not take into account many factors.

Typically, a VRP model contains a single criterion to be minimized, which is the cost proportionate to the total trip distance or time. In fact, most of route planning problems are multi-criteria, and there is no unified solution, which simultaneously satisfies all the objectives.

The purpose of the research is to design a mathematical model of multi-objective VRP and a scalarization approach for reducing decision-maker participation. This paper considers more complex and generalized variant of the VRP, that can be categorized as “rich” VRP [5, 6], that is closer to the practical distribution problems. It makes the model universally applicable and allows for exclusion of extra specifics and characteristics, if needed.

2 PROBLEM OVERVIEW

First of all, let us consider the classical VRP to demonstrate the need for multiple criteria or constraints.

2.1 The classical VRP

The solution of the classical VRP problem is a set of routes, which all begin and end in one depot, and which satisfy the constraint that all the customers are served only once (Figure 1). The transportation cost can be diminished by reducing the total travelled distance and by reducing the number of the required vehicles.

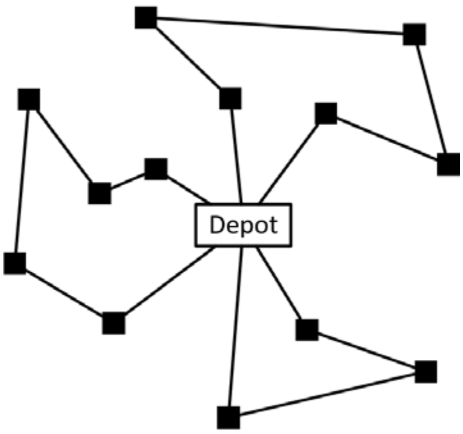

Figure 1: Example of a classical VRP solution: the black squares and the connecting lines represent clients and routes respectively.

2.2 Downside of a single criterion

In the case of the described classical VRP with one depot, the minimized sum of weights of all edges that make up the routes leads to the only optimal solution of having a single route for a single vehicle, i.e. VRP is reduced to less difficult TSP.

This conclusion follows from the structural features of the road network and its graph: the weight of any path between two vertices must be greater than or equal to the weight of the edge connecting them, therefore the triangle inequality is met for all edges: $c_{ik} + c_{kj} \geq c_{ij}, \forall i, j, k \in V$.

Since the increase in number of vehicles leads to an increase in number of the edges entering routes, it is possible to reduce the solution cost by connecting the last vertex of the previous route with the first vertex of the next (Figure 2).

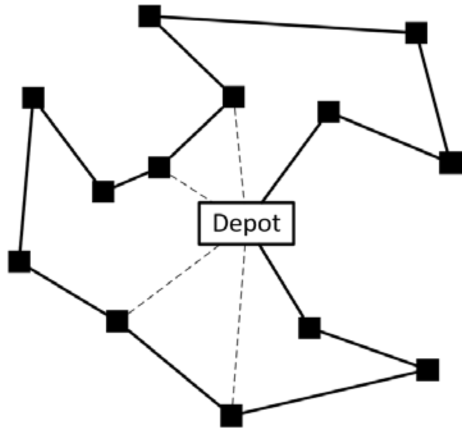

Figure 2: The total cost of the solution is reduced by connecting routes into one.

Obviously, in this case, the total value of all routes is reduced, but clients are served sequentially and in general, the implementation takes more time than using several vehicles. Thus, overall performance is reduced and another criteria or constraints are is required to resolve the dilemma.

Notice that the conclusion is valid for both symmetric and asymmetric matrix of costs if the triangle equality is satisfied.

2.3 Constraints instead of multiple objectives

Some objectives can be replaced by constraints in order to consider multiple criteria. It is suitable for some specific cases and usually greatly facilitates the search for solutions, but it poorly corresponds to reality in general.

For example, one of widespread approaches to obtain approximately equivalent routes is route balancing. There are different ways to balance routes by restrictions, like balancing the number of customers served by each active vehicle, balancing the distance of routes travelled by vehicles or balancing the waiting time required for the route.

In the first instance, the number of vertices in each route must not differ by more than specified (one, in extreme cases). This restriction allows to find solutions effectively even for a large number of vertices [7].

Thus, routes are balanced in the number of vertices, but not always distance-balanced (it can be seen in Figure 3 that routes are incommensurable in length).

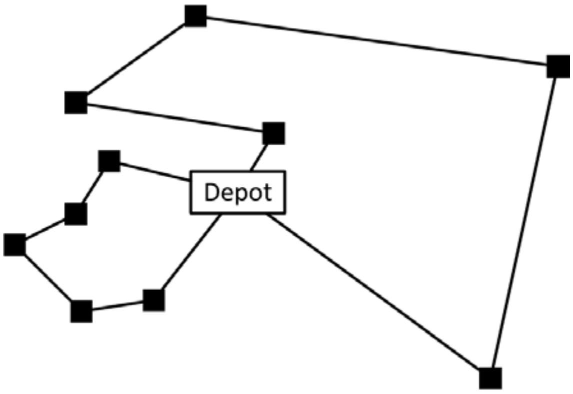

Figure 3: Balancing the number of vertices.

Contrariwise, the equalization of route distances may lead to irrational result, when a vertex is forced to belong to the route (Figure 4 shows that two vertices at the bottom are in different routes).

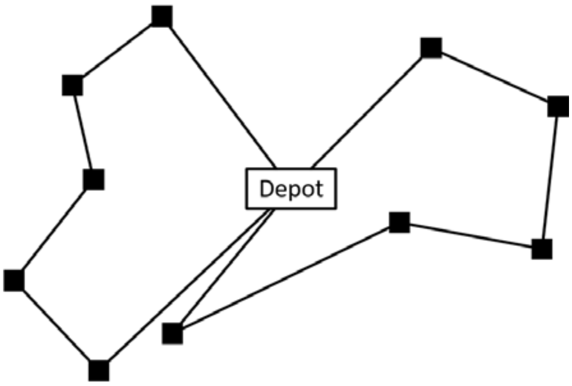

Figure 4: Balancing the distances.

It should be noted that balancing is sometimes proposed as an objective function [2]. However, it is rarely justified and associated with costs. Furthermore, it is clear, that a compromise among different balancing criteria is needed.

3 GENERALIZATIONS

In this paper, an asymmetric Heterogeneous Multi-Depot VRP (HMDVRP) mathematical model is used to formalize the described general multi-objective problem. It is a variant of the VRP characterized by multiple depots, multiple vehicle types and multiple asymmetric matrixes of initial data for each vehicle. Figure 1 provides an example for the solution with the use of four vehicles (a, b, c, d) and different depots.

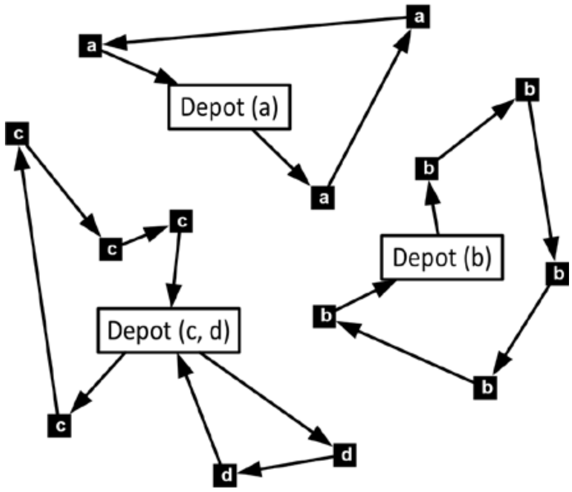

Figure 3: The solution for combined generalizations.

3.1 Symmetric and asymmetric

In the symmetric VRP, the distance between two customers is the same in each opposite direction, forming an undirected graph [8]. This assumption does not correspond to real conditions [9] and often leads to a certain gap between a theoretical project and practical application. Actually, the shortest path between two points of the road network usually depends on directions. Such differences are most noticeable at small scales, for example, in an urban environment. Therefore, from a practical point of view, it is advisable to consider the asymmetric VRP, which assumes different distances in each opposite direction, forming a directed graph. In addition, many software tools for routing provide data according to chosen direction.

On the other hand, the use of a directed graph significantly increases the solution space and, consequently, complicates the search for the optimum. To avoid this, depending on the particular application, costs (weights of edges) for opposite directions can be reduced to a certain average value.

Thus, symmetric problem can be considered as a particular case of asymmetric one, and since the purpose of this article is to pose a generalized version of multi-objective VRP, asymmetric variant will be used further.

3.2 Single and multiple depots

As noted above, route planning from depots to customers is a common and challenging task. Nevertheless, there is a rigid assumption that there may be only one depot. Although the single-depot VRPs have attracted so much attention, they are not suitable for some cases where a company has more than one depot, in which vehicles start and end their routes [10].

To resolve this limitation, this paper focuses on the VRP with multiple depots, or Multi-depot VRP (MDVRP). Multi-depot VRP is a generalization of the classical VRP, so it does not rule out the case of single depot.

Because there are additional depots, the decision makers usually have to determine which depots serve which customers, which is a grouping problem to be solved prior to the routing and scheduling problems. Obviously, this type of problem is more challenging and sophisticated than the single-depot VRPs.

3.3 Homogenous and heterogeneous fleet

Commonly, the fleet in VRP models is homogeneous, which does not always correspond to reality. Decisions relating routing heterogeneous fleets of vehicles are frequently taken into consideration in logistics operations [11].

The Heterogeneous Fleet VRP (HVRP) is a generalization of the classical VRP in which customers are served by several different types of vehicles with various characteristics. It is assumed in proposed model that the number of vehicles of each type is fixed and equal to a constant (for it to be unlimited the number of vehicles just should be big enough).

It is harder to solve heterogeneous fleet problem than the homogeneous one. Therefore, if the difference among vehicles is not significant, characteristics can be considered the same.

4 MATHEMATICAL MODEL

Taking into account everything above, we have constructed a mathematical model of the problem based on a linear programming formulation in terms of graph theory.

4.1 Problem formulation

Let $G = (V, A)$ be a directed graph, where $V = V_C \cup V_D$ is the set of vertices $\{1, \ldots, n\}$, V_C represents clients, V_D represents depots and $A = \{(i, j) : i, j \in V, i \neq j\}$ is a set of arcs defined between each pair of vertices. Heterogeneous fleet K of vehicles available and there is a bijection between the set K of vehicles and the set V_D of depot-vertices. Note that technically the location of depot-vertices can coincide if some vehicles belong to a single depot. The demand D_j is set for each customer j and the carrying capacity Q_k for each vehicle k. The end goal is to determine a minimum-cost set of routes in the feasible region considering all n criteria.

The formulation uses a set X of binary variables x_{ijk} equal to 1 if vehicle k travels directly from i to j, and to 0 otherwise.

According to the established assumptions, the Generalized Multi-Objective Vehicle Routing Problem can be stated as follows:

$$\min\left\{ f_1(X),\ldots, f_n(X)\right\}, \tag{1}$$

subject to:

$$\sum_{k\in K}\sum_{j\in V} x_{ijk} = 1, \quad \forall j \in V_C; \tag{2}$$

$$\sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, \quad \forall k \in K \wedge \forall i \in V; \tag{3}$$

$$\sum_{i\in V_D}\sum_{j\in V} x_{ijk} \le 1, \quad \forall k \in K; \tag{4}$$

$$\sum_{j\in V} x_{ijk} = 0, \quad \forall i \in V_D \wedge \forall k \in K, \quad i \neq k; \tag{5}$$

$$\sum_{j\in V_C} D_j \sum_{i\in V} x_{ijk} \le Q_k, \quad \forall k \in K. \tag{6}$$

Equation (1) contains a vector of objective functions to minimize. Constraints (2) guarantee that each customer will be visited exactly once. Flow conservation constraints are expressed in (3). Constraints (4) mean that each vehicle departs from the depot once or doesn’t depart at all. It is given that the fleet is heterogeneous, therefore it is important to consider that each vehicle belongs to its own depot (5). Finally, the limitation of the carrying capacity, which cannot be less than the total demand of the visited customers for each vehicle, presented in (6).

Such flexible formulation makes it possible to exclude insignificant limitations from consideration leaving only the needed constraints.

4.2 Problem parameters

Roads and clients are characterized by several parameters for each type of the vehicle, and defined as the initial data of the problem:

- c_{ijk} – fixed fare between vertices i and j for vehicle k;
- d_{ijk} – distance from vertex i to vertex j for vehicle k;
- t_{ijk} – travel time between vertices i and j for vehicle k, including the service time.

Such choice of parameters is based on real basic information about the path between two points, which can be obtained using modern navigation tools. It is supposed that data on roads are defined for various types of transport; therefore, an extra parameter index k corresponding to a particular type of vehicle is used.

4.3 Objectives

Let us form a vector of objective functions (1) allowing to take into account the basic criteria for estimating the cost of a solution. In construction of the mathematical model, the following principle was adopted: all decisions ultimately affect the enterprise profits and costs, which can be predicted at least roughly. Otherwise, additional methods of decision-making are required to find the weights of objective functions. Integrated expert estimates for decision-making support can simplify this problem [12].

1. Number of involved vehicles:

$$f_1(X) = \sum_{k\in K} \lambda_{1k} \sum_{i\in V_D} \sum_{j\in V} x_{ijk}.$$

Minimizing the number of involved vehicles is one of the key objectives of VRP [13]. If the route contains a single vertex, the vehicle does not leave the depot and is considered uninvolved. The penalty value λ_{1k} is applied to each involved vehicle k. Expenses on preparation of the vehicle and the courier determine the size of a penalty.

2. Total travelled distance:

$$f_2(X) = \sum_{k\in K} \lambda_{2k} \sum_{i\in V} \sum_{j\in V} d_{ijk} x_{ijk}.$$

The total distance of all routes determines mainly expenses on fuel and transport servicing. As a rule, the fuel consumption per distance unit and the type of used gasoline are known for all vehicles. Based on these data, fuel cost per distance unit can be obtained for each vehicle k. Maintenance costs are estimated per distance unit according to particular vehicle characteristics. In addition, different risk costs are calculated with consideration of distance. For instance, the risk associated with traffic accidents can be estimated as a multiplication of the probability of the accident and the average cost of its consequences. In a similar manner, other expenses associated with the distance travelled can be estimated. Thus, the coefficient of the objective function λ_{2k} is the sum of all components.

3. Total travelled time:

$$f_3(X) = \sum_{k\in K} \lambda_{3k} \sum_{i\in V} \sum_{j\in V} t_{ijk} x_{ijk}.$$

The cumulative time to be spent by all vehicles is necessary to take into account for the time rate wage payment calculation. So, λ_{3k} is estimated as wages per unit time for each courier.

4. The completion time:

$$f_4(X) = \max\left(\lambda_4 \sum_{i\in V} \sum_{j\in V} t_{ijk} x_{ijk} \right).$$

The time since departure of the first vehicle to return of the last vehicle determines costs of maintaining the transportation system as λ_4 and allows for calculation of the prior basic compensation of all couriers regardless of their employment. All involved vehicles move simultaneously therefore the completion time is determined by the most prolonged route and is evaluated using Chebyshev scalarization function. Coefficient λ_4 corresponds to the enterprise costs per time unit and does not depend on specific routes.

5. Fixed costs:

$$f_5(X) = \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c_{ijk} x_{ijk}.$$

The movement between two vertices may be associated with a priori costs, for instance, toll road taxes. In some cases, the cost depends on the type of vehicle, thence coefficient value is set for each.

Thus, for all considered objective functions, the value is now expressed in uniform units; therefore, a linear scalarization (weighted sum) can be used:

$$F = \sum_{i=1}^{n} f_i(X).$$

The proposed criteria are not exhaustive for all practical problems, but as analysis has shown, they have a significant impact on the cost of the solution. Any other criteria can be added in a similar way, but it is important to consider that the difficulty of finding a solution depends, among other things, on the complexity of an objective function. If it is impossible to relate the significance of the criterion to costs, other decision-making methods may be needed, especially for nonlinearity.

5 SOLUTION METHOD

In the field of combinatorial optimization, the VRP is regarded as one of the most challenging problems. It is indeed NP-hard, so that the task of finding the best set of vehicle tours by solving optimization models is computationally prohibitive for real-world applications [14]. As a result, different types of heuristic methodologies are usually applied. Furthermore, in view of conflicting objective functions it is difficult to accomplish the task of clustering. Clustering is likely to eliminate the optimal before the start of the search and keep suboptimal. Existing algorithms are not designed to solve the proposed generalized multicriteria vehicle routing problem.

Therefore, to solve this problem a modified multi-objective ant colony optimization algorithm (ACO) is being developed. ACO is a probabilistic technique for finding good paths through graphs and it is suitable for multi-objective problems [15, 16]. Swarm metaheuristics like ACO are useful for a large search space, especially using methods of increase in effectiveness [17]. The results obtained using the algorithm allow us to conclude that the solution of the problem depends substantially on the chosen weighting coefficients of the objective function.

Figures 5-6 show an example of how different solutions of the problem with two of the above objectives, the total travelled distance and the completion time. These criteria are highly conflicting and therewith illustrative. In the first case (Figure 5), the sum of distances is minimized, but the routes are not balanced and the time required to completion is excessive.

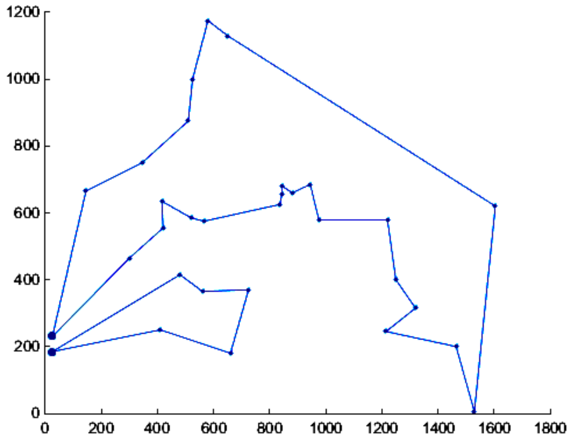

Figure 5: Minimization of the total travelled distance.

Alternatively, in the second case, the time of the longest route is minimized primarily, so the routes are approximately of equal length though they do not look optimal separately.

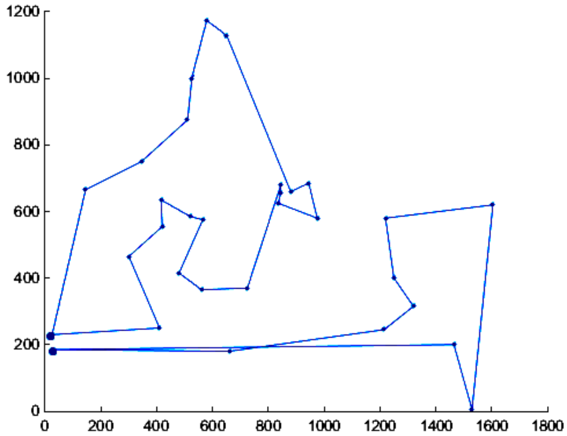

Figure 6: Minimization of the completion time.

6 CONCLUSIONS

A general mixed-integer linear mathematical programming formulation of Multi-Objective Vehicle Routing Problem with multiple depots and heterogeneous vehicles fleet is designed. The proposed set of objective functions considers factors affecting costs, but the considered list is not exhaustive. The presented approach allows supplementing the model with other objectives in the same way reducing decision-maker participation. In addition, it is possible to exclude some extra parameters, which are caused by generalization of the model, if required.

The model stated in this paper can be used effectively not only to solve problems concerning the delivery or collection of goods but for the solution of different real-world applications rising in transportation systems as well. Several examples of specific real-life situations [18, 19, 20] involving multi-objective routing problems are presented below:

- cargo transportation;
- fast food delivery;
- school bus routing;
- solid waste and trash collection;
- merchandise transport routing;
- tour planning for mobile healthcare facilities;
- postal services;
- maintenance engineering.

Thus, the new mathematical model, unlike existing ones, expands the practical applicability in cases of distributed depots and customers, and, at the same time, aims to global cost savings. In the forthcoming work, an algorithm for solving the problem will be reviewed in detail.

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