Detection of Changes in Oil Well Power Consumption Profile on the Basis of Dynamic Time Warping Algorithm

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Abstract: At present oil companies are forced to continually decrease electric power inputs. However, energy efficiency of oil well equipment decreases in time. Well re-equipment enables to stop energy efficiency loss but it requires large additional inputs. The possible solution of this problem is development of the energy efficiency growth strategy that does not include equipment replacement. To do this the oil well model that is able to precisely estimate energy efficiency of every element in electric power system needs to be constructed. Oil well technological and mechanical parameters, determining production efficiency, are strongly connected to the electric parameters of equipment. Therefore, they need to be included in the model. Models used in oil companies for energy efficiency estimation reflect dependencies between described parameters but they do not consider instant changes of electric parameters caused by changing of electric power system regime. Mathematical models of electric power systems that consider instant changes of electrical parameters are based on differential equations which have complicated solutions. The paper considers a method for instant changes analysis in power consumption profiles of oil well equipment that is based on dynamic time warping algorithm. It is demonstrated that instant changes of electrical parameters at the short time period caused only by electric power system regime changes and are independent from well production conditions. Based on this thesis it is proposed to study instant changes of electrical parameters in wells with similar production conditions. The comparison of two modifications of dynamic time warping algorithm is presented. Investigation of the properties of given modifications when applying to power consumption profiles exposes limitations of using the method. However, the study of other algorithm modifications allows to find possible ways of overcoming the restrictions.

1 INTRODUCTION

When operating oil field, two processes occur: on the one hand, depleting of the oil reserves causes changing of extraction conditions, on the other hand, ageing of well equipment causes increase of electrical energy loss in elements of electric power systems (EPS) and hydraulic loss in tubing strings. These factors lead to increase of the operation and maintenance expenses of the oil wells.

To ensure a stable profit, oil companies are forced to yearly increase the oil extraction while reducing the cost of operation and maintenance of oil fields. Unreasonable selection and misuse of well electrical equipment causes inefficient EPS operating regimes (e.g. underload and overload) and also leads to an increase of the electrical power inputs due to losses.

When operating the well, it is necessary to maintain parameters of the technological process that ensure maximal flow rate to the well under given geological, climatic and technological conditions.

The technological parameters restrict operating of mechanical and electrical equipment of the well.

Since full-scale experiments in oil fields are not allowed, the methods of studying the well are based on mathematical modeling [1], simulation [2][3] and time series analysis and prediction [4][5].

At present, different technological, hydraulic and electrical models of oil well are developed. These models allow to make decisions on rational choice and effective operation of well equipment. However, they are based on theoretical equations and do not completely meet real operational conditions.

Moreover, to increase precision of these models the object parameters identification needs to be done. It requires obtaining of internal parameters of equipment (e.g. motor flux linkage, rotor and stator resistances and others) that is impossible in real conditions.

In these conditions, the task of evaluating the equipment parameters subject to its operation features under the conditions of uncertainty and data incompleteness becomes important. To solve this, oil field statistical model can be constructed. This model makes possible to analyze the object by indirect method based on statistical data representing changes in electrical, mechanical, and technological parameters.

Since the well production conditions and the operating practice vary depending on many factors, it is necessary to understand the nature of these changes and the leverage of various factors on them. Based on this information it is possible to make a data clustering for identification of typical regimes.

The main indicator that determines the economic efficiency of the oil well is the specific power consumption. This is the ratio of the amount of electrical power, consumed by oil extraction equipment, to the mass of the produced oil or the volume of the liquid produced. Consumption level depends on electrical equipment type, EPS regime parameters and control algorithm applied to the pump electrical drive. The volume of extracted liquid depends on geological, climatic and technological parameters. The mass of produced petroleum is determined by chemical composition of the formation fluid and the content of water and gas in it.

The paper considers questions of analysis of changes in well parameters under different operational conditions.

The aim of the research is studying of power consumption changes when changing electrical parameters of well equipment and pump control parameters under different fixed values of technological and mechanical parameters.

The degree of relationship between available electrical, mechanical and technological parameters and the intensity of their changes is studied in the research. Based on these data, the analysis of changes in power consumption profiles will be carried out.

2 OIL FIELD PARAMETERS

Oil well is a vertical, inclined or horizontal bore connecting surface with reservoir.

When productive formation keeps shut in, the reservoir pressure is equal at every point and liquid does not flow. When formation exposing the pressure at the wellbore becomes less than reservoir pressure and liquid starts flowing to the well [6]. Flowing continues until the difference between reservoir and wellbore pressures becomes less than the sum of hydraulic resistances in a tubing string.

The main operational characteristics of oil well are production condition and lifting type.

Production conditions define energy sources that provide maintenance of reservoir pressure sufficient for lifting liquid to the surface. The study assumes production conditions are given by oil field operating practice.

Lift type defines tools used for lifting the liquid. This study considers wells with pumping based on electrical submersible pumps (ESP) with induction motors (IM) placed inside the well.

Production rate (measured in barrels per day (BPD)) determines volume of liquid potentially being extracted from the well at a given time period. BPD depends on well inflow and determines total company profit obtaining from the well.

Well operational expenses depend on different parameters. When pumping, the most expenses are electrical energy costs (up to 40% of total costs). Therefore, well operational efficiency is measured by specific power consumption described above. In these conditions, increase of operational efficiency can be obtained either by BPD increasing or by decreasing energy consumption. This paper considers abilities of energy consumption decreasing when fixed BPD values.

The next subsections describe main well parameters and their dependencies.

2.1 Technological parameters

The main technological parameter that determines liquid extraction efficiency is well inflow. It can be obtained using Darcy equation [1, 6]. The general solution of this is complex; therefore in practice the specific solution is used. It holds when the following assumptions:

- flowing is radial around the well;
- reservoir characteristics and liquid composition do not change in sufficiently long time period.

When following above assumptions, geological parameters of the productive formation do not significantly change in a short time period. In these conditions, the pressure drawdown (calculated as the difference between reservoir and bottomhole pressures) determines well inflow. The value of it has to be maintained a constant in accordance with technological process.

2.2 Mechanical Parameters

The subject of this study is well equipped with electrical submersible pumps with induction motors (ESP). The ESP provides lifting of the reservoir liquid to the wellhead and maintaining wellhead pressure sufficient for moving liquid to the booster pumps.

Main parameters of ESP are head (h) and flow rate (q). Head is the height of vertical column of liquid generating at the discharge of the pump.

Flow rate defines liquid volume that pump is able to lift to the height equal to h under given hydrodynamic parameters of tubing string. It depends on pipe diameter, flow velocity and pipe hydraulic resistance.

When head is given, flow rate can be obtained using H-Q curve. This curve is presented in ESP manuals.

ESP converts kinetic energy of shaft rotation into pressure energy. The following equations describe connections between pump parameters and rotational speed of the motor:

$$q_2 = q_1\left(\frac{n_2}{n_1}\right) , \quad (1)$$

$$h_2 = h_1\left(\frac{n_2}{n_1}\right)^2 , \quad (2)$$

$$BHP_2 = BHP_1\left(\frac{N_2}{N_1}\right)^3 . \quad (3)$$

In the above formulas h_1, h_2 are pump heads, q_1, q_2 are flow rates, BHP_1, BHP_2 are pump break horsepowers, and n_1, n_2 are rotational speeds in two different operational conditions respectively.

When substituting nominal values of corresponding parameters to the (1-3), pump characteristics for any given rotational speed can be obtained.

2.2 Electrical Parameters

ESP is driven by induction motor that installed in one shaft with a pump stages. Therefore, it can be assumed that motor torque is equal to the pump torque:

$$\tau = \tau_r , \quad (4)$$

where τ is a motor torque, τ_r is a pump torque.

IM consumes power of two types: active power (P) that is spent on the shaft rotation and reactive power (Q) that is spent on electric field generation. Reactive power is usually compensated by special equipment, therefore this study considers only active power consumption.

The frequency converters are usually used to control IM in oil wells. They change the rotational speed of the model by changing both mains frequency and voltage. The equation (5) describes dependency between synchronous speed of the motor and AC power frequency.

$$n_{synch} = \frac{120 \cdot f}{p_{poles}} , \quad (5)$$

where n_{synch} is synchronous speed, f is frequency of AC power, p_{poles} is number of poles in stator.

Active power of the IM is calculated by the following formula:

$$P = \tau_r \cdot \omega \cdot \eta_m \cdot \eta_p , \quad (6)$$

where ω is angular velocity, τ_r is pump torque, η_m is motor efficiency, η_p is pump efficiency.

Synchronous speed and angular velocity of IM is connected by the following expression:

$$\omega_{synch} = \frac{\pi \cdot n_{synch}}{30} \quad (7)$$

The shaft rotation speed of the induction motor is less than synchronous rotation speed of magnetic field by the value of $\Delta\omega$ depending on the slip value:

$$slip = \frac{\omega_{synch} - \omega}{\omega} , \quad (8)$$

where ω_{synch} is synchronous IM speed, ω is IM shaft speed.

The slip value depends on the torque developed by the engine, however, under given conditions, the slip can be assumed constant and equal to the nominal value. The bases of this assumption are given below.

Dependencies between the IM torque and the pump torque and induction motor rotational speed when different values of voltage and frequency are shown in Figure 1.

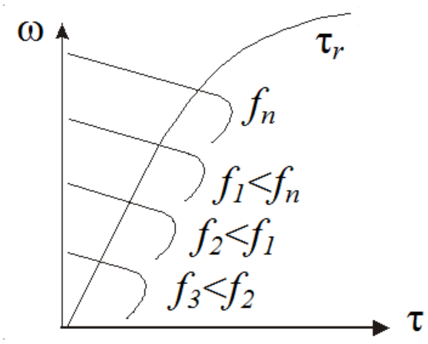

Figure 1: Motor and pump torque curves under different frequency values.

The figure shows that for the given load type, the IM torque required to rotate the pump shaft is significantly less than the critical torque at the entire frequency range. Under operating conditions, the frequency control range is sufficiently small (30-60 Hz), and the torque developed by the motor does not change without changing the frequency. The intersections of the IM and pump torque curves are at the segment where the torque curve has sufficiently slight slope. Thus, the slip in the whole control range has insignificant changes in comparison with the nominal value. In this case, it is possible to not consider slip changes when change the motor torque.

Formulas considered in the section, describe dependencies between main parameters of oil well but the typical models based on them are robust and do not allow to study tiny changes in power consumption profiles of wells [2][6][7].

3 STATISTICAL MODEL OF OIL FIELD

For estimating power consumption changes in oil fields the average consumption values are used (in energy units, kWh).

Standard averaging intervals are day, month, quarter, and year. Energy efficiency is estimated in a whole field and is determined by average annual integrated consumption index. For increasing energy efficiency, energy consumption of a field is yearly decreasing on fixed value.

The expected values of the well energy consumption are calculated based on the formulas described in the section above. Calculated parameters are averaging then by the whole field.

Current and expected power consumption are calculated based on actual volume of produced oil and expected values of production rates. The annual electricity consumption reduction is determined by these parameters.

This technology has the following shortcomings: the potential optimizing abilities of a single well are not considered; averaging over long periods does not allow to determine the cause of ineffective operating regimes of the well electric equipment. Ineffective regimes are both regimes with high power consumption and emergency regimes.

The average energy consumption is used as the main parameter for energy efficiency estimation. To determine the optimal averaging interval of this parameter, 32 wells placed in two fields with different geological characteristics and lifecycle stages were analyzed. The profiles of well energy consumption were built with the averaging intervals of month, day, hour, minute and second. When analyzing profiles with shorter averaging intervals, the average values of power consumption over the previous interval were used as a template for comparing. The study showed that the optimal averaging interval is an hour.

Table 1: Oil Well Parameters.

Parameter (Symbol)	Units
Active power (P)	kW
Frequency (f)	Hz
Motor Rotational Speed (n)	RPM
Motor temperature (T)	°C
Intake pressure (p_{in})	bar
Wellhead pressure (p_{wh})	bar
Head (h)	ft.
Liquid production rate (q)	BPH

During preliminary study changes intensity analysis as well as correlation analysis of given parameters were carried out. It was found that electrical parameters are changed intensively while other parameters are subject to weak changes. Correlation analysis showed that all electrical parameters and motor temperature have strong relations with correlation coefficients of 0.7 (Spearman correlation) and 0.9 (Pearson correlation). Technological and electrical parameters as well as mechanical parameters have weak correlation (with correlation coefficients of about 0.02 for both methods). Based on obtained data it was concluded that the volume of information is insufficient for determining statistical dependencies between electrical and non-electrical parameters of oil well. The preliminary study showed that energy consumption changes in a single well are not caused by changes of geological, technological and mechanical processes in a short time period. The changes come from internal electrical and thermal processes in equipment and external parameters of EPS regimes.

On the ground of above analysis it was suggested to divide full energy consumption profile into two parts. The first part is caused by technological process. It is relatively stable for a single well in a short time period. The second part is caused by external changes of EPS regimes and internal changes in equipment. This part has significant changes even in a short time period. Minimizing of energy consumption can be obtained by reducing them. To do this the variable part of the signal is to be extracted from the whole profile. The extraction is based on comparative analysis of test energy consumption profile and reference one. Profiles with known production conditions and EPS regimes were selected as reference signals.

At the main research phase the deviation analysis of power consumption profiles from the template profile was carried out. The profiles that demonstrate consumption of oil well in the known production regimes and under the same operating conditions as the investigated wells were selected as template profiles.

To study the changes in electricity consumption, eight identical samples were generated for four wells (two samples per well on October and March, respectively). This choice was made based on the results of enterprise inspection that showed the most unfavorable changes in electricity consumption in the autumn and spring. Selected wells were in operation during the given time intervals and the biggest amount of data was obtained from them.

Each sample consists of 30 columns corresponding to the day of month. Each column has 24 rows where average hour consumption values are placed. The day consumption change graphs were built using the samples. These graphs were used both as reference and as test signals in the analysis procedure described in the next section.

4 ENERGY CONSUMPTION ANALYSIS

Dynamic time warping algorithm (DTW) was originally introduced as a tool for similarity measurement of complex signals [8] but it is also possible to use it for measuring differences between test signal and given template [9]. Algorithm transforms test signal into template by stretching and shrinking different segments of time axis. Algorithm accuracy depends on similarity of test signal and template after warping. When warping, optimal warping path is constructed. Optimal path is a matrix that contains minimal amount of transformations providing maximal similarity of warped signals. Full description of the DTW and its features is given in [8] – [13].

Warping path is defining points of signals being shifted when warping and the shifting distances. To do this optimally, the weighting matrix is used. Weighting matrix constrains possible ways of points shifting and maximal shifting distances. Weighting matrix influences the accuracy of algorithm and its ability to give right similarity measures. Different types of weighting matrices are considered in [14] – [16].

In the study two different weighting matrices are used (9) and (10). Below expressions describe possible shifting ways and distances for classical and modified DTW respectively.

$$\begin{bmatrix} m & n-1 \\ m-1 & n-1 \\ m-1 & n \end{bmatrix}, \tag{9}$$

$$\begin{bmatrix} m & n-1 \\ m-1 & n-2 \\ m-1 & n-1 \\ m-2 & n-1 \\ m-1 & n \end{bmatrix}, \tag{10}$$

To estimate the deviation of signals by the shape of the optimal path curve, the method of analyzing the deviations of the path from the diagonal, proposed in [9], was used.

During the analysis, the following parameters of the algorithm were evaluated: the matching accuracy of the test reference signals after warping, the distribution of the matched points of the test and reference signals, distribution of distances between warping path curve and diagonal. The following graphs were built: signals before and after warping, matching diagrams, warping path diagrams along with diagonal and lines showing distances between path and diagonal.

The study results for the classical DTW when the test power consumption profile corresponding to the stationary regime with small deviations are showed in Figures 2-5.

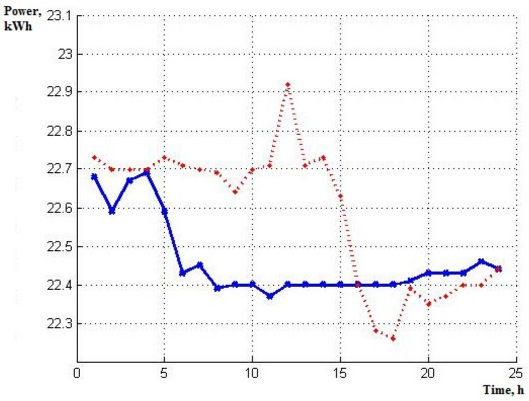

Figure 2: Power consumption profile of oil well. Dotted line - reference signal, solid line - test one.

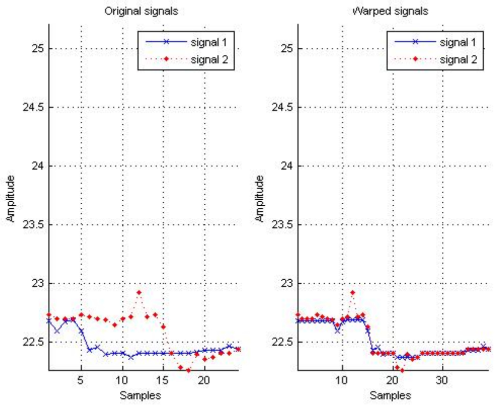

Figure 3: Warping diagrams for classical DTW. Left graph – signals before warping, right graph – signals after warping.

The graph of signals after warping shows that the curves of the test and reference signals are close to each other. It indicates sufficiently high accuracy of the algorithm for this type of curves.

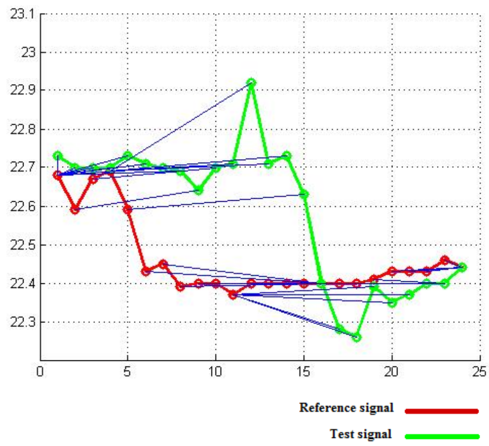

Figure 4: Matching diagram for classical DTW. Reference signal is green, test signal is red. Blue lines show shifting distances of points.

Matching diagram has multiple matching points. There are the points of a signal where more than one matching lines come (in the figure matching lines are blue). The multiple matching points reduce the accuracy of the algorithm and the information capability of the warping path curve. They produce long straight sections on the warping path.

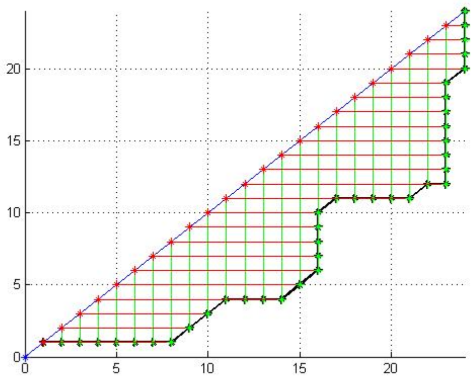

Figure 5: Warping patch along with diagonal for classical DTW. Lines show distances between path and diagonal.

Deviations of the warping path from the diagonal show the discrepancy between the reference and test signals at each point. The better the signals match each other after warping, the more accurately changes are reflected in the warping path. If a certain optimal power consumption profile is used as a reference signal (e. g., obtained by mathematical model), the discrepancy can be used to find the points where potential problems exist. These problems, then, need to be investigated by other methods. Thus, the algorithm can be used to find the sections of power consumption profiles to be optimized.

Figures 6-8 illustrate the warping results with modified weighting matrix (10).

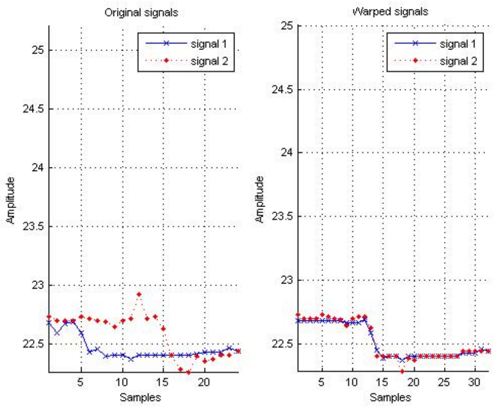

Figure 6: Warping diagrams for modified DTW. Left graph – signals before warping, right graph – signals after warping.

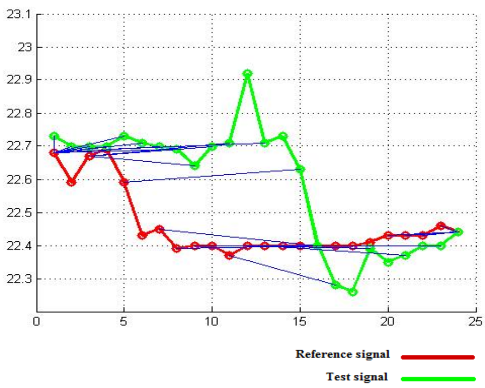

Figure 7: Matching diagram for modified DTW. Reference signal is green, test signal is red. Blue lines show shifting distances of points.

The matching diagram of the modified algorithm has fewer multiple matching points, but it also has unconnected points that can lead to the loss of significant points.

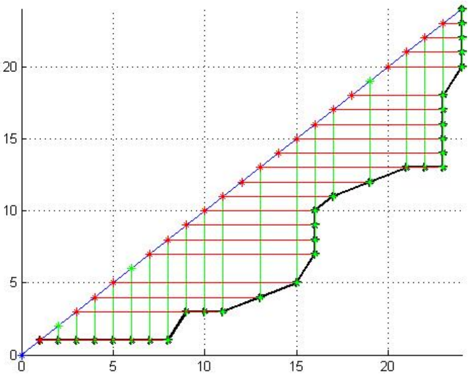

Figure 8: Warping patch along with diagonal for modified DTW. Lines show distances between path and diagonal.

The study showed that the algorithm with a modified weight matrix is more accurate than the classical one. In addition, this algorithm has nearly no cases of multiple matching, but there are points at the matching diagram that do not have connections. This gives potential ability for skipping these points. If the skipped point is significant (e.g. it demonstrates a significant decrease of power consumption), skipping the point leads to incorrect interpretation of the warping path curve.

Further studies showed that if the discrepancy of signals increases, both the number of multiple matching cases in the classical algorithm and the number of missing points in the modified algorithm increase (Figures 9-13).

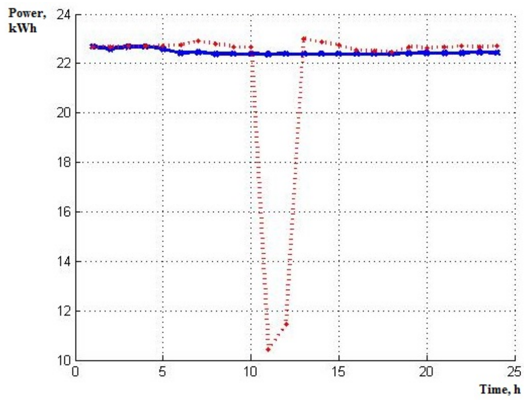

Figure 9: Example of a profile with a big deviation. Dotted line - reference signal, solid line - test one.

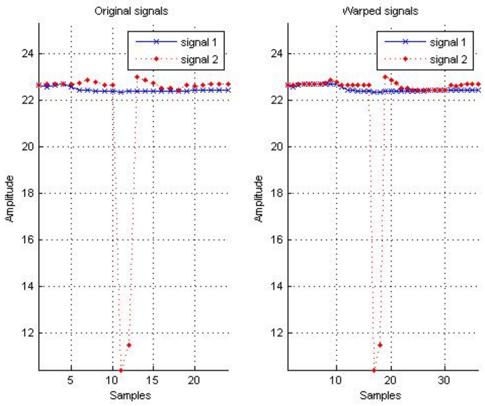

Figure 10: Warping diagrams for classical DTW. Left graph – signals before warping, right graph – signals after warping.

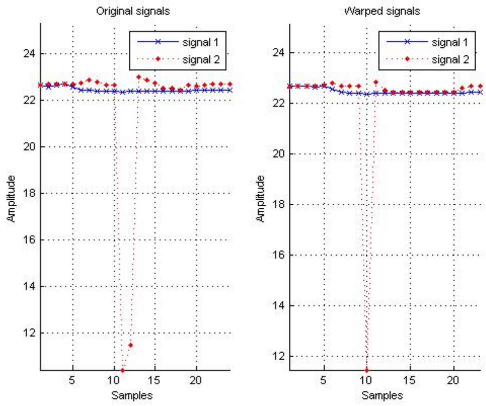

Figure 12: Warping diagrams for modified DTW. Left graph – signals before warping, right graph – signals after warping.

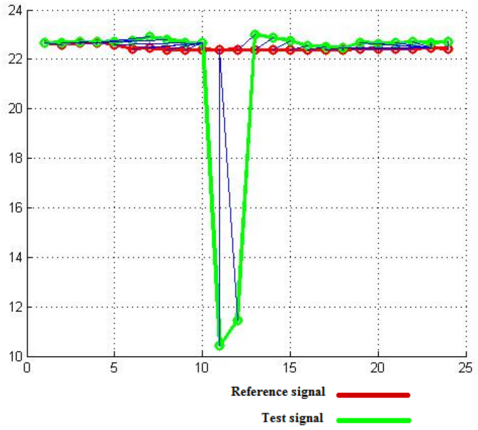

Figure 11: Matching diagram for classical DTW. Reference signal is green, test signal is red. Blue lines show shifting distances of points.

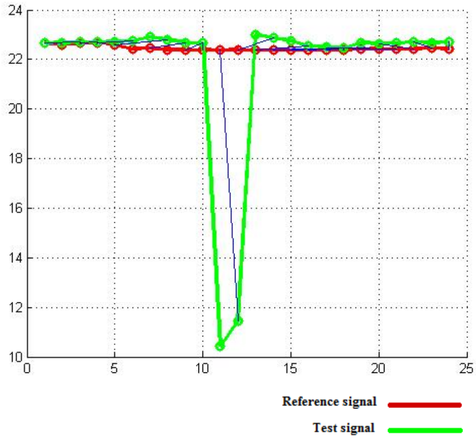

Figure 13: Matching diagram for modified DTW. Blue lines show shifting distances of points.

The accuracy of both algorithms decreases.

As a limiting case, the transient process of an emergency motor shutdown was considered (Figure 14).

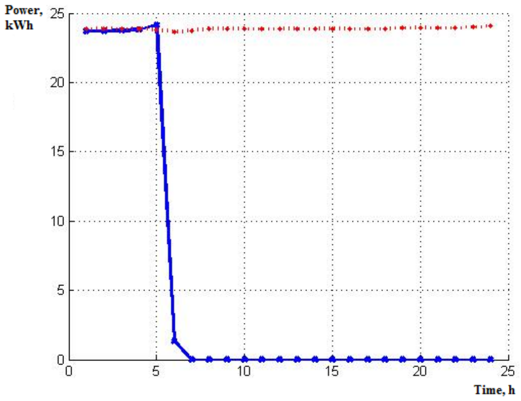

Figure 14: Example of profile with zero-valued segment. Dotted line - reference signal, solid line - test one.

The specified signal in this case becomes zero. The accuracy of both algorithms decreases significantly in this case (Figures 14-15). In addition, at the zero-valued segment of the test signal, the warping path curve in classical DTW algorithm matches with the diagonal (Figure 16), which makes it uninformative.

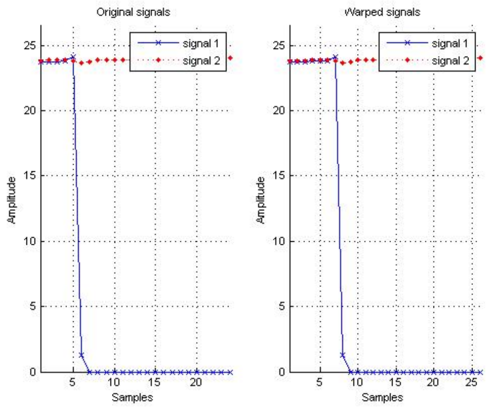

Figure 15: Warping diagrams for classical DTW. Left graph – signals before warping, right graph – signals after warping.

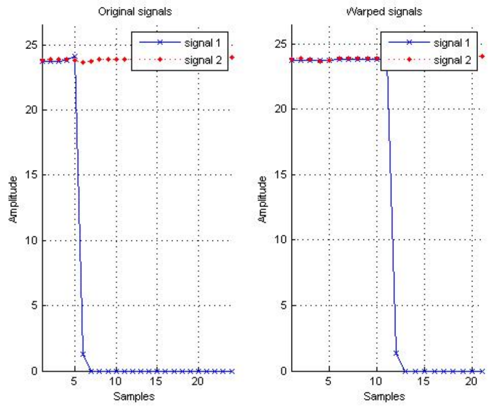

Figure 16: Warping diagrams for modified DTW. Left graph – signals before warping, right graph – signals after warping.

An additional study showed that classical DTW is not appropriate for comparing signals with straight lines. Modified algorithm in this case gives unreliable results and is also not able to be used.

The effects described appear because only the time axis is warped, so the algorithm recognizes properly the horizontal changes in the signal, but vertical changes are not recognized well. One possible solution of this problem is warping the whole plane [17] that allows to transform both time and value axes. The accuracy of the modified algorithm exceeds the accuracy of the classical one. When difference between signals is not significant, the modified DTW has more correct results but this algorithm allows unconnected points that can lead to significant change loss.

5 CONCLUSIONS

The study showed that parameters of the technological process and mechanical pump characteristics have weak influence to the instant changes of well equipment electric parameters. These changes caused only by changing EPS operational regime. It corresponds to theoretical statements described in [1][6].

Cross-sectional analysis of signals obtained from wells with similar production conditions was carried out to study instant changes patterns in electric power consumption profiles. Two modifications of DTW algorithm with different weighting matrices were used for the analysis. The methodic based on measuring distances between warping path and diagonal was used for DTW results interpretation.

Study allowed to define restrictions of described algorithm modifications on precision of changes detection when recognizing signal differences. When analyzing tiny magnitude changes in signals both modifications of the algorithm had precise results. This fact corresponded to the conclusions given in [9], [10]. However, none of these modifications was able to correctly recognize large magnitude changes between signals. In addition, when studying signals with large straight sections (both zero-valued and not), presence of which is a feature of considered profiles, results interpretation is impossible due to incorrect form of warping curve.

The study showed that classical DTW algorithm recognized changes with less precision then modified one. This problem also considers in [14] [16]. Nevertheless, when using modified algorithm, mismatching points appears. This may cause significant decreasing of recognition precision when mismatched point corresponds to significant regime change. Although in several works [8][10] – [13] these effects are not consider, they constrain use of this algorithm for described task.

Experiments showed that algorithm better recognizes changes in width of signals (shifting points along the time axis) than in magnitude. The possible solution for this problem is use of two-dimensional warping algorithm [17]. Moreover, this algorithm has variety of modifications [14] – [17] eliminating some negative effects when analyzing signals with different specific features.

The research highlighted features of DTW algorithm that restricted its use for analyzing changes in power consumption profiles. It also depicted basic features of the power consumption profiles themselves. Obtained results will be the basis for further investigations that will conform the algorithm to specific features of studied signals.

REFERENCES

[1] L. Hailong, “The numerical simulation for multistage fractured horizontal well in low-permeability reservoirs based on modified Darcy’s equation”, Journal of Petroleum Exploration and Production Technology, vol. 7(3), pp. 735-746, 2017, DOI:10.1007/s13202-016-0283-1

[2] V. V. Alekseev, A. P. Emel’yanov, and A. E. Kozyaruk, “Analysis of the dynamic performance of a variable-frequency induction motor drive using various control structures and algorithms”, Russian Electrical Engineering vol. 87, no. 4, pp. 181-188, DOI: 10.3103/S1068371216040027

[3] D. C. Montgomery, “Design and Analysis of Experiments”, Wiley, 2012, 752 p.

[4] Chatfield, C. “Time-series forecasting”, New York: Chapman and Hall, 2001;

[5] Gurol Irzik “Causal Modeling and the Statistical Analysis of Causation”, 1996;

[6] G. Takacs, Electrical submersible pump manual: design, operations, and maintenance, Gulf Professional Publishing, 2009, 420 p.

[7] J.F.Gülich, Centrifugal pumps, 2nd Edition, Springer, 2010, 998 p.

[8] Sakoe H., Chiba S., A Dynamic Programming Approach to Continuous Speech Recognition, In Proceedings of the 7th International Congress on Acoustics, vol. 3, 1971, pp. 65-69.

[9] I. Luzyanin, A. Petrochenkov, B. Krause, Problems of tiny changes analysis in complex time series using dynamic time warping algorithm, Proceedings of the XIX International Conference on Soft Computing and Measurements. SCM`2016, 2016, pp. 419-422, DOI: 10.1109/SCM.2016.7519799

[10] Y. Zhang, T. F. Edgar, A Robust Dynamic Time Warping Algorithm for Batch Trajectory Synchronization, American Control Conference, Seattle, June 2008, pp. 2864-2869.

[11] C. Cassisi, P. Montalto et al. (2012). Similarity Measures and Dimensionality Reduction Techniques for Time Series Data Mining, in: A. Karahoca (Ed.), “Advances in Data Mining Knowledge Discovery and Applications”, InTech, 2012, pp. 71-94.

[12] L. R. Rabiner, A. E. Rosenberg and S. E. Levinson Considerations in Dynamic Time Warping Algorithms for Discrete Word Recognition, IEEE Transactions on Acoustics, Speech and Signal Processing, vol. ASSP-26, Dec. 1978, pp. 575-582.

[13] 10. M. Müller, Dynamic Time Warping, Information Retrieval for Music and Motion, Berlin Heidelberg: Springer 2007, pp. 69-84.

[14] Y. Jeong, M. K. Jeong, O. A. Omitaomu, Weighted Dynamic Time Warping for Time Series Classification, Pattern Recognition, No. 44, 2011, pp. 2231-2240.

[15] M. Kotas, J. M. Leski, and T. Moró, (2016). Dynamic time warping based on modified alignment costs for evoked potentials averaging DOI:10.1007/978-3-319-23437-3_26.

[16] T. Giorgino, Computing and Visualizing Dynamic Time Warping Algorithms in R: The DTW Package, Journal of Statistical Software, Vol. 31, Issue 7, 2009, pp. 1-25.

[17] M. Schmidt, M. Baumert et al., Two-Dimensional Warping for One-Dimensional Signals — Conceptual Framework and Application to ECG Processing, IEEE Transactions on Signal Processing, Vol. 62, No. 21, Nov. 2014, pp. 5577-5588.