Vertical Competition and Multi-Period Interaction: A Theoretical and Behavioral Analysis on the Impact of Strategic Inventories in Supply Chain Management

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II Symbols

a	Parameter of the linear demand function that describes the maximum price that is paid on the external market
b	Parameter of the linear demand function that describes how much the price reduces if an additional item is sold to the external market
C	Commitment contract in which the supplier commits on setting identical wholesale prices in both periods (i.e. $w_1^C = w_2^C$)
DCF	Discounted cash flow model
e	Type of the solution
$FB1$	First-Best solution of the single-period model
FB	First-Best solution of the two-period model
FW	Forward contract model
F^{TP1}	Fixed fee of the two-part tariff in the single-period model
F_t^{TP}	Fixed fee of the two-part tariff in period $t = 1, 2$ in the two-period model
HCM	Horizontal competition model
h	Holding costs per unit and per period
h^e	Holding costs per unit and per period in solution e
$\overline{h_D^{DCF}}$	Upper bound for the holding cost under which the dynamic solution is feasible in the discounted cash flow model
$\overline{h_B^{DCF}}$	Upper bound for the holding cost under which the buyer prefers the dynamic solution in the discounted cash flow model
$\overline{h_S^{DCF}}$	Upper bound for the holding cost under which the supplier prefers the dynamic solution in the discounted cash flow model
$\underline{h}^{FW}$	Minimum level of the holding cost value that allows the unlimited forward solution in the forward contract model

i^e	Interest rate (i.e. holding cost per purchase price, per unit and per period) of model $e = IIR, DCF$
IO	Single-period model under individual optimization
IIR	Inventory interest rate model
I	Inventory
I^e	Inventory in model e
I_j^{HCM}	Inventory of buyer $j = 1,..,N$ in the horizontal competition model
j	General buyer in the horizontal competition model
k	A specific buyer in the horizontal competition model
L	Lagrange function
N	Number of buyers in the supply chain in the horizontal competition model
o^{FW}	Number of forwards (preorders) in the forward contract model
p	p-value of the corresponding statistical test
p	End customer price (based on demand function)
$\hat{p}^e$	Weighted end customer price over both periods of model e
p^e	End customer price in a single-period model e
p_t^e	End customer price of period $t = 1, 2$ of two-period model e
q	Selling quantity of the buyer to the external market
q^e	Selling quantity of the buyer to the external market of single-period model e
q_t^e	Selling quantity of buyer to the external market in period $t = 1, 2$ of two-period model e
$q_{j,t}^{HCM}$	Selling quantity of buyer $j = 1,..,N$ to the external market in period $t = 1, 2$ of the horizontal competition model
$q^{\min}$	Minimum for the buyers selling quantity that is set by the supplier in a “resale price maintenance” contract
Q	Purchasing quantity of the buyer

Q^e	Purchasing quantity of the buyer in the single-period model e
Q_t^e	Purchasing quantity of the buyer in period $t = 1, 2$ of a two-period model e
$Q_{j,t}^{HCM}$	Purchasing quantity of buyer $j = 1,..,N$ in period $t = 1, 2$ of the horizontal competition model e
SM	Solutions of the standard model
$TP1$	Single-period model with the two-part tariff
TP	Two-period model with the two-part tariff
t	Period ($t = 1,\ 2$)
u	Lagrange parameter
v^{FW}	Forward price to which the buyer may preorder items from the supplier for the next period
w	Wholesale price to which the buyer can purchase items from the supplier
w^e	Wholesale price to which the buyer can purchase items from the supplier in the single-period model e
w_t^e	Wholesale price of period $t = 1,\ 2$ to which the buyer can purchase items from the supplier in the two-period model e
λ	Lagrange parameter
π_B^e	Profit of the buyer in model e
π_S^e	Profit of the supplier in model e
π_{SC}^e	Profit of the supply chain in model e
$\pi_{B,\ j}^{HCM}$	Profit of buyer *j* in the horizontal competition model
$\pi_B^{DCF,dynamic}$	Profit of the buyer in the dynamic solution of the discounted cash flow model
$\pi_S^{DCF,dynamic}$	Profit of the supplier in the dynamic solution of the discounted cash flow model

$\pi_{SC}^{DCF,dynamic}$	Profit of the supply chain in the dynamic solution of the discounted cash flow model
$\pi_{B}^{DCF,static}$	Profit of the buyer in the dynamic solution of the discounted cash flow model
$\pi_{S}^{DCF,static}$	Profit of the supplier in the static solution of the discounted cash flow model
$\pi_{SC}^{DCF,static}$	Profit of the supply chain in the dynamic solution of the discounted cash flow model
$\pi_{SC}^{SM,dynamic}$	Profit of the supply chain in the dynamic solution of the standard model
$\pi_{SC}^{SM,static}$	Profit of the supply chain in the dynamic solution of the standard model

III Abbreviations

HC	High Cost (Treatment)
HCM	Horizontal Competition Model
DCF	Discounted Cash Flow Model
FB	First-Best
FW	Forward Contract Model
IIR	Inventory Interest Model
LC	Low Cost (Treatment)
LD	Labor Dollar
LFW	Limited Forward Solution
SM	Standard Model
UFW	Unlimited Forward Solution

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1 Introduction

Although companies are linked to each other through a supply chain, they often only focus on their own business in order to achieve their goals. Such individual optimization is known to be a major source of inefficiencies within supply chains, because the goals of the individual supply chain members are typically contradictory. This leads to individually optimal decisions that harm the overall supply chain performance (Lee et al. (1997), Cachon (2003), Arshinder et al. (2011)). A common example for a suboptimal supply chain performance caused by individual optimization within supply chains is the double marginalization effect that was first identified by Spengler (1950). Consider a supply chain, in which a supplier sells goods to a buyer, who, in turn, sells these goods to end-customers. In order to maximize his profit, the supplier will charge a prize that is above his respective marginal cost. The buyer, in turn, will again charge a surplus on his marginal costs (i.e. on the selling price of the supplier) leading to a suboptimally high end customer price. Hereby, the caused loss of welfare is more pronounced the higher the degree of the supply chain member's market power is.

In the research area of supply chain management companies within a supply chain are no longer viewed at in isolation. Instead, the focus gets extended to the interactions (e.g. delivery of goods, sharing of information) between those companies. This way, the previously wasted potential of the supply chain can get revealed and advanced supply chain mechanisms can be developed that enable the supply chain to utilize this hidden potential. Such advanced supply chain mechanisms consist of vertical contracts between the supply chain parties (e.g. a quantity discount, a buyback contract, a revenue sharing contract (Cachon (2003)) or a screening contract (Corbett and de Groote (2000), Ha (2000), Corbett (2001)), information technology and sharing (e.g. sharing demand, orders, inventory and/ or point of sale data) or other forms of collaboration between the supply chain parties (e.g. vendor managed inventory (Cheung and Lee, 2002), collaborative forecasting and planning (Skjoett-Larsen et al. (2003), quick response (Choi and Sethi (2010)). In an ideal case, all supply chain members benefit from implementing advanced mechanisms as the additional profit is shared amongst them. In this case, all supply chain members would be willing to accept the implementation of those mechanisms.

As discussed, the inefficiency is caused by uncoordinated optimization within supply chains. Hence, the advanced mechanisms from the area of supply chain management aim to coordinate the decisions within the supply chain by aligning the objectives of the supply chain members with the objectives of the supply chain. This way an individually optimal decision will be identical to the decision that is optimal from the perspective of the entire supply chain. One typical mechanism to achieve this alignment are contracts that are concluded between the supply chain members and define the conditions for interaction within the supply chain (e.g. pricing schemes or the commitment to share specific information). A contract that perfectly aligns the incentives of the supply chain members and, therefore, unlocks the full potential of the supply chain gets referred to as a coordinating contract.

Another source of inefficiencies within supply chains is caused by uncoordinated inventory decisions. Despite causing holding costs, companies utilize inventory for a number of well documented reasons. For example, classical reasons to hold inventory refer to the decoupling of production/ procurement and demand. This way companies can smooth their production/ procurement (e.g. in the case of high seasonal demand) or utilize economies of scale in production/ procurement by producing/ procuring more items than needed to satisfy the actual demand. Moreover, companies might build up inventory to hedge against uncertainties. A common example for this is the build-up of safety stocks in case of stochastic demand. This way the company ensures that it will be able to serve its customers even if the demand exceeds the expectations. Further, companies might hedge themselves against increasing prices by building up a speculative stock. Last, during the transportation of items pipeline inventory emerges.

Focusing on these classical reasons to hold inventory, a major fraction of research in the area of supply chain management has analyzed the deficits[1] that occur under uncoordinated inventory management within supply chains.[2]

However, a recently emerging branch of research on the effects of non-cooperative behavior in supply chains is concerned with the effects of multi-period interaction in supply chains with vertical competition between a supplier and a buyer, who both possess a high degree of monopoly power (e.g. Anand et al. (2008), Desai et al. (2010), Arya and Mittendorf (2013)). One of the surprising findings in this literature is that buyers possess an incentive to build up an inventory caused by purely strategically

[1] For example, a buyer might prefer to order small quantities from his supplier in order to reduce holding costs. But, the supplier, in turn, might prefer a larger delivery size aiming to reduce the frequency of orders, which reduces the costs of delivery.

[2] Tsay et al. (1998) give a good review on the literature including deterministic and stochastic demand models, while Cachon’s (2003) more recent review focuses on models with stochastic demand.

considerations, even if other motivations do not exist. Hence, this so called *strategic inventory* that was first discovered by Anand et al. (2008) clearly differs from the classical reasons to hold inventory, because it is built up by buyers solely to offset the strategic advantage that a monopolistic supplier otherwise has. Regarding the supply chain performance, this might be good news as strategic inventory can partly offset the disadvantages of the double marginalization effect[3] that is caused by individual optimization. As a result, strategic inventory can be advantageous to the overall supply chain performance although creating additional holding cost. However, at the same time the buyer's possibility to utilize inventory in a strategic manner hinders the implementation of contracts that were developed without taking multi-period interaction into account from continuing to coordinate the supply chain. More precisely, Anand et al. (2008) have shown that the supplier is no longer able to propose a <u>dynamic</u> vertical contract that simultaneously coordinates the supply chain and allows a free distribution of the supply chain profit between supplier and buyer.

Based on the findings of Anand et al. (2008) various studies focusing on strategic inventory have been conducted. For example, Keskinocak et al. (2008) introduce a capacity constraint for the supplier's first-period production/ procurement quantity. Desai et al. (2010) integrate a discount factor into the model of Anand et al. (2008) and investigate duopoly settings both for suppliers as well as for retailers. However, the existing literature on strategic inventories focuses on expansions of the theoretical findings. Hence, strategic inventories lack a validation regarding the practical relevance. This is especially important as previous studies on supply chain interaction have shown that real decision makers do not simply maximize their profit but also have a tendency to consider fairness consequences (Cui et al., 2007; Loch and Wu, 2008; Pavlov and Katok, 2011). In order to get an empirical foundation of the utilization of strategic inventory in vertical competition, an experiment with real decision makers who have the option to apply strategic inventory gets presented in this thesis.

Moreover, the current literature (Desai et al. (2010), Viswanathan and Jang (2009)) only examines horizontal competition in duopoly settings. However, in practice, a higher number (than two) of horizontal competitors often exists in the market. An examination

[3] Intuitively, strategic inventory in a multi-period supply chain game reduces the monopoly power of a supplier, because the inventory acts as a "virtual competitor" and leads to a reduced equilibrium wholesale price, which in turn allows the buyer to reduce the market price and serve a larger number of customers.

of the effects of horizontal competition between an arbitrary number of buyers is conducted in this thesis.

In the study of Anand et al. (2008), a constant per unit holding cost rate is used. This assumption is critical, as in practice holding costs partially depend on the purchasing cost. As long as the purchasing costs maintain constant within the model this is no thread for the general validity of the model. However, a major effect of strategic inventories is a shift in the supplier's wholesale price determination. To fully cover the price effects of strategic inventory, purchasing cost depending holding costs have to be used. Desai et al. (2010) already studied strategic inventory within a discount rate model (i.e. later cash flows get discounted). In this thesis, an inventory interest rate model is additionally considered and compared to both the standard model by Anand et al. (2008) as well as to the discounted cash flow model by Desai et al. (2010).

While strategic inventory may increase supply chain performance, it also causes holding cost and, therefore, impairs supply chain performance. As physically holding inventory does not seem to be mandatory for its positive effect, it is unclear if an obligation of the supplier to deliver goods at a predefined price to the buyer does not lead to better supply chain performance.

This thesis is organized as follows. In chapter 2, companies' reasons to build up inventory will be explained. Thereby, the companies' incentive to build up inventory will be divided into two sections. In the first section, classical reasons like decoupling of procurement, production and demand (anticipation stock, cycle inventory), hedging against uncertainty (safety stock) or price speculation will be introduced in more detailed form. In the second chapter, an overview on the latest research regarding strategic reasons to build up inventory is given.

In the first section of chapter 3, the double marginalization effect will be demonstrated by analyzing a single-period model with one supplier and one buyer. Using the solution of an integrated channel as a benchmark, it will be shown that the double marginalization effect is caused by individual and uncoordinated optimization in a vertical supply chain. Moreover, using a two-part tariff as an example, the coordinating effect of more sophisticated contracts is demonstrated. In the second section of chapter 3, the two-period model of Anand et al. (2008) that serves as the baseline model of the forthcoming theoretical expansions will be reviewed. Hereby, the dynamics that are caused by the buyer's possibility to utilize inventory as a strategic tool will be explained.

In chapter 4, a laboratory study will be presented. Although, the utilization of strategic inventory is predicted in theory it is not self-evident that it also gets utilized by human decision makers. In order to utilize inventory as a strategic tool as described in theory, decision makers are required to demonstrate a high degree of strategic sophistication in their behavior. Given the extensive literature on behavioral biases in single-period supply chain interactions, however, it is not self-evident that theoretically predicted efficiency gains are behaviorally sustained in this type of multi-period interplay.

The results of the laboratory investigation give strong empirical support for the utilization of strategic inventory and, moreover, identifies behavioral effects that top off the purely strategic effect of inventory. Seeking a more equitable payoff distribution in the supply chain, the buyers get empowered through strategic inventory and may harm the supply chain performance by choosing suboptimal small inventories. But, this negative effect of buyer empowerment on supply chain performance is generally offset by a positive effect of lower first-period prices.

Since the behavioral study in chapter 4 gives strong empirical support for the theory of strategic inventory, the theoretical analysis will be extended to scope a broader range of scenarios that are closer to real world scenarios.

In chapter 5, the validity of the findings regarding strategic inventory will be tested considering the case of a diverging supply chain structure with multiple buyers. Hence, the vertical competition that caused the utilization of strategic inventory in multi-period interaction will be supplemented by horizontal competition between the buyers. This allows to test whether strategic inventory continues to play a pivotal role, even if the market power on the side of the buyers decreases. The analysis proves that strategic inventory remains relevant in a diverging supply chain structure. However, as the monopoly power of the buyers decreases under increasing horizontal competition, the supply chain profit converges to the first-best solution. Consequently, the relevance of strategic inventory decreases as less space for an improvement of the supply chain profit exists.

In chapter 6, holding costs that depend on the buyer's purchasing costs are introduced. This is an advisable modification, since the original constant per unit holding cost rate like it is postulated in Anand et al. (2008) does not take into account that the purchasing costs of the buyer are affected by strategic inventory. Hence, the buyer would likely also face changing holding costs that cannot be represented by constant per unit holding costs. In the first section of chapter 6, the holding costs get replaced by an inventory interest rate. This way, fluctuations of the purchasing costs are directly reflected in the holding costs. The results show that the function of strategic inventory persists under this assumption.

In the second section of chapter 6, a discounted cash flow approach gets applied. Hereby, all cash flows that are generated in the second period will be discounted. Hence, also the supplier has an incentive to generate a higher profit in the first-period. In contrast to the model with an inventory interest rate, the discounted cash flow approach causes strong structural changes in stocking decisions.

In chapter 7, the theoretical analysis aims to eliminate the holding costs that occur through the physical build-up of strategic inventory. Thereby, the supplier is enabled to offer the buyer the option to preorder items at a specific price. Hence, the buyer can acquire items for the second period without having to pay costs for holding an inventory. The analysis will show that the solution strongly depends on the size of the holding cost parameter and that preorders indeed are able to replace physical strategic inventory. Surprisingly, the supply chain profit is larger if holding costs are high as this enables the supplier to implement a solution, in which he does no longer cause an increase of the double marginalization effect in the first-period like in the model of Anand et al. (2008).

Finally, chapter 8 summarizes the results of the theoretical and behavioral findings of this thesis and outlines directions for future research in the area of strategic inventory.

2 Companies' Reasons to hold Inventory

Holding inventory causes costs for companies that can be separated into different components. First, capital costs arise because holding inventory locks up capital for the companies. As this capital cannot be used for other purposes, it causes opportunity costs (in the simplest case, these could be foregone interest on the locked capital). Second, the value of stored goods might decrease as they become obsolete or be forfeited completely as the goods deteriorate. Third, the company has expenses to provide the storage place (i.e. cost for the building and the necessary equipment). Last, running a warehouse causes operating costs like personal expenses, electricity costs or the insurance of the stored goods.

In terms of cost cutting, it could be assumed that companies should avoid using inventory at all. In fact, a common denominator of definitions of the just-in-time approach contains the philosophy of pursuing zero inventories[4] or a bare minimum of work-in-process inventory[5]. In practice, however, it can be observed that a major fraction of companies' locked up capital is caused by their inventory. In Germany (2012), for instance, the locked up capital through inventory on the total assets averages at 10.1% for affiliated groups and at 16.6% for the subsidiaries (i.e. without the holding).[6] Hence, companies likely gain benefits from holding inventory that outweigh their holding costs. Reasons to build up inventory may either be motivated by the procurement, production and/ or selling processes of a company itself (referred to as classical reasons to build up inventory) or by strategic considerations regarding the other parties within a company's supply chain. Following Moellgaard et al. (2000), the latter case, in which the explicit purpose of holding inventory is to influence the decisions of horizontal or vertical supply chain members, gets referred to as strategic inventory.

The various classical reasons to carry inventory will be described in section 2.1, while inventory that is build up due to strategic considerations within supply chains will be presented in section 2.2.

[4] Vollmann et al. (2005) p.301

[5] Nahmias (2009) p. 369

[6] "Monthly Report July 2014" of the Deutsche Bundesbank, p. 58

2.1 Classical Reasons for Holding Inventory

The functions[7] that inventory fulfils within the procurement, production and/ or selling processes of a company are summarized in Figure 1 and will be separately described in detail in the following subsections.

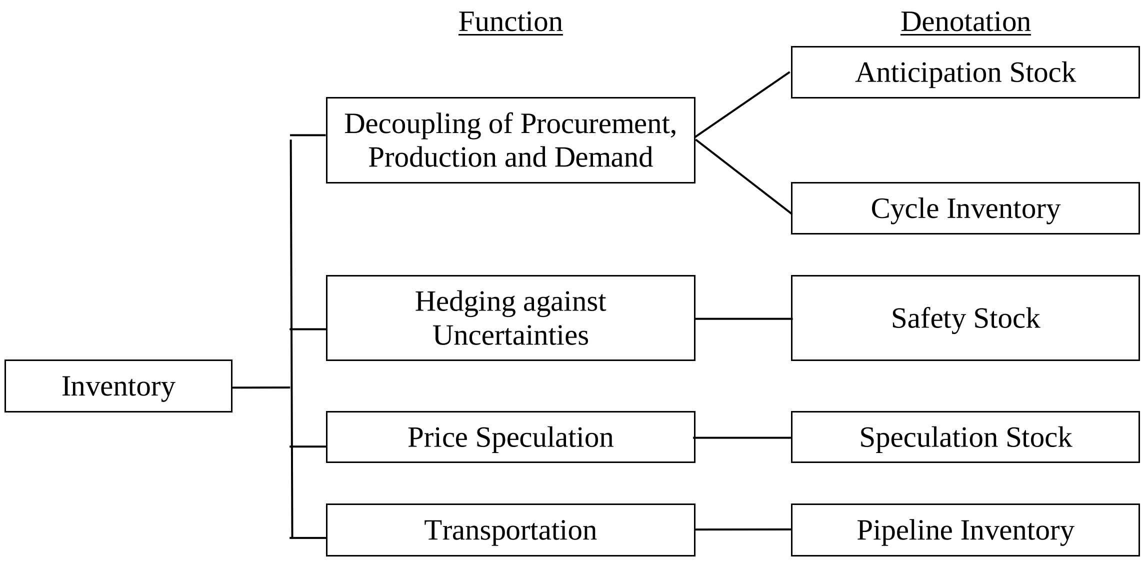

Figure 1: Classical Reasons to Hold Inventory

2.1.1 Anticipation Stock

Under ideal conditions, companies would start the production of items in a way that the production is finished at exactly the time at which demand occurs. Moreover, the necessary input factors for the production would be delivered exactly at the start of the production. This way, both an inventory for input factors as well as for finished goods could be avoided. However, such a just-in-time approach often is not the optimal approach in a real world scenario, where it might be beneficial to use inventory to implement the possibility for a temporal separation of purchasing, production and shipment processes.

A common reason to separate production and demand is seasonality of the demand. Products like air conditioners, winter jackets or Christmas gifts have a known seasonal peak (i.e. a predictable variation). Producing such items just-in-time would cause a high variation of production quantities. As a result the capacity (workers and machines) of the facility must be sufficiently high to produce enough items even in high peak periods. At the same time, in the low demand periods (i.e. out of the season) the capacity remains largely unused. To alleviate the described disruptions caused by variable

[7] Vollman et al. (2005) pp. 135-136, Nahmias (2009) p. 198, pp. 202-203

production rates, anticipation stock can be used. This enables the company to smooth the production by building up inventory in the low demand periods that then gets depleted during the peak demand periods. Therefore, companies have to solve a trade-off between holding and capacity costs to determine the optimal level of anticipation stock.[8]

2.1.2 Cycle Inventory

Economies of Scale exist, if the average procurement or production costs per unit are decreasing with an increase of the procurement or production quantity. Reasons for economies of scale in procurement are, for example, fixed cost per order or a quantity discount of the supplier. In the production process, economies of scale may, for example, arise because companies may have to clean the equipment or to reconfigure the machines before starting the production of a certain product. By increasing the quantity of a production batch, the setup costs that arise from setting up a production lot are shared between a larger number of units. Therefore, an increase of the production lot leads to lower average unit costs.

In order to benefit from economies of scale, it could be economical for companies to order or produce larger quantities than needed to satisfy the immediate requirement. Hence, companies have to solve a trade-off between holding and order costs. The so-called economic order quantity[9] that aims to solve this trade-off may be one of the most prominent inventory models. Stock that is built up to use benefits from economies of scale is referred to as cycle inventory.

2.1.3 Safety Stock

Next to predictable variations in demand, companies are also confronted with uncertainties regarding their supply, their production process or their demand (stochastic variations).

Although creating forecasts, the exact amount of customer's demand is usually unknown. If production exactly covers the forecasted demand and the customers demand exceeds the forecast, the company could no longer satisfy the entire demand of its customers. This would result in lost sales and/ or forfeit of customers' goodwill. By holding an additional safety stock, companies can reduce the risk of being out of stock.

[8] A good overview, how to solve this trade-off , is given in Chapter 3 (pp. 124-162) of Nahmias (2009).

[9] See Chapter 4.5 (pp. 210-217) of Nahmias (2009) for a detailed description of the economic order quantity model.

The newsvendor problem, as one of the most popular inventory models, considers this trade-off between cost of having too few items (i.e. lost sales/ loss of goodwill) and having too much items (holding or salvage costs).[10]

While demand uncertainty is perhaps the most important reason to hold safety stock, a multitude of additional reasons exist. One of them is the uncertainty of production/ delivery lead times. Regarding the supply, the lead time describes the span between placing an order and its arrival. If the lead time is higher than expected, companies face the risk of having too few input items, which would disrupt the production process. Regarding the production, lead time describes the span that is required to produce an item. However, if the production process takes longer than expected, the company might fail to fulfill the customer’s demand. By building up safety stock a smooth flow of the production and sales processes can be ensured. Other uncertainties include the risk of disruption, yield risk of a production/ procurement process[11] or poor availability of input factors (e.g. oil in the 1970s[12]).

2.1.4 Speculation Stock

If a company expects that the price of one of their input factors increases in future, it might buy a larger amount of items at the current price and build up a stock at the current (low) price. This stock then gets depleted, when the prices are higher. Examples why a price increase might be expected include an expected shortage of an input factor, the ending of a low price contract with a supplier. Companies that use speculative inventory to hedge against a price increase of their input factors, therefore, face a trade-off between the holding costs and the risk of having to buy items at higher prices.

2.1.5 Pipeline Inventory

Another reason for the occurrence of inventory is the transportation times that are needed to transport items from one location to another. The average level of the pipeline inventory that sometimes is also called in-transit inventory directly depends on the distance between the transportation points. An illustrating example is oil pipelines: The longer the transportation distance is, the more oil will be stored within the pipeline. Hence, pipeline inventory can be reduced in the long term by reducing the distance between starting and end location (e.g. building a factory closer to the source of the

[10] See Chapter 5 (pp. 248-291) of Nahmias (2009) for a detailed description of the newsvendor problem.

[11] Yano, Lee (1995) give a review on the literature on determining lot sizes under random yields.

[12] Nahmias (2009), p. 202

input factors) or in the short term by choosing a faster transportation method (e.g. switching from rail to air freight). Hence, regarding pipeline inventory there exist two trade-offs. In the short term, it is holding versus transportation costs (i.e. method of transporting materials). In the long term, it is holding cost against the costs of choosing another supplier or relocation of the factory.

2.2 Strategic Reasons to Build up Inventory

While the classical reasons to build up inventory are tied to considerations regarding the operations of a single company, strategic inventory intends to influence the decisions of other supply members. Hence, strategic inventory has an effect on the competitive conditions between the supply chain members. Herby, it has to be distinguished between horizontal and vertical competition.

2.2.1 Horizontal Competition within Supply Chains

Horizontal competition exists between supply chain members (referred to as retailers in this section) that are on the same level within the supply chain (see grey shaded area of Figure 2). Regarding the upstream level (left side of Figure 2), the retailers may compete against each other in order to achieve a better (i.e. cheaper) supply condition. This may either be a lower purchasing price or a more reliable supply. A lower price could be achieved by purchasing a larger quantity due to economies of scale. Reliability of supply is especially pronounced if the supplier can only deliver a limited amount of goods and has to decide how to split his capacity among the retailers.

Regarding the downstream level (right side of Figure 2), competition between multiple retailers arises if they are active in the same (external) market. Hereby, products do not necessarily need to be identical as it is sufficient if the sold products are substitutes (e.g. cars and motorcycles, juice and soft drinks). If horizontal competition exists because retailers are active in the same market, their individual actions do not only have an impact on themselves but also on their competitors. If, for example, retailer 1 decides to lower the selling price in order to sell more goods, other retailers might lose customers as they prefer to buy goods from retailer 1 now. In order to control the loss of customers, the other retailers might lower their selling prices as well. Hence, the decision of retailer 1 also affects the decisions and profits of the other companies.

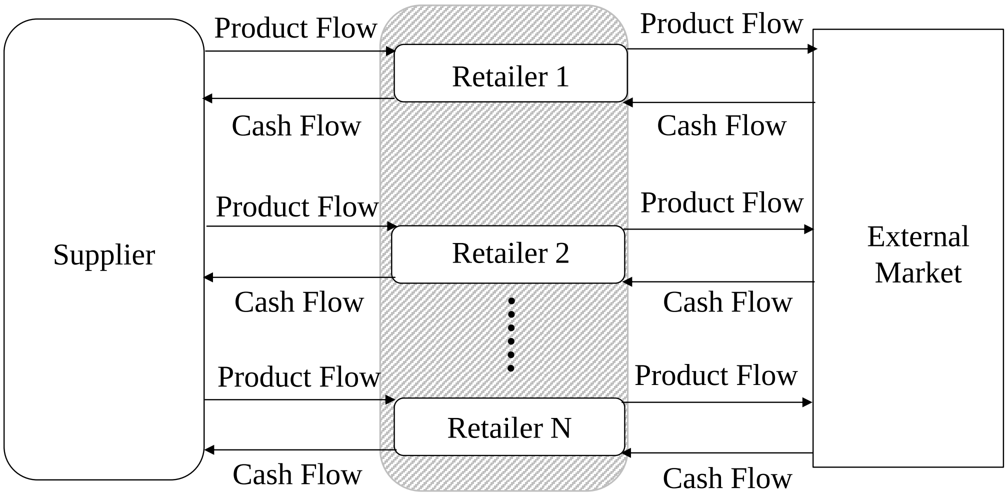

Figure 2: Supply Chain with horizontal Competition

A large number of literature exists that incorporates either upstream, downstream or a combination of both factors of horizontal competition between retailers. However, the major fraction of literature regarding horizontal competition is limited to a single-period examination (a brief review of recent studies on single-period horizontal competition is given in chapter 5). Typically, no incentive exists to carry inventory at the end of the last period, as it cannot be used in subsequent periods. Consequently, the utilization of (strategic) inventory is out of consideration in most single-period studies.

However, a minor faction of the literature on horizontal competition considers multi-period supply chain interaction and, therefore, the possibility to utilize strategic inventory. Ware (1985) shows that a monopolistic retailer can hold a strategic inventory in the post-entry game. Hereby, strategic inventory can be used as a credible threat to temporary sell goods below marginal production cost as inventory has zero supply costs at the moment of the selling quantity decision. Therefore, inventory can be used strategically by the monopolistic retailer in order to hinder other retailers to join the market.

Saloner (1986) analyzes a supply chain setting with a retailer with a first-mover advantage and a second retailer that moves afterwards (Stackelberg competition). The first-movers production quantity can be observed by the second-moving retailer. However, as the first-mover may partly carry over goods from the first to the second period as inventory, the commitment regarding his selling quantity is reduced. If holding costs are low (i.e. the likelihood of inventory usage is high), the Stackelberg leader loses his first mover advantage. Hence, in this specific setting, allowing the first-mover to build up inventory is a threat to his first-moving advantage.

Rotenberg and Saloner (1989) incorporate limited production capacities for the retailers. In their study retailers can improve their profit if they agree on selling fewer units to the customers. If a retailer's competitor deviates from cooperation (i.e. sells more units than under cooperation), the other retailer can punish this retailer by producing and selling more units in future periods. However, the ability to punish the competitor is limited by the retailer's production capacity. However, by using their excess capacity the retailers are able to build up a strategic inventory which improves their ability to punish their competitor. Hence, strategic inventory serves as a tool to punish competitors that deviate from cooperation and strengthens the power of a commitment to cooperate.

Pal (1991) extends the model of Rotenberg and Saloner (1989) by allowing production cost functions to vary across periods. Under rising marginal production costs one out of two competing retailers acts as a Stackelberg leader and builds up a strategic inventory in the first-period to establish a price-advantage in the second period in comparison to the second moving retailer. Moellgaard et al. (2000) support this finding by showing that strategic inventory may also get used by simultaneously moving retailers with diverse production cost functions in order to achieve a future price-advantage to their competitor.

Even though the number of studies that consider the impact of inventory under horizontal competition is small, the existing literature clearly shows that strategic inventory can strongly affect the interaction between supply chain members. However, the existing literature mainly focuses on downstream competition, while competition regarding the upstream needs closer examination in future research.

2.2.2 Vertical Competition within Supply Chains

The competition that exists between supply chain members that are on different levels within the supply chain is referred to as vertical competition. In its simplest version, an upstream supply chain member (denounced as supplier) sells goods to a downstream supply chain member (denounced as buyer) which, in turn, sells these goods to the consumers (Figure 3). Because the supplier and the buyer are the only source for their respective customer, both supply chain members (supplier and buyer) possess a high degree of market power: As there are no horizontal competitors (i.e. alternative suppliers or buyers), both supplier and buyer can freely decide about the price they are offering their respective customer. Both supply chain members independently try to maximize their individual profits by setting their prices. As a result, this leads to a suboptimal high consumer price and the supply chain profit is below the first-best solution (i.e. highest possible supply chain profit under centralized decision making). This effect that reduces supply chain efficiency is called double marginalization.

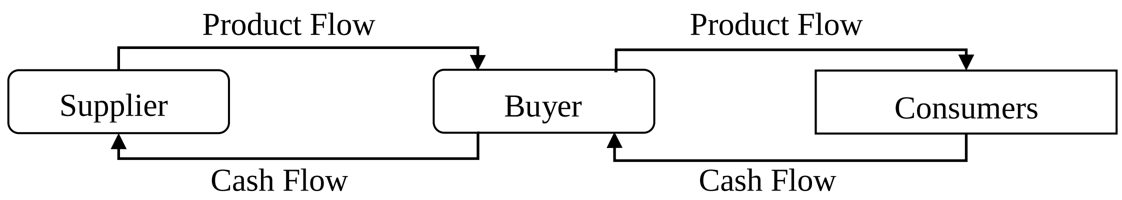

Figure 3: Supply Chain Structure with vertical Competition

As under horizontal competition, the majority of literature regarding the interaction within supply chains with vertical competition does not consider a strategic utilization of inventory. A good review on the literature including deterministic and stochastic demand models is given by Tsay et al. (1998), while a more recent review by Cachon (2003) focuses on models with stochastic demand.

Recently, Anand et al. (2008) proved that strategic inventory plays a pivotal role under vertical competition. A buyer who faces a supplier with a high degree of monopoly power can build up inventory for future periods. This inventory then serves as an alternative supply source and, therefore, can dampen the monopoly power of the supplier. This leads to a lower price setting of the supplier in future periods. The supplier, in turn, anticipates the strategic behavior of the buyer and increases the price, if he expects the buyer to build up a strategic inventory. Overall, if holding costs are not extremely high, both supplier and buyer can achieve an improvement of profits compared to a situation in which holding inventory is not feasible.

However, the profit enhancing effects of strategic inventory only hold for the linear wholesale price as payment scheme between supplier and buyer. The study examines the effect of a two-part tariff that consists of a fixed fee plus a per unit wholesale price in each period. In the single-period case, a two-part tariff solves the double marginalization problem. However, if the length of the horizon is extended, the contract is no longer able to achieve a coordination of the supply chain caused by the buyer's possibility to build up a strategic inventory. By building up strategic inventory in the first-period the buyer forces the supplier to lower the fixed fee in the second period. As a result, the supplier will set a first-period wholesale price that is above his marginal costs in order to prevent the buyer from building up extensive amounts of inventory. Nevertheless, the buyer will continue to build up a considerable amount of strategic inventory. Because of the inventory holding costs and the first-period wholesale price that is above the supplier marginal costs, the supply chain outcome can no longer achieve the first-best outcome in multi-period interaction.

Anand et al. (2008) also consider commitment contracts in which the supplier credibly commits himself to set a specified prices in the future. In the case of the simple wholesale price contract, this effectively eliminates an inventory use of the buyer as she can no longer influence the committed second period wholesale price of the supplier. However, as the supplier strictly prefers the solution without commitment, he has no incentive to implement such a contract. Moreover, in most cases even the buyer prefers the solution without a commitment contract.

In the case of the two-part tariff, however, a commitment of the supplier is no longer able to generally prevent an inventory build-up of the buyer as the buyer may try to reduce the payment of the fixed fee. Depending on the holding cost two scenarios for the commitment version of the two-part tariff exist. If holding costs are low, the supplier will not hinder the buyer from building up strategic inventory. Instead, he will not allow the buyer to purchase items in the second period. As a result, the buyer will pre-purchase her entire second period selling quantity and will not buy additional units in the second period. However, the supplier will extract the entire profit from the buyer with the first-period fixed fee. For higher holding costs, the supplier will control the strategic inventory build-up of the buyer by setting a first-period wholesale price that is above his marginal costs which effectively eliminates strategic inventory. Still, the contract fails to coordinate the supply chain because of the increased first-period wholesale price that leads to below-optimal first-period selling quantities.

Additionally, Anand et al (2008) implemented various extensions to their standard model to further test the relevance of strategic inventory under multi-period interaction. First, they replaced the linear demand function of their standard model by a convex, piecewise-linear, and decreasing demand function. They show that buyers maintain to build up strategic inventory and, therefore, prove the relevance of strategic inventory for that class of demand functions. For the class of concave demand functions, however, inventory is not carried under certain conditions (i.e. on the intercept and slopes of the linear segments). Nevertheless, the threat that the buyer might use a strategic inventory can influence the supplier's wholesale price decision and, therefore, impacts the equilibrium solution.

Second, they extended the horizon of the model to three periods to test the robustness of their result regarding an end of (last) period effect. The results show that strategic inventories are not just an artifact caused by an end of period effect, but instead are also pivotal under general horizon length. Moreover, they consider the case with an infinite horizon whereby future periods get discounted and show that the existence of strategic inventories maintains.

Third, they generalize their finding regarding more sophisticated contracts. As a result of the buyers extended action space of strategic inventory, the supplier is no longer able to implement a <u>dynamic</u> vertical contract that simultaneously implements the first-best solution and extracts away all of the buyer's residual profits.[13] However, by using a more sophisticated commitment contract the supplier can eliminate the buyer's inter-temporal link between periods as strategic inventory no longer impacts the previously committed prices of the supplier. As a result, a sophisticated commitment contract enables the supplier to implement the first-best solution and to obtain the entire supply chain channel profit.

Following the findings of Anand et al. (2008), several studies that examine the utilization of strategic inventory under vertical competition have been conducted.

Keskinocak et al. (2008) introduce a capacity constraint for the supplier's first-period production/ procurement quantity that was unlimited in the standard model. They show that strategic inventory only gets utilized, if the supplier's capacity level is above a critical threshold. However, if the supplier's capacity is above the critical threshold but lower than the equilibrium purchasing quantity of the standard model, the buyer has to balance his selling quantity and inventory size against each other reducing her strategic power. Moreover, they show that, if the supplier faces positive capacity cost, he might prefer lower capacity levels than the buyer. As a result, certain capacity costs will cause the supplier to choose capacity levels that harm the performance of the supply chain. In these cases both buyer and supply chain profit would be higher under a commitment contract.

The working paper of Viswanathan and Jang (2009) extends the standard model of Anand et al. (2008) to a duopoly with one supplier selling through two retailers under a

[13] See: Anand et al. (2008) p.1802: Theorem 4

simple wholesale price contract. The working paper of Viswanathan and Jang (2011) additionally studies the impact of a commitment contract for the one supplier, two buyer's oligopoly. However, both working papers report different solutions for the wholesale price contract and both solutions seem to be erroneous as a comparison to a more generalized horizontal competition model with a single supplier who sells through an arbitrary number of buyers will show.

The study of Desai et al. (2010) addresses the utilization of strategic inventory under a variety of settings. First, they integrate a discount factor into the model. This way, the value of cash flows is determined in dependence of the period in which it was realized. The general function of strategic inventory maintains in the discounted cash flow approach. However, as the supplier favors to realize a higher degree of his profit if the discount rate increases, some structural differences to the solution of model of Anand et al. (2008) occur. A detailed recap on the discounted cash-flow approach is given in section 6 of this thesis, where the discounted cash flow approach of Desai et al. (2010) gets compared to a new developed model with an inventory interest rate.

Moreover, Desai et al. (2010) analyze three different supply chain structures: A base model and two duopoly cases (i.e. with horizontal competition within the supply chain). In the base model, a single supplier sells his goods to a single buyer (as in Anand et al. 2008). In the first duopoly setting, two competing suppliers face a single buyer while in the second duopoly setting, a single supplier faces two competing buyers. Moreover, they present an extended model in which the buyer can invest in merchandising that increases the demand of the end customers and in which the supplier is allowed to give a trade promotion. The trade promotion grants the buyer a special, lower first-period wholesale price which is only available, if the buyer has accomplished a certain level of merchandising effort. Last, they also analyze whether strategic inventories are also used under demand uncertainty.

With their one supplier, two buyers setup, Desai et al. (2010) show that the utilization of strategic inventory is not always beneficial from the buyers' perspective. However, as they are caught in a prisoner's dilemma, they are enforced to continue to build up strategic inventory. In their two suppliers, one buyer setup, however, the retailer is better off through forward buying in comparison to the situation with only one supplier. Regarding trade promotions, the authors show that the supplier can influence the buyer's level of merchandising effort by implementing trade promotions that are only accessible for the buyer if she commits to fulfilling a specified level of merchandising effort. By allowing the buyer to utilize strategic inventory, the necessary price discount of the first-period trade promotion that is needed to encourage the buyer to choose the higher level of merchandising effort gets reduced. Hence, both supplier and buyer are able to benefit from the possibility to utilize strategic inventory.

As previously mentioned, Desai et al. (2010) also take an approach on stochastic demand in their study on trade promotions. Thereby, they mix strategic and operational (i.e. safety stocks) reasons to build up inventory. In their setup, the demand is either low or high (i.e. they use two different demand functions) and gets revealed to all supply chain members after the buyer has placed her first-period purchasing order. Hence, the actual realization of demand is common knowledge across the supply chain members. Whether the buyer will utilize strategic inventory depends on the level of the holding costs. If they are sufficiently low, the buyer will keep an inventory regardless of the realization of the demand function. For medium holding cost, the buyer will only keep inventory, if the demand is low (i.e. if prices are low), else he will sell all of her items. Finally, for high holding costs values the buyer will never utilize a strategic inventory.

In Zhang et al. (2010) the end customer demand is stochastic in each period. The buyer's unsold units are carried over to the next period as inventory. However, the supplier cannot observe the sales quantity and the buyer does not share this information. Hence, the buyer's amount of strategic inventory is private information. In contrast to the standard model of Anand et al. (2008), the supplier is allowed to use dynamic short-term contracts (i.e. the supplier offers a purchasing contract to the retailer that is only available in the specific period). While the supplier is unable to observe the buyer's size of strategic inventory, he can estimate it by using his information about the distribution of the end customers demand function (common knowledge) and recent buyer's purchasing quantity. The study shows that the non-observable inventory size may cause information distortions within the supply chain and, therefore, negatively impacts supply chain efficiency. They show that a batch order contract can solve the distortions in their infinite-horizon model.

Li et al. (2015) implement production costs for the supplier. Hereby, the second period production costs are declining linearly with the first-period production quantity because of the experienced gained by first-period production. The learning curve parameter, however, is determined by a random factor. The study shows that the double marginalization problem is more pronounced in the presence of learning than without. Using a revenue sharing contract, the supplier can reestablish a first-best supply chain outcome. However, the supplier's share of profit decreases the lower the holding costs are, while the buyer's share of profit increases.

Arya and Mittendorf (2013) analyze how end customers rebates that are offered directly by the supplier affect the interaction in multiple period, vertical competition and the utilization of strategic inventory. They show that the supplier can increase the buyer's (opportunity) costs of holding inventory by offering customers rebates in the first-period that increase the customer's willingness to pay. Hence, the buyer can set higher prices that get compensated by the supplier's rebates and, thus, do not decrease the customers demand and the level of strategic inventory gets slightly reduced. Because of the reduction of double marginalization due to the rebates the overall supply chain performance also gets enhanced. Surprisingly, both supply chain members benefit from the introduction of supplier-to-customer rebates. Moreover, Arya and Mittendorf (2013) also investigate a nonlinear pricing scheme (i.e. a quantity-contingent pricing scheme) that is supplemented by supplier-to-customer rebates. By implementing such a contract, the supplier can eliminate strategic inventory and achieve channel coordination.

Arya et al. (2014) investigate the impact of strategic inventories under centralized and decentralized procurement structures of a retailer that consists of multiple divisions. Each division sells an identical product on its individual market. The identical product gets purchased from a single supplier. In the case of a centralized structure, purchasing decisions are made by a central planner. Consequently, the centralized model and the solution are identical to the standard model of Anand et al. (2008). In the decentralized structure, however, the individual divisions of the retailer make the procurement choices. As the incentive to build up inventory is smaller for the individual division than for the entire company, a decentralized procurement structure generates a free-rider problem leading to lower inventory levels and higher second period prices. At the same time, the lower willingness to build up inventory forces the supplier to set a lower first-period price than under centralization. The study shows that the benefit from getting a lower first-period wholesale price outweighs the disadvantage of a higher second period wholesale price, if holding costs are sufficiently low. Hence, if holding costs are sufficiently low, the retailer prefers a decentralized procurement structure over a centralized one. Moreover, the study analyzes the impact of a delegation factor that the retailer uses to virtually increase the divisions holding costs. This way, the retailer can influence the individual divisions to build up fewer inventories. As a higher delegation factor commits the retailer to build up fewer inventories, the supplier is forced to set a lower first-period wholesale price, but will set a higher second period wholesale price. Overall, the delegation factor proves being beneficial for a certain range of low holding costs. More precisely, under very low holding costs the retailer prefers a solution with strategic delegation. If holding costs are sufficiently high, the retailer prefers a decentralized structure.

3 Strategic Inventory and its Impact on Supply Chain Coordination under Vertical Competition

Anand et al. (2008) were the first showing analytically that in supply chains with vertical competition a strategic benefit can be obtained by using inventory. In such a supply chain as shown in Figure 4, one supplier (he) produces and sells goods to one buyer (she) that, in turn, sells these goods to customers with price dependent demand.

As there are no horizontal competitors (i.e. alternative suppliers or buyers), both supplier and buyer possess a high degree of monopoly power and are independently trying to optimize their individual profits by competing with their prices. This individual optimization may lead to a suboptimally high end customer price and to a supply chain profit that is below the first-best solution (i.e. the highest possible supply chain profit). This so-called called double marginalization effect that reduces supply chain efficiency was first shown by Spengler (1950).

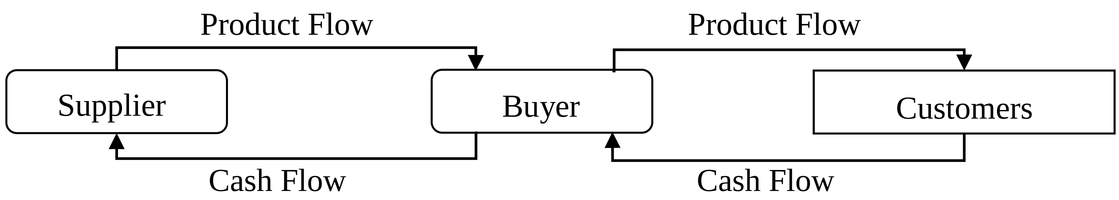

Figure 4: Supply Chain with vertical Competition

In the first subsection, the well-known single-period model (Spengler 1950) will be reviewed first in order to demonstrate the disadvantage that is caused by the coordination failure (i.e. the double marginalization effect) under individual optimization. In this model, the supply chain consists of a single supplier selling to a single buyer. By analyzing an integrated supply chain, the first-best solution can be obtained. Comparing the result of the integrated supply chain with the solution under individual optimization demonstrates the coordination failure. In a last step, the two-part-tariff will be shown as an example for a more sophisticated contract that can restore the first-best solution even under individual optimization by realigning the incentives of the supply chain members with those of an integrated supply chain.

In the second subsection, the results of the study of Anand et al. (2008), in which the single-period model was extended to a two-period model, will be recapped. Thereby, the effects of strategic inventory will be demonstrated: After perceiving benchmarks from the single-period model, the impact of strategic inventory under individual optimization will be shown. Moreover, it will be shown that the possibility to use strategic inventory hinders the two-part tariff from providing a solution with first-best supply chain profits.

Technically, the solution of the various scenarios (e) can be obtained by backwards induction. This means that the optimization starts with the last decision point in the model. Optimizing the corresponding target function for this decision delivers a reaction function for the next decision variable. This decision function then gets inserted into the objective functions. Next, the model gets optimized for the second last decision variable and the obtained reaction function also gets inserted into the target functions and, additionally, into the already calculated reaction function(s) of the previous step(s). These steps get repeated until the first decision within the model gets optimized. This no longer results in a reaction function. Instead, the final result for this variable is obtained. Inserting the result into the reaction functions and the target functions yields the final results for the decision functions as well as the profits of the supply chain members.

3.1 Supply Chain Performance in the Single-Period Horizon Model

In this section, the first-best solution ($e = FB1$) of the single-period model is obtained by analyzing the supply chain under vertical integration (Figure 6). Then, the solution of the single-period model under individual optimization ($e = IO$) will be derived (Figure 5). A comparison to the first-best outcome will show the profit reducing effect caused by double marginalization. Last, the two-part tariff ($e = TP1$) will be analyzed as an example of a more sophisticated contract. This will demonstrate that such a contract can align the incentives of supply chain members and, thus, allows a first-best supply chain outcome.

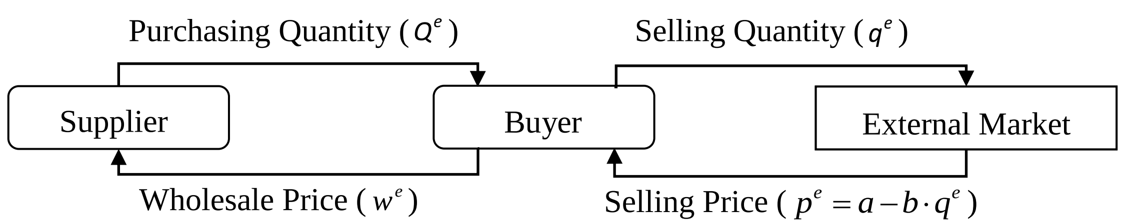

Figure 5: Single-Period Model

In the classical single-periodic setting, the supplier first determines the wholesale price (w^e). Then, the buyer chooses his purchasing (Q^e) and selling quantity (q^e). It is assumed that unsold units possess a salvage value of zero.[14] Hence, the buyer will never order more units than he will sell (i.e. $Q^e = q^e$). Both supplier and buyer possess perfect information[15]. In this model setup the customer demand of the external market is assumed to be described by a linear, quantity-dependent price function:

$$p^e\left(q^e\right) = a - b \cdot q^e \tag{3.1}$$

Moreover, it is assumed that the supplier does not have any production/ purchasing cost and neither supplier nor buyer face any handling costs.[16] Therefore, the supplier's profit function (π_S^e) is given by:

$$\pi_S^e\left(w^e\right) = w^e \cdot q^e\left(w^e\right) \tag{3.2}$$

And, the buyer's profit function (π_B^e) is given by:

$$\begin{aligned}\pi_B^e\left(q^e\right) &= p^e\left(q^e\right) \cdot q^e - w^e \cdot q^e \\ &= \left(a - b \cdot q^e\right) \cdot q^e - w^e \cdot q^e,\end{aligned} \tag{3.3}$$

Combining the supply chain members profit functions ((3.2), (3.3)) results in an overall supply chain profit function (π_{SC}^e) of:

$$\begin{aligned}\pi_{SC}^e &= \pi_B^e + \pi_S^e = p^e\left(q^e\right) \cdot q^e \\ &= \left(a - b \cdot q^e\right) \cdot q^e.\end{aligned} \tag{3.4}$$

3.1.1 Solution of the Integrated Supply Chain (First-Best)

In an integrated supply chain, all decisions are made by a single player in order to maximize the supply chain profit. Therefore, the solution of the integrated channel delivers the first-best ($e = FB$) solution. Note that the wholesale prices only define the transfer payments between the supplier and the buyer and are, therefore, not relevant for optimizing the overall supply chain profit. The supply chain model, therefore, can be simplified as shown in Figure 6:

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[14] As long as the salvage value is below or equal to the sum of the suppliers production cost and the handling costs of both supplier and buyer, this assumption is without loss of loss of generality. Else, a situation with the possibility to create infinite profits would exist.

[15] The term perfect information describes that both supply chain members know the previously made decisions of their supply chain member as well as knowing that their supply chain knows about that.

[16] The results stay structurally the same if production costs are constant. However, under increasing/ decreasing marginal cost, the supplier might have an incentive to discourage or encourage an inventory build-up of the buyer in order to achieve lower average production costs.

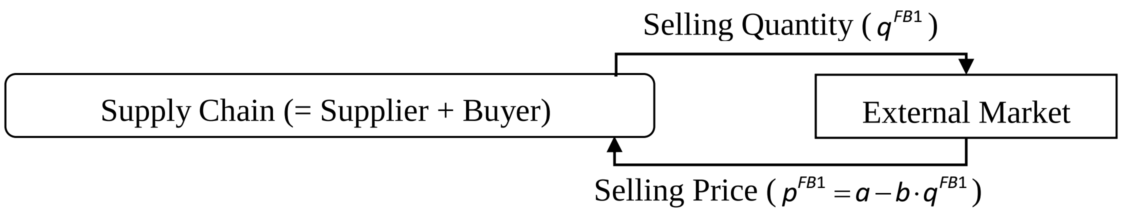

Figure 6: Simplified Model with an Integrated Supply Chain

Determining the solution of the integrated supply chain the supply chain profit (3.4) must be optimized:[17]

$$\max \ \pi_{SC}^{FB1} = \left(a - b \cdot q^{FB1}\right) \cdot q^{FB1} \tag{3.5}$$

Hence, the first-best solution can be obtained by solving:

$$\frac{\partial \pi_{SC}^{FB1}}{\partial q^{FB1}} = a - 2b \cdot q^{FB1} = 0 \tag{3.6}$$

The optimal selling (and buying) quantity is, therefore, given by:

$$q^{FB1} = Q^{FB1} = \frac{a}{2b} \tag{3.7}$$

Inserting the first-best order quantity (3.7) in the demand function (3.1) and the supply chain profit function (3.4) leads to an external market price of

$$p^{FB1} = \frac{a}{2} \tag{3.8}$$

and a first-best supply chain profit of:

$$\pi_{SC}^{FB1} = \frac{a^2}{4b} \tag{3.9}$$

A summary of the first-best solution is given in Table 1.

3.1.2 Solution under Individual Optimization

Under individual optimization, the solution of the model can be found by backwards induction. The corresponding optimization problems of supplier and buyer are:

$$\max \ \pi_S^{IO}\left(w^{IO}\right) = w^{IO} \cdot q^{IO} \tag{3.10}$$

[17] The effects of individual optimization in comparison to the solution of an integrated supply chain were first derived by Spengler (1950).

$$\pi_B^{IO}\left(q^{IO}\right)=\left(a-b\cdot q^{IO}\right)\cdot q^{IO}-w^{IO}\cdot q^{IO} \tag{3.11}$$

As the buyer's selling quantity (q^{IO}) is the last decision within the model, the buyer's reaction function ($q^{IO}\left(w^{IO}\right)$) has to be determined first:

$$\frac{\partial\pi_B^{IO}\left(q^{IO}\right)}{\partial q^{IO}}=a-2b\cdot q^{IO}-w^{IO}=0 \tag{3.12}$$

Solving the buyer's first-order condition (3.12) yields the buyer's reaction function in dependence of the supplier's wholesale price decision:

$$q^{IO}\left(w^{IO}\right)=\frac{a-w^{IO}}{2b}. \tag{3.13}$$

As the supplier anticipates the decision of the buyer, the reaction function (3.13) must be inserted into the supplier's profit function (3.2):

$$\pi_S^{IO}\left(w^{IO}\right)=w^{IO}\cdot\frac{a-w^{IO}}{2b}. \tag{3.14}$$

Solving the supplier's first-order condition

$$\frac{\partial\pi_S^{IO}}{\partial w^{IO}}=\frac{a-2w^{IO}}{2b}=0 \tag{3.15}$$

delivers an optimal wholesale price of:

$$w^{IO}=\frac{a}{2}. \tag{3.16}$$

Inserting the supplier's wholesale price (3.16) into the buyer's reaction function (3.13) gives the buyer's selling (and buying) quantity under individual optimization:

$$q^{IO}=\frac{a}{4b}. \tag{3.17}$$

This leads to the following profits:

$$\pi_S^{IO}=\frac{a^2}{8b} \tag{3.18}$$

$$\pi_B^{IO}=\frac{a^2}{16b} \tag{3.19}$$

$$\pi_{SC}^{IO} = \frac{3a^2}{16b}. \tag{3.20}$$

The results of the single-period model under individual optimization are summarized in Table 1 alongside with the corresponding values of the first-best solution. Comparing these solutions shows that under individual optimization only 75% of the first-best supply chain profit can be achieved (π_{SC}^{IO} versus π_{SC}^{FB1}).

Table 1: Comparison of Solutions of the One Period Model

	First-Best $e = FB1$	Individual Optimization $e = IO$		First-Best $e = FB1$	Individual Optimization $e = IO$
Wholesale Price w^e	-	$\frac{a}{2}$	Profit Supplier π_S^e	-	$\frac{a^2}{8b}$
Purchasing Quantity Q^e	$\frac{a}{2b}$	$\frac{a}{4b}$	Profit Buyer π_B^e	-	$\frac{a^2}{16b}$
Selling Quantity q^e	$\frac{a}{2b}$	$\frac{a}{4b}$	Profit Supply Chain π_{SC}^e	$\frac{a^2}{4b}$	$\frac{3a^2}{16b}$
External Market Price p^e	$\frac{a}{2}$	$\frac{3a}{4}$			

This profit reduction is caused by the individual surcharge on the marginal cost which is done by both supplier and buyer. In order to maximize his profits, the supplier chooses a wholesale price that already matches the supply chain profit maximizing market price of the first-best solution ($w^{IO} = p^{FB1}$). Hence, in order to maintain the supply chain profit of the first-best solution, the buyer would have to pass through this wholesale price to the end-customers. In this situation, the supplier would receive the whole supply chain profit leaving nothing for the buyer. However, to gain a part of the overall supply chain profit, the buyer also charges a surplus on her marginal costs (i.e. the wholesale price). In comparison to the first-best solution, this leads to a higher market price ($p^{IO} > p^{FB1}$) and a lower selling quantity ($q^{IO} < q^{FB1}$). Overall, under a linear demand function this effect causes a drop in supply chain profit of $\pi_{SC}^{FB1} - \pi_{SC}^{IO} = a^2/16b$, which is a quarter of the first-best solution (i.e. a deadweight loss 25% of the first-best supply chain profit). Hereby, the supplier manages to get two thirds of the overall supply chain profit (equivalent to 50% of the first-best profit), while the buyer's share is only one third (equivalent to 25% of the first-best profit).

3.1.3 Achieving First-Best Supply Chain Profit through a Coordinating Contract

To avoid the profit loss that is caused by double marginalization, more sophisticated pricing schemes, that cause the downstream party (buyer) to charge the supply chain optimal external market price[18], can be used.[19] A straightforward approach to accomplish this task is the so called "resale-price maintenance", in which the upstream supply chain member (i.e. supplier) controls the price that the downstream part (i.e. buyer) can charge. Equivalently, the supplier could also explicitly set a minimum for the buyer's selling quantity that matches the supply chain optimal amount ($q \geq q^{\min} = q^{FB1}$). However, both mechanisms require a direct control of the downstream member's decision, which is not always a realistic scenario.

Alternatively, the supplier could use a non-linear pricing scheme in order to eliminate the deadweight loss caused by double marginalization. A common example for a non-linear pricing scheme is the two-part tariff ($e = TP1$), which consists of the payment of a fixed fee (F^{TP1}) and a uniform price (w^{TP1}) per sold unit (Figure 7). Hence, the buyer has to pay the fixed fee, if she wants to purchase goods from the supplier. This fixed fee is independent from the actual amount of items that the buyer purchases from her supplier, while the uniform price is paid per purchased unit (i.e. a wholesale price).

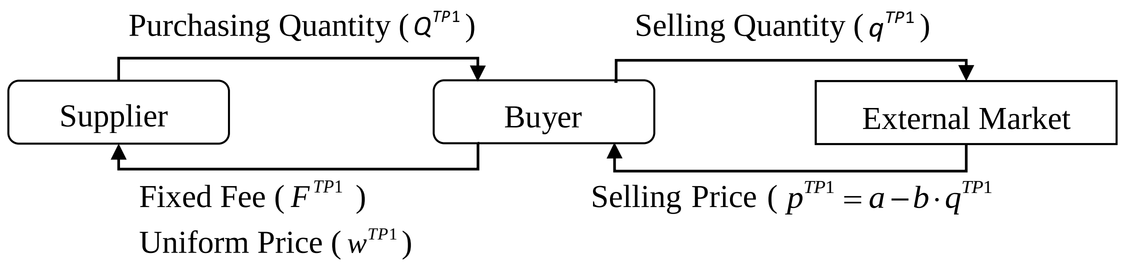

Figure 7: Two-Part Tariff in the Single-Period Model

Anand et al. (2008) have shown that under a two-part tariff, the supplier's optimal strategy is to set the wholesale price to zero (i.e. to the marginal costs). This way the double marginalization effect gets prevented and the supply chain profit equals the first-best outcome. Moreover, the supplier is able to use the fixed fee to extract the entire supply chain profit from the buyer.

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[18] This condition is necessary to achieve first-best supply chain profits.

[19] A good overview of such pricing is given in Tirole (1988).

3.2 The Impact of Strategic Inventory on Supply Chain Performance in Multi-Period Horizon Models

Anand et al. (2008) were the first that extended the single-period model, which was reviewed in chapter 3.1 to multiple periods. With their extended two-period model (Figure 8) they were able to demonstrate that the buyer exhibits an incentive to carry inventory between both periods due to purely strategic considerations (i.e. to influence the price setting of the supplier). The supplier, in turn, reacts to this strategic inventory by adjusting his price setting.

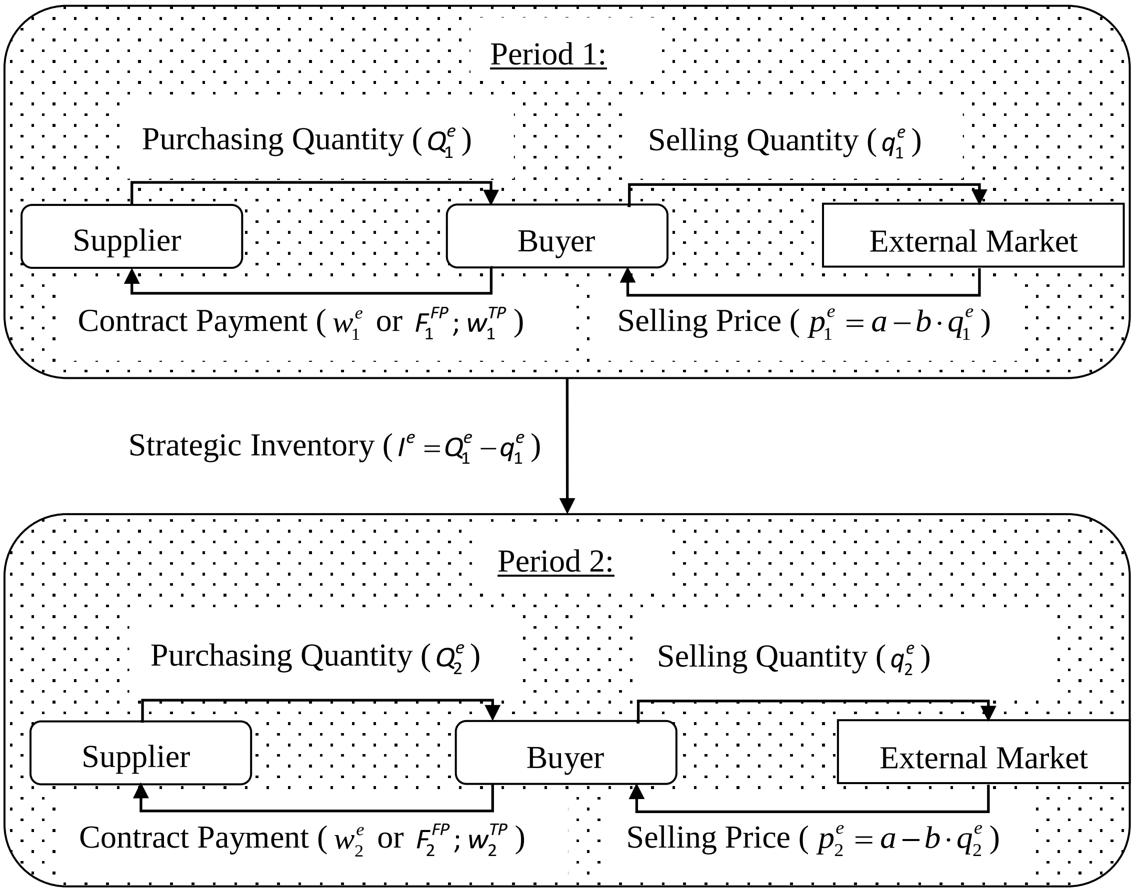

Figure 8: Two-Period Model

The underlying two-period model of Anand et al. (2008) simply duplicates the decisions of the standard single-period model. Hence, in each period ($t = 1, 2$) the supplier determines the contract parameters (wholesale price contract: w_t^{WP}, two-part tariff: F_t^{FP} and w_t^{TP}) at which the buyer can purchase goods. The buyer, in turn, chooses both her purchase quantity (Q_t^e) and the quantity of units that she sells to an external market (q_t^e) for each period. The sales price of each period (p_t^e) at the external market is again determined by a linear inverse demand function: $p_t^e\left(q_t^e\right) = a - b \cdot q_t^e$.

In contrast to the one-period model, the buyer can carry over inventory from period one to period two by choosing a first-period purchase quantity that is larger than the first-period selling quantity (i.e. $Q_1^e > q_1^e$). The buyer can then use this inventory ($I^e = Q_1^e - q_1^e$) to reduce the second period purchase quantity that is required to optimize her profit ($Q_2^e = q_2^e - I^e$). For each stored unit the buyer faces holding cost (h^e). At the end of period two, unsold units have a salvage value of zero. Hence, the buyer will sell all of her goods at the end of the second period. Also, the production cost of the supplier and any handling costs are normalized to zero. Both supplier and buyer possess perfect information. Note, although the model is extended to two-periods, still none of the classical reasons to build up inventory as described in chapter 2 exist.

Anand et al (2008) analyzed several supply chain scenarios including the solutions of an integrated channel, under individual optimization with a simple wholesale price contract, and under individual optimization with a two-part tariff. For the latter contract types, solutions under commitment were analyzed as well (i.e. the supplier commits to set identical wholesale prices in both periods: $w_1^C = w_2^C$). This way, the buyer would have no incentive to build up any inventory as he cannot influence the committed second period wholesale price of the supplier.[20]

Both the solution of an integrated channel as well as the solution of the simple wholesale price with commitment ($e = C$) can be easily obtained from the single-period model.

3.2.1 Benchmarks from the One-Period Model

In an integrated supply chain, the wholesale prices only define the transfer payments between the supplier and the buyer and are, therefore, not relevant for optimizing the overall supply chain profit. Furthermore, inventories incur holding costs without providing any benefit to the supply chain as a whole. Thus, building up inventory is not reasonable, if no conflict of interest between the two parties exists. Therefore, in an integrated supply chain, only the optimal sales quantities that maximizes the joint profits need to be determined:

$$\begin{aligned}\pi_{SC}^{FB}\left(q_1^{FB}, q_2^{FB}\right) &= p_1^{FB}\left(q_1^{FB}\right)\cdot q_1^{FB} + p_2^{FB}\left(q_2^{FB}\right)\cdot q_2^{FB} \\ &= \left(a - b\cdot q_1^{FB}\right)\cdot q_1^{FB} + \left(a - b\cdot q_2^{FB}\right)\cdot q_2^{FB}.\end{aligned} \tag{3.21}$$

[20] Analogously, the buyer could also commit to not build up any inventory.

As no inventory is built up, the periods are no longer connected and the problem can be separated into two single-period models. Hence, the selling quantity of each period is unaffected by the horizon extension and identical to the selling quantity of the single-period model:

$$q_t^{FB} = q^{FB1} = a/2b \, . \tag{3.22}$$

Consequently, the first-best supply chain profit of the two-period model is simply twice as large as the first-best profit of the single-period model:

$$\pi_{SC}^{FB} = 2 \cdot \pi_{SC}^{FB1} = a^2/2b \, . \tag{3.23}$$

The results of the first-best solution that is needed as a benchmark to the following conducted solutions are summarized in Table 2 at the end of section 3.2.2.

Similar to the way the first-best solution was determined, the results of the solution under a commitment contract can be obtained from respective solution of the single-period model. As the commitment contract disables the strategic utilization of inventory, the periods get decoupled and can be optimized separately. Consequently, as both supply chain members optimize individually, the results of each period mimic the solution of the single-period model under individual optimization:

$$w_1^C = w_2^C = w^{IO} = \frac{a}{2} \tag{3.24}$$

$$Q_1^C = Q_2^C = q_1^C = q_2^C = q^{IO} = \frac{a}{4b} \tag{3.25}$$

Again, the profits can be simply obtained by doubling the respective profits of the single-period model under individual optimization:

$$\pi_S^C = 2 \cdot \pi_S^{IO} = \frac{a^2}{4b} \tag{3.26}$$

$$\pi_B^C = 2 \cdot \pi_B^{IO} = \frac{a^2}{8b} \tag{3.27}$$

$$\pi_{SC}^C = 2 \cdot \pi_{SC}^{IO} = \frac{3a^2}{8b} \tag{3.28}$$

However, Anand et al. (2008) have shown that the supplier will never offer a commitment contract, as he would not obtain any profit enhancement by doing so. Hence, the values of the commitment solution only apply, if the holding costs are high enough to prohibit a build-up of strategic inventory. This so called static solution will be mathematically examined in the following chapter. The results of the static solution that are identical to the commitment solution are summarized in Table 2.

3.2.2 Individual Optimization in the Two-Period Model and the Role of Strategic Inventory

By analyzing their two-period model under individual optimization and simple wholesale price contracting Anand et al (2008) show that buyers have an incentive to utilize inventory as a strategic tool. This setup will serve as the baseline model for the following theoretical and behavioral studies within this thesis and is, therefore, referred to as the standard model (SM).

In the standard model ($e = SM$), the supply chain is not integrated. Therefore, supplier and buyer independently choose their decision variables (w_t^{SM} and q_t^{SM} , I^{SM}) to optimize their individual profits. Analyzing the standard model will show that in equilibrium strategic inventory is built up, if the holding costs are not prohibitively high (i.e. $I^{SM} > 0$ if $h^{SM} < a/4$). These cases will be denounced as the *dynamic solutions*, because the supplier has an incentive to choose dynamic prices (i.e. $w_1^{SM} \neq w_2^{SM}$). However, if the holding costs are too high (i.e. if $h^{SM} \geq a/4$), the buyer no longer has an incentive to build up a strategic inventory. This leads to constant wholesale prices over both periods (i.e. $w_1^{SM} = w_2^{SM}$) and to equilibria that resemble the solution of the commitment contract (i.e. decisions within each period are identical to those of the one period model under individual optimization). These cases will be called the *static solutions*. The closed-form static and dynamic solutions can be derived using backward induction as shown by Anand et al. (2008) and are described below.

Using a simple wholesale price contract, the supplier only has to determine the prices of period one and two (w_1^{SM} , w_2^{SM}) within his profit function:

$$\begin{aligned} \pi_S^{SM}\left(w_1^{SM}, w_2^{SM}\right) &= w_1^{SM} \cdot Q_1^{SM} + w_2^{SM} \cdot Q_2^{SM} \\ &= w_1^{SM} \cdot \left(q_1^{SM} + I^{SM}\right) + w_2^{SM} \cdot \left(q_2^{SM} - I^{SM}\right). \end{aligned} \tag{3.29}$$

The profit function of the buyer, who has to determine q_1^{SM}, I^{SM} and q_2^{SM} is given by:

$$\begin{aligned}\pi_B^{SM}\left(q_1^{SM},I^{SM},q_2^{SM}\right) &= p_1^{SM}\left(q_1^{SM}\right)\cdot q_1^{SM} - w_1^{SM}\cdot Q_1^{SM} - h^{SM}\cdot I^{SM} \\ &\quad + p_2^{SM}\left(q_2^{SM}\right)\cdot q_2^{SM} - w_2^{SM}\cdot Q_2^{SM} \\ &= \left(a-b\cdot q_1^{SM}\right)\cdot q_1^{SM} - w_1^{SM}\cdot\left(q_1^{SM}+I^{SM}\right) - h^{SM}\cdot I^{SM} \\ &\quad + \left(a-b\cdot q_2^{SM}\right)\cdot q_2^{SM} - w_2^{SM}\cdot\left(q_2^{SM}-I^{SM}\right).\end{aligned} \tag{3.30}$$

And the overall supply chain profit is described by:

$$\begin{aligned}\pi_{SC}^{SM}\left(q_1^{SM},q_2^{SM}\right) &= p_1^{SM}\left(q_1^{SM}\right)\cdot q_1^{SM} - h^{SM}\cdot I^{SM} + p_2^{SM}\left(q_2^{SM}\right)\cdot q_2^{SM} \\ &= \left(a-b\cdot q_1^{SM}\right)\cdot q_1^{SM} - h^{SM}\cdot I^{SM} + \left(a-b\cdot q_2^{SM}\right)\cdot q_2^{SM}.\end{aligned} \tag{3.31}$$

At the beginning of the second period the buyer chooses her selling quantity q_2^{SM} given the decisions of period 1 and the supplier's pricing in period 2:

$$\frac{\pi_B^{SM}\left(q_1^{SM},I^{SM},q_2^{SM}\right)}{\partial q_2^{SM}} = a - 2b\cdot q_2^{SM} - w_2^{SM} = 0 \tag{3.32}$$

However, if the buyer possesses inventory, she will always sell it as it has purchasing cost of zero (previous purchasing and holding costs are sunk costs). Therefore, the optimal response function to the supplier's wholesale price of the second period is

$$q_2^{SM}\left(w_2^{SM}\right) = \max\left\{I^{SM}, \frac{a-w_2^{SM}}{2b}\right\}. \tag{3.33}$$

Hence, the buyer will only purchase additional goods in the second period if $w_2^{SM} < a - 2b\cdot I^{SM}$. Anand et al. (2008) have shown mathematically that this condition is always fulfilled (i.e. the supplier always chooses a wholesale price in period two that is sufficiently low to incentivize the buyer to purchase additional goods).[21] To keep the results concise, the description is limited to the case that $Q^{SM} > 0$ (i.e. $w_2^{SM} < a - 2b\cdot I^{SM}$).[22]

The supplier integrates the buyer's response function (3.33) into his profit function:

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[21] See e-companion to Anand et al. (2008) pp. 2–4.

[22] This is without loss of generality as the supplier's second period wholesale price response function (3.35) shows.

$$\begin{aligned}\pi_S^{SM}\left(w_1^{SM},\, w_2^{SM}\right) &= w_1^{SM}\cdot Q_1^{SM} + w_2^{SM}\cdot Q_2^{SM}\\ &= w_1^{SM}\cdot\left(q_1^{SM}+I^{SM}\right)+w_2^{SM}\cdot\left(\frac{a-w_2^{SM}}{2b}-I^{SM}\right).\end{aligned} \tag{3.34}$$

The respective first order condition and response function for the period 2 wholesale price are:

$$\begin{aligned}\frac{\pi_S^{SM}\left(w_1^{SM},\, w_2^{SM}\right)}{\partial w_2^{WP}} &= \frac{a-w_2^{SM}}{2b}-I^{SM}=0\\ w_2^{SM}\left(I^{SM}\right) &= \max\left\{0,\frac{a}{2}-b\cdot I^{SM}\right\}\end{aligned}. \tag{3.35}$$

The supplier's response function shows that by building up an inventory, the buyer can influence the period two equilibrium wholesale price. Hence, the buyer has a strategic incentive to build up inventory that distinguishes from the classical reasons to hold inventory. The buyer anticipates this strategic effect of her inventory on the second period wholesale price into her profit function (3.30):

$$\begin{aligned}\pi_B^{SM}\left(q_1^{SM},I^{SM}\right) &= \left(a-b\cdot q_1^{SM}\right)\cdot q_1^{SM}-w_1^{SM}\cdot\left(q_1^{SM}+I^{SM}\right)-h^{SM}\cdot I^{SM}\\ &+\left(a-b\cdot\frac{\frac{a}{2}+b\cdot I^{SM}}{2b}\right)\cdot\frac{\frac{a}{2}+b\cdot I^{SM}}{2b}-\left(\tfrac{a}{2}-b\cdot I^{SM}\right)\cdot\left(\frac{\frac{a}{2}+b\cdot I^{SM}}{2b}-I^{SM}\right).\end{aligned} \tag{3.36}$$

The first order conditions and optimal response functions for the first-period selling quantity and inventory size are:

<u>selling quantity:</u>

$$\frac{\partial\pi_B^{SM}\left(q_1^{SM},I^{SM}\right)}{\partial q_1^{SM}} = a-2b\cdot q_1^{SM}-w_1^{SM}=0 \tag{3.37}$$

$$q_1^{SM}\left(w_1^{SM}\right)=\max\left\{0,\,\frac{a-w_1^{SM}}{2b}\right\} \tag{3.38}$$

<u>inventory size:</u>

$$\frac{\partial\pi_B^{SM}\left(q_1^{SM},I^{SM}\right)}{\partial I^{SM}} = -w_1^{SM}-h^{SM}+\frac{3}{4}a-\frac{3}{2}b\cdot I^{SM}=0 \tag{3.39}$$

$$I^{SM}\left(w_1^{SM}\right)=\max\left\{0,\frac{a}{2b}-\frac{2}{3b}\cdot h^{SM}-\frac{2}{3b}\cdot w_1^{SM}\right\}. \tag{3.40}$$

Hence, the buyer only builds up inventory, if

$$w_1^{SM} + h^{SM} < \tfrac{3}{4} \cdot a \tag{3.41}$$

Since a and h^{SM} are constant, the supplier's choice of the wholesale price determines whether or not the buyer builds up strategic inventory.

Case 1 – Static Solution: Supplier chooses a period one wholesale price that prevents the buyer from building up inventory (i.e. $w_1^{SM} \geq \tfrac{3}{4} \cdot a - h^{SM} \quad \Rightarrow I^{SM} = 0$*):*

In this case, no inventory is built up and, therefore, the two periods are independent from each other. Hence, each decision is identical to the corresponding decision of the one-period model. Table 3 summarizes the results for this case (static solution). Note, in comparison to the one-period model the profits of the two-period model have to be doubled because of the doubled horizon length.

Case 2 – Dynamic Solution: Supplier chooses a period one wholesale price that leads to an inventory build-up by the buyer (i.e. $w_1^{SM} < \tfrac{3}{4} \cdot a - h^{SM} \quad \Rightarrow I^{SM} > 0$*):*

In this case, the supplier anticipates the behavior of the buyer ((3.38), (3.40)) and the optimal period 2 decisions concerning w_2^{SM} and q_2^{SM} from (3.33) and (3.35). Inserting in (3.34) results in the following profit function:

$$\pi_S^{SM}\left(w_1^{SM}\right) = \frac{18a \cdot w_1^{SM} - 17\left(w_1^{SM}\right)^2 - 4w_1^{SM} \cdot h^{SM} + 4\left(h^{SM}\right)^2}{18b}. \tag{3.42}$$

Then, the supplier chooses his optimal period one wholesale price:

$$\begin{aligned} \frac{\pi_S^{SM}\left(w_1^{SM}\right)}{\partial w_1^{SM}} &= \frac{18a - 34w_1^{SM} - 4h^{SM}}{18b} = 0 \\ w_1^{SM} &= \frac{9a - 2h^{SM}}{17} \end{aligned} \tag{3.43}$$

Inserting the period one wholesale price (3.43) into the condition for case 2 ($w_1^{SM} < \tfrac{3}{a} \cdot a - h^{SM}$) directly shows that $h^{SM} < \tfrac{a}{4}$ must hold. Else, no inventory is built up and the solution of case 1 is relevant. Hence for $h^{SM} \leq a/4$ the optimal first-period wholesale price is:

$$w_1^{SM} = \max\left\{\frac{a}{2}, \frac{9a - 2h^{SM}}{17}\right\}. \tag{3.44}$$

The analysis shows that for $h^{SM} < a/4$ the buyer will build up a strategic inventory with a size of $I^{SM} = \frac{5 \cdot \left(a - 4h^{SM}\right)}{34b}$ that influences the wholesale price setting of the supplier. Following Anand et al. (2008), this solution will be referred to as dynamic solution, because of the different wholesale prices of period one and two. In contrast, the solution without strategic inventory and constant wholesale prices ($h^{SM} \geq a/4$) will be called static solution (Note, that the static solution also mirrors the solution of the one period model). The complete solution of the model can be obtained by inserting w_1^{SM} into the respective first-period selling quantity and inventory response function. Then the obtained results are inserted into the second period wholesale price response function. Next, these results are inserted into the second period selling quantity response function. This way all decision variables are known and can be inserted into the profit functions. The results of the dynamic alongside with the static and first-best solution are summarized in Table 2:

Table 2: Solutions of the Two-Period Model under Wholesale Price Contracting[23]

	First-Best	Standard Model	
		Static	Dynamic
	$e = FB$	$e = SM$	$e = SM$
		$h^{SM} \geq a/4$	$h^{SM} < a/4$
Wholesale Price Period 1 (w_1^e)	-	$\frac{a}{2}$	$\frac{9a - 2h^{SM}}{17}$
Purchase Quantities Period 1 (Q_1^e)	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{13a - 18h^{SM}}{34b}$
Sales Quantities Period 1 (q_1^e)	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{4a + h^{SM}}{17b}$
Inventory (I^e)	0	0	$\frac{5 \cdot \left(a - 4h^{SM}\right)}{34b}$
Retail Prices Period 1 (p_1^e)	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{13a - h^{SM}}{17}$
Wholesale Price Period 2 (w_2^e)	-	$\frac{a}{2}$	$\frac{6a + 10h^{SM}}{17}$
Purchase Quantities Period 2 (Q_2^e)	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{3a + 5h^{SM}}{17b}$
Sales Quantities Period 2 (q_2^e)	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{11a - 10h^{SM}}{34b}$
Retail Prices Period 2 (p_2^e)	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{23a + 10h^{SM}}{34}$
Profit Supplier (π_S^e)	-	$\frac{a^2}{4b}$	$\frac{9a^2 - 4ah^{SM} + 8\left(h^{SM}\right)^2}{34b}$
Profit Buyer (π_B^e)	-	$\frac{a^2}{8b}$	$\frac{155a^2 - 118ah^{SM} + 304\left(h^{SM}\right)^2}{1156b}$
Profit Supply Chain (π_{SC}^e)	$\frac{a^2}{2b}$	$\frac{3a^2}{8b}$	$\frac{461a^2 - 254ah^{SM} + 576\left(h^{SM}\right)^2}{1156b}$

[23] These results were first derived by Anand et al (2008) and a recap of Table 1 of their study.

3.2.3 The Impact of Strategic Inventory on Supply Chain Performance

Table 2 provides a summary of the first-best solution as well as the solution under individual optimization (standard model) that splits into the static and the dynamic solution. The supply chain profit of the standard model is displayed in Figure 9.

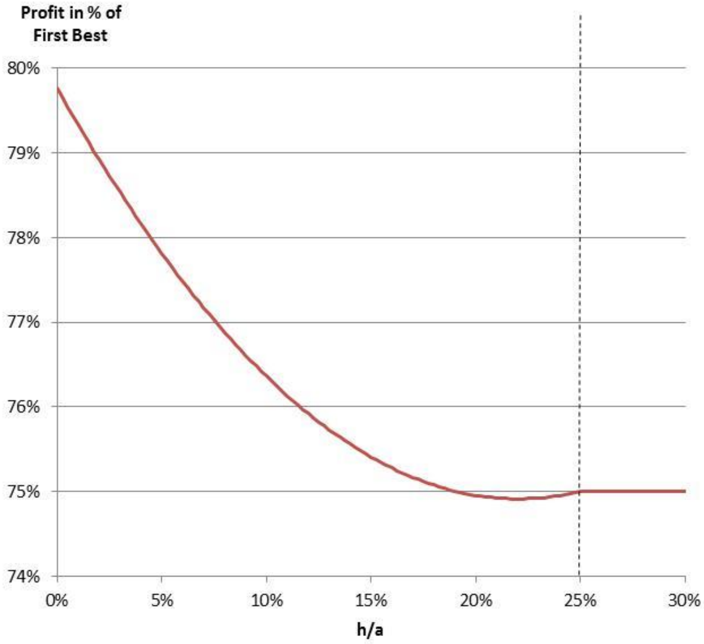

Figure 9: Supply Chain Profit in the Standard Model

In the static solution ($h^{SM} / a \geq 0.25$), the total supply chain profit corresponds to only 75% of the total supply chain profit of the first-best solution:

$$\frac{\pi_{SC}^{SM}}{\pi_{SC}^{FB}} = \frac{3a^2}{8b} \bigg/ \frac{a^2}{2b} = 0.75 \tag{3.45}$$

In the dynamic solution ($h^{SM} / a < 0.25$), the total supply chain profit depends on the holding costs:

$$\frac{\pi_{SC}^{SM}}{\pi_{SC}^{FB}} = \frac{\dfrac{461a^2 - 254ah^{SM} + 576\left(h^{SM}\right)^2}{1156b}}{\dfrac{a^2}{2b}} = \frac{461a^2 - 254ah^{SM} + 576\left(h^{SM}\right)^2}{578a^2} \tag{3.46}$$

Determining the first- and second-order derivatives of (3.46) shows that a minimum exists at $h^{SM} \approx 0.22a$:

$$\frac{\partial \frac{\pi_{SC}^{SM}}{\pi_{SC}^{FB}}}{\partial h^{SM}} = -254a + 1156h^{SM} = 0 \tag{3.47}$$

$$h^{SM} = \frac{127}{578}a \approx 0.22a \tag{3.48}$$

$$\frac{\partial^2 \frac{\pi_{SC}^{SM}}{\pi_{SC}^{FB}}}{\partial \left(h^{SM}\right)^2} = 1156 \tag{3.49}$$

Inserting (3.48) into (3.46) determines the worst case supply chain profit of the dynamic solution (weighted with the first-best outcome):

$$\frac{\pi_{SC}^{SM}\left(h^{SM} = \frac{127}{578}a\right)}{\pi_{SC}^{FB}} \approx 0.7491 \tag{3.50}$$

As the maximum supply chain profit of the dynamic solution must be either at the lower ($h^{SM} = 0$) or upper bound ($h^{SM} = 0.25$), the bounds must be inserted into (3.46):

$$\frac{\pi_{SC}^{SM}\left(h^{SM} = 0\right)}{\pi_{SC}^{FB}} = \frac{461}{578} \approx 0.7976 \tag{3.51}$$

$$\frac{\pi_{SC}^{SM}\left(h^{SM} = a/4\right)}{\pi_{SC}^{FB}} = \frac{461a^2 - \frac{127}{2}a^2 + 36a^2}{578a^2} = 0.75 \tag{3.52}$$

Hence, the minimum profit of the dynamic solution corresponds to 74.91% of the first-best outcome and will not exceed about 79.76%, even if the holding cost are zero. This also holds for the entire standard model as the minimum and maximum profit levels of the dynamic solution are below/ above the profit level of the static solution.

Moreover, the utilization of strategic inventory enables higher supply chain profits if holding cost are sufficiently low ($h^{SM}/a < 55/288 \approx 0.19$):

$$\begin{aligned}
&\pi_{SC}^{SM,dynamic} \overset{?}{>} \pi_{SC}^{SM,static} \\
&\frac{461a^2 - 254ah^{SM} + 576\left(h^{SM}\right)^2}{1156b} \overset{?}{>} \frac{3a^2}{8b} \\
&\frac{55}{2312}a^2 - \frac{127}{578}ah^{SM} + \frac{144}{289}\left(h^{SM}\right)^2 \overset{?}{>} 0
\end{aligned} \tag{3.53}$$

$$\rightarrow \qquad h^{SM} < \frac{55}{288}a \qquad \vee \qquad h^{SM} > \frac{a}{4}$$

The lower supply chain profit of the standard model in comparison to the first-best solution is obviously due to the double marginalization effect that arises, because both supply chain members individually maximize their profits, by placing monopoly surcharges on their marginal costs. However, although additional holding cost occur in the dynamic solution due to the utilization of strategic inventory, the overall supply chain profit is larger for reasonably low holding costs. Hence, strategic inventory seems to dampen the double marginalization effect.

If the holding costs are sufficiently low for strategic inventories to be applied (i.e. in the dynamic solution), then the first-period's wholesale price is greater and the second period's wholesale price is smaller than in the static solution:

First-period wholesale price:

$$\begin{aligned}
&w_1^{SM,dynamic} \overset{?}{>} w_1^{SM,static} \\
&\frac{9a - 2h^{SM}}{17} \overset{?}{>} \frac{a}{2} \\
&a - 4h^{SM} \overset{?}{>} 0
\end{aligned}$$

$$\rightarrow h^{SM} < \frac{a}{4} \tag{3.54}$$

Condition (3.54) is always fulfilled in the dynamic solution.

Second period wholesale price:

$$\begin{aligned}
&w_2^{SM,dynamic} \overset{?}{>} w_1^{SM,static} \\
&\frac{6a + 10h^{SM}}{17} \overset{?}{>} \frac{a}{2} \\
&-5a + 20h^{SM} \overset{?}{>} 0
\end{aligned}$$

$$\rightarrow h^{SM} < \frac{a}{4} \tag{3.55}$$

Condition (3.55) is always fulfilled in the dynamic solution.

The intuition is that the supplier sets a higher price in the first-period to reduce the buyer’s incentives to build up an inventory. In the second period, the wholesale price is lower than in the static solution, because the buyer only needs to satisfy her residual demand, given the inventory. Thus, the strategic inventory reduces the monopoly power of the supplier. Nevertheless, the comparison also shows that the supplier is always better off in the dynamic solution (Figure 10), because he is able to implement an indirect form of price differentiation for the buyers selling quantity in period two (i.e. the second period selling quantity consists of the strategic inventory that was purchased at a high price (w_1^{SM}) and the second period purchasing quantity which was purchased at a lower price (w_2^{SM})).

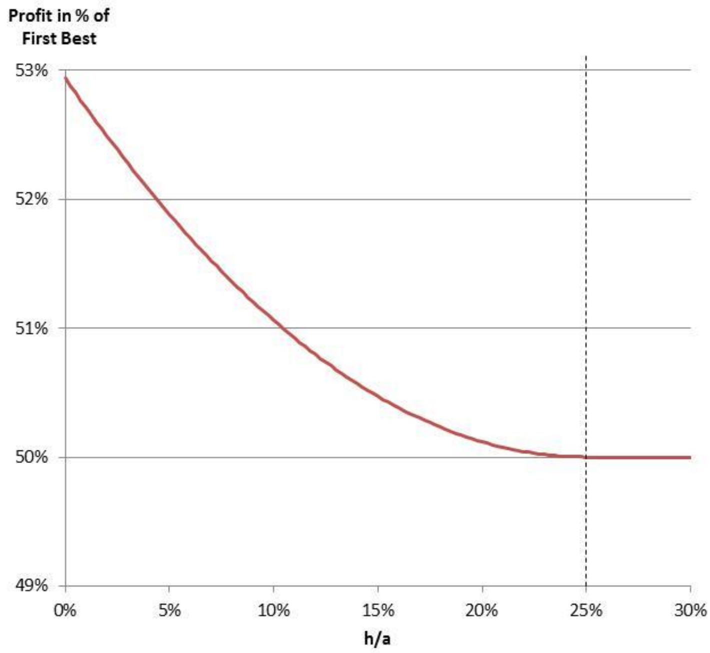

Figure 10: Supplier Profit in the Standard Model

These differentiated prices yield a lower average wholesale price across both periods and, therefore, reduce the degree of double marginalization. The buyer (Figure 11) is also better off with a strategic inventory (i.e. in the dynamic solution), as long as the holding cost are not too high (i.e. as long as $h^{SM} < {}^{21}\!/_{152}\, a \approx 0.138a$):

$$\pi_B^{SM,dynamic} \overset{?}{>} \pi_B^{SM,static}$$

$$\frac{155a^2 - 118ah^{SM} + 304\left(h^{SM}\right)^2}{1156b} \overset{?}{>} \frac{a^2}{8b}$$

$$21a^2 - 236ah^{SM} + 608\left(h^{SM}\right)^2 \overset{?}{>} 0$$

$$\rightarrow h^{SM} < \frac{21}{152}a \;\vee\; h^{SM} > \frac{a}{4} \tag{3.56}$$

Hence, her profits in the dynamic solution are only less than those in the static solution if $\frac{21}{152}a\left(\approx 0.138a\right) < h^{SM} < 0.25a$.

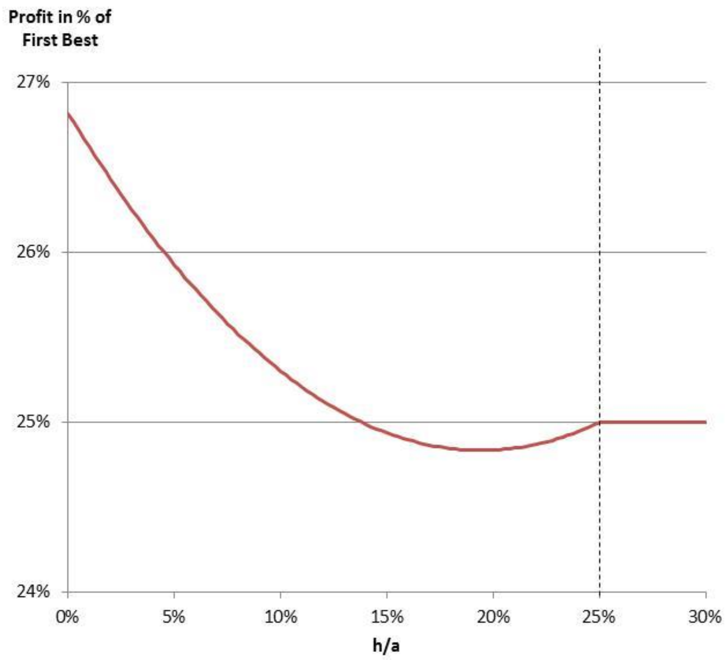

Figure 11: Buyer Profit in the Standard Model

In summary, the overall performance of a non-integrated supply chain in the dynamic solution is superior to the performance of the static solution for sufficiently low holding costs ($h^{SM} < {}^{21a}\!/_{152} \approx 0.138a$). For holding cost above this threshold, the benefits of the lower wholesale prices (i.e. the benefits from the reduction of the double marginalization) are offset by the increase of the total costs due the inventory holding costs. The largest improvement in supply chain performance (about 6.34% more than in the static solution) can be achieved with a strategic inventory, when the holding cost is zero. Nevertheless, even at zero holding cost, the first-best solution cannot be achieved, because the double marginalization effect is only diminished, but not fully eliminated.

3.2.4 Strategic Inventory in Case of a Two-Part Tariff

In the one period model, the supplier could implement the first-best supply chain outcome by using a two-part tariff consisting of a fixed fee (F^{TP1}) and a marginal wholesale price (w^{FP1}). With this contract design, the supplier was able to achieve the coordination of the channel by setting the marginal wholesale price to his marginal cost (i.e. eliminate double marginalization) and to freely distribute the supply chain profit via the fixed fee (i.e. he could extract the whole supply chain profit from the buyer). However, the channel coordinating effect of the two-part tariff in the one period model does not carry over to the two-period model as shown by Anand et al. (2008).

In the two-period model, the supplier has to decide both about the fixed fee (F_t^{TP}) and wholesale price (w_t^{TP}) in each period, while the buyer still has to decide about the purchasing (Q_t^{TP}) and selling quantity (q_t^{TP}) of each period (with $Q_1^{TP} = q_1^{TP} + I^{TP}$ and $Q_2^{TP} = q_2^{TP} - I^{TP}$). Note, that the decision to purchase goods in a specific period involves the decision whether to pay the fixed fee of that period, as the buyer does not have to pay the fixed fee if he forgoes ordering items in the respective period.

In the two-period model, the two-part tariff needs to fulfill two criteria in order to enable a first-best supply chain outcome: First, just as in the one period model, the supplier needs to set the wholesale price at the level of his marginal cost to prevent double marginalization. Second, for $h^{TP} > 0$ the buyer must not carry any strategic inventory because inventory would cause holding costs and, therefore, a deadweight loss for the supply chain. Anand et al. (2008) have shown that for $0 < h^{TP} < \frac{a}{4}$ a profit maximizing supplier will not be able to satisfy both criteria at the same time.[24] As under wholesale price contracting, strategic inventories force the supplier to price for the residual demand of the buyer in period two. Additionally, the buyer might try to save the fixed fee of period two by building up a sufficiently large strategic inventory. As a reaction, the supplier will adjust both his fixed fees as well as his wholesale prices to control for strategic inventory (i.e. he will cause double marginalization because he will no longer set his wholesale prices to his marginal costs.

[24] Anand et al. (2008) limit their analysis of the two-part tariff to $h^{TP} < a/4$. At this upper bound strategic inventory is not used anymore under wholesale price contracting. The mathematical analysis can be found in the online Appendix of Anand et al. (2008).

The supplier can choose between two pricing schemes: Either, he can commit himself to prices at the beginning of the first-period (i.e. the buyer cannot influence the second period price by building up inventory) or he can set dynamic prices (i.e. react on the buyers inventory build-up in the second period).

In the commitment two-part tariff, the supplier will announce all prices (F_t^{TP}, w_t^{TP}) at the beginning of period one. By credibly committing himself to these prices, the buyer is no longer able to use strategic inventory to influence the supplier's prices. However, the supplier still must consider the buyer's possibility to use inventory to save the fixed fee, when determining the commitment prices. Depending on the holding cost, the two-part commitment contract splits into two different solutions:

If holding costs are low $\left(h^{TP} < \left(1 - 2/\sqrt{6}\right)a \approx 0.184a\right)$, the supplier will not try to control the inventory build-up of the buyer. Hence, he will set the wholesale price of period one to his marginal cost, allowing the buyer to build up a large strategic inventory in combination with a high fixed fee that extracts the entire supply chain profit from the buyer. For the second period, he will commit himself to prohibitively high prices, forcing the buyer to accept the fixed fee of period one and to purchase the entire second period selling quantity in period one which will cause holding costs and prohibits the supply chain from achieving the first-best solution.

If holding costs are intermediate or high $\left(h^{TP} \geq \left(1 - 2/\sqrt{6}\right)a \approx 0.184a\right)$, the supplier will avoid an inventory build-up of the buyer and set the first-period wholesale price above his marginal costs, while charging marginal costs in the second period. However, although the supplier charges a fixed fee in both periods, he is unable to extract the entire profit from the buyer (i.e. also the buyer will achieve a small profit).

In the dynamic two-part tariff (i.e. no commitment), the supplier controls the inventory build-up of the buyer by charging a wholesale price above marginal costs in the first-period. The buyer will build up inventory and force the buyer to set a low fixed fee in the second period. The supplier is still able to extract the entire profit from the buyer, but the supply chain profit is lower in comparison to the first-best solution because of the suppliers pricing scheme (i.e. w_1^{TP} is above the supplier's marginal costs). More details can be found in Anand et al. (2008).

4 Experimental Analysis and Behavioral Insights on Strategic Inventory in Supply Chains

The theoretical analysis of Anand et al. (2008) summarized in chapter 3 has shown that strategic inventory can have a major impact on the decisions of supply chain members in multi-period interactions. Strategic inventory enables the buyer to dampen the monopoly power of her supplier. In the case of a simple wholesale price contract, the utilization of strategic inventory can lead to a reduction of double marginalization and, therefore, enhance the supply chain efficiency. In most cases (i.e. as long as holding costs are not relatively high) both supplier and buyer benefit from this enhancement.

Since most supply chain interactions in reality take place in multi-period settings and the wholesale price contract is commonly used in industry[25], the efficiency enhancing effects of strategic inventories may be good news for the economy. For the phenomenon to be effective, however, the players are required to demonstrate a high degree of strategic sophistication in their behavior. Given the extensive literature on behavioral biases in single-period supply chain interactions, however, it is not self-evident that theoretically predicted efficiency gains are behaviorally sustained in this type of multi-period interplay. Especially the frequently observed failure to identify profit maximizing order quantities or wholesale prices (Schweitzer and Cachon, 2000; Katok and Wu, 2009) and the tendency to consider fairness consequences of supply chain decisions (Cui et al., 2007; Loch and Wu, 2008; Pavlov and Katok, 2011) may behaviorally interfere with the theoretical predictions of Anand et al. (2008).

In this chapter, a laboratory experiment that allows testing for the empirical relevance of the concept of strategic inventories is presented. The laboratory investigation delivers overwhelmingly clear evidence for the behavioral relevance of strategic inventories and the efficiency enhancing effect that they have on the overall supply chain performance. Using a control treatment, in which strategic inventories are out of equilibrium, demonstrates that the subjects (management and economics undergraduates) of the laboratory investigation use the inventories in a strategically, sophisticated manner and not just because they are given the opportunity to do so.

The strong evidence that is found for the behavioral relevance of strategic inventories is surprising, given the interplay between strategic behavior and strategic uncertainty, which is inherent in this multi-period interaction. In equilibrium, both the supplier's

[25] Cachon (2003), p. 12

wholesale price and the buyer's order quantity in the first-period are greater than in the case without strategic inventories (Anand et al., 2008). Increasing both the price and the quantity, not only requires a clear understanding of the strategic situation on the side of both parties, but also a mutual trust in each other's strategic sophistication. Hence, for the equilibrium to be behaviorally relevant, both supplier and buyer must trust that the other party deliberates with a high degree of strategic sophistication and plays the equilibrium strategy. A substantial part of the literature on behavior in supply chains, however, shows that players may fail to optimize or fail to believe that their counterparts optimize (e.g. Schweitzer and Cachon (2000), Katok and Wu (2009), Özer et al. (2011), Croson et al. (2012)). In contrast to the persistent out of equilibrium behavior found in that literature, a high degree of behavioral stability that is very close to the equilibrium is observed. Hence, the results indicate that strategic inventories are a robust phenomenon of supply chain interaction, as long as holding costs are not prohibitively high.

While it is observed that strategic inventories are adopted whenever predicted by theory, they are significantly smaller than in equilibrium. By choosing smaller inventories the buyer can establish a more equitable distribution of the profits. As one explanation of this phenomenon, *buyer empowerment* is defined to be the possibility of reducing the inequity of the payoff distribution via inventory choices. It is shown that suppliers facing empowered buyers are willing to reduce average wholesale prices as long as they can keep their profits above a certain threshold. This behavioral effect leads to a supply chain performance that is even more enhanced than in the game theoretic prediction of Anand et al. (2008).

4.1 Literature Review

The underlying laboratory investigation contributes to the literature on the effects of strategic decision making in inter-temporal supply chains, by examining the behavioral validity and reliability of the game theoretic predictions. Recently, a rather large body of literature on the behavioral aspects of the newsvendor's problem (e.g. Keser and Paleologo (2004), Bolton et al. (2012)) and the bull-whip effect (e.g. Croson and Donuhue (2006), Croson et al. (2012)) has emerged, demonstrating the contribution of experimental research to a better understanding of strategic interaction in supply chains. The main findings of this literature can be summarized in several behavioral phenomena, each interfering in a different manner with the game theoretic predictions.

In the following, the observed behavioral phenomena are summarized and related to this study.

Frequently observed behavioral phenomena are concerns for fairness and reciprocity. The notion that fairness generally plays an important role in human interaction has been common knowledge in social sciences for centuries. But, an elaborate research of the concept and its consequences for economic performance only started after a series of early economic experiments had documented that concerns for fairness persistently affect economic behavior (e.g. Güth et al. (1982), Forsythe et al. (1994), Berg and Dickhaut (1995), Bolton (1991), Fehr et al. (1998)). The research has culminated in a number of theoretical papers modeling different facets of fairness, including a preference for equity in income distribution (Fehr and Schmidt (1999), Bolton and Ockenfels (2000)), a preference for reciprocal responses to acts of intentional kindness and spite (Rabin (1993), Dufwenberg and Kirchsteiger (2004)), a preference for increasing mutual benefits, or any combination of the preferences listed above (Charness and Rabin (2002), Falk and Fischbacher (2006)). Fairness concerns in ultimatum type games (as in the underlying laboratory study) have the following effects: First, the proposer's fairness concerns might lead to offers that are more equitable than offers in the absence of fairness concerns. Second, the receiver may reject offers that are perceived as unfair. The proposer may try to reduce this rejection risk by offering a larger share of profits.

While it is rather difficult to clearly separate the different facets of fairness preferences in supply chain settings,[26] it is important to note that in most cases all facets of the concern for fairness will have the same type of impact on behavior. Such concerns generally drive the wholesale prices down leading to a decrease in the payoff differences in the supply chain (Cui et al. (2007), Pavlov and Katok (2011)). Loch and Wu (2008) conduct an experimental study of supply chains with wholesale price contracts under deterministic, price sensitive demand. They show that the profits in all treatments are more evenly distributed than predicted by standard theory, because the suppliers set lower wholesale prices than predicted. Additionally, if an inter-personal tie has been created between the supplier and the buyer, the buyer tends to increase sales

[26] A supply chain game in terms of an ultimatum game can be interpreted as follows. The supplier is the proposer who offers a contract (e.g., a wholesale price contract). The supplier affects the buyer's share of profits by the contract terms, e.g, setting a lower wholesale price results in a higher share of profits for the buyer. The buyer is the receiver who determines the sales quantity in a double marginalization context. The buyer may lower the supplier's profits by choosing suboptimal low sales quantities.

boosting the overall efficiency gains. In another experimental study, Keser and Paleologo (2004) examine the behavior of supply chain members in a newsvendor setting under a wholesale price contract. They observe a tendency towards an equitable distribution of profits. As the buyers tend to terminate games with high wholesale prices, suppliers seem to voluntarily choose lower wholesale prices that split the profits approximately equal.

While the above mentioned studies cannot distinguish between the two underlying behavioral effects that drive suppliers' behavior (i.e. suppliers' fairness concerns and/or perception of contract failure risk), Katok and Pavlov (2013) as well as Katok et al. (2014) perform experiments highlighting that suppliers' behavior is mainly driven by incomplete information about the risk of contract rejection (i.e. suppliers fear contract rejection, if the wholesale prices are set too high). In addition, they show that the buyers' behavior is mostly driven by their fairness concerns (i.e. if the suppliers do not suffer from a rejection because they have an outside option, then the buyers' tendency to reject offers is significantly lower).

The underlying study of this chapter contributes to this literature by examining the influence of other regarding preferences such as fairness concerns in the context of a two-period supply chain interaction with strategic inventories.

4.2 Experimental Design and Procedure

The underlying laboratory study is based on the standard model by Anand et al. (2008) that was described in chapter 3 and consists of two treatments: A low cost treatment (LC) with holding cost of $h^{SM} = 4$ and a high cost treatment (HC) with holding costs of $h^{SM} = 42$. Both treatments have an identical, inverse demand function: $p_t^{SM}\left(q_t^{SM}\right) = 152 - 2 \cdot q_t^{SM}$. In contrast to the standard model, two modifications were conducted. First, the buyer does no longer have to decide about her selling quantities. Instead, the profit maximizing selling quantity is automatically chosen in each period. This modification was made in order to facilitate the decision problem of the buyer and to allow him to fully concentrate on her inventory decision.[27] Second, to resupply the buyer with an outside option, he is able to abort the game. In this case, both supplier and buyer would get zero profits. Technically, this solution is identical to the case in which

[27] Previous studies of the one-period model have shown mixed results regarding the selling quantity. Lim and Ho (2007) report slightly below profit maximizing end customer prices (i.e. above profit maximizing selling quantities). Loch and Wu (2008) report slightly above profit maximizing end customer prices (i.e. below profit maximizing selling quantities).

the buyer purchases zero items from the supplier.[28] The resulting theoretical predictions for the described parameters of the two experimental treatments are shown in Table 3.

In the low cost treatment, due to the relatively low inventory holding cost, the game theoretic model predicts a dynamic solution, with falling wholesale prices, a strategic inventory, and higher payoffs both for the supplier and the buyer, when compared to the static solution without strategic inventory. The distribution of supply chain payoffs is asymmetric in equilibrium, with about two-thirds going to the supplier and one-third to the buyer. In the high cost treatment, the relatively high holding cost prohibits the profitable adoption of a strategic inventory, so that the game theoretic model predicts a static solution with constant wholesale prices and order quantities. As in the other treatment, the distribution of supply chain payoffs is asymmetric in equilibrium, with about two-thirds going to the supplier and one-third to the buyer. Hence, while the treatments are very different concerning the strategic situation, they are very similar in the distribution of equilibrium payoffs. This similarity is important, because it guarantees that differences in the frequency of equilibrium play are not due to the ex-ante differences in the equilibrium payoff distributions.

Table 3: Theoretical Predictions

	Low Cost Treatment (LC) $h^{SM} = 4$		High Cost Treatment (HC) $h^{SM} = 42$	
	Period 1	Period 2	Period 1	Period 2
w_t^{SM*}	80	56	76	76
p_t^{SM}	116	104	114	114
q_t^{SM}	18	24	19	19
I^{SM*}	10		0	
π_S^{SM}	3,024 (66.55%)		2,888 (66.67%)	
π_B^{SM}	1,520 (33.45%)		1,444 (33.33%)	
π_{SC}^{SM}	4,544		4,332	

* decision variables in the experimental analysis

[28] An experimental examination of the two-period model with the full decision space (and no explicit outside option for the buyer) was recently conducted by Hartwig et al (2014). Enabling the buyers to choose their selling quantities seems to have only a minor effect on the general behavioral findings. Therefore, the presented findings of the experiment with automated selling decisions get supported by the results of the unlimited experiment.

The experiment was conducted at a German University using the software z-tree (Fischbacher (2007)). A total of 96 subjects, 24 suppliers and 24 buyers in each of the two treatments, participated in the experiment. At the outset of the experiment, each subject was randomly assigned either the role of a supplier or of a buyer. Subjects maintained their roles throughout the 15 decision rounds. Hence, a total of 360 games were played per treatment. The subjects were divided into matching groups of three suppliers and three buyers and randomly re-matched within their matching groups in every decision round to avoid reputation (i.e. repeated game) effects. As each matching group forms an independent observation, a total of 8 observations per treatment were used for the statistical analysis.

The instructions were handed out to the subjects upon arrival and were read aloud. Then, after a short individual re-reading time, the subjects had the possibility to ask questions that were answered privately. Communication between the subjects was prohibited. The subjects were then given a computerized comprehension quiz to ensure that they had fully understood the rules of the game. Subjects were paid the sum of their profits in all rounds in cash, immediately after the experiment.

The course of events in each decision round is displayed in Figure 12. First, the supplier determines the first-period wholesale price (w_1). Next, the buyer decides whether to terminate the game (corresponds to $q_1 = q_2 = I = 0$) or to continue. If the buyer terminates the game, the payoffs of both the buyer and the supplier are set to zero and the round ends immediately. If the buyer continues, she chooses the size of her strategic inventory (I) and the system automatically supplements the optimal sales quantity of the first-period (q_1). Providing the optimal quantity choices allows buyers to focus on their choice of the strategic inventories (I) that constitute the essential strategic element of the game. In period 2, the supplier sets his wholesale price (w_2) and the round ends with the automatic choice of the buyer's optimal second period sales quantity (q_2).

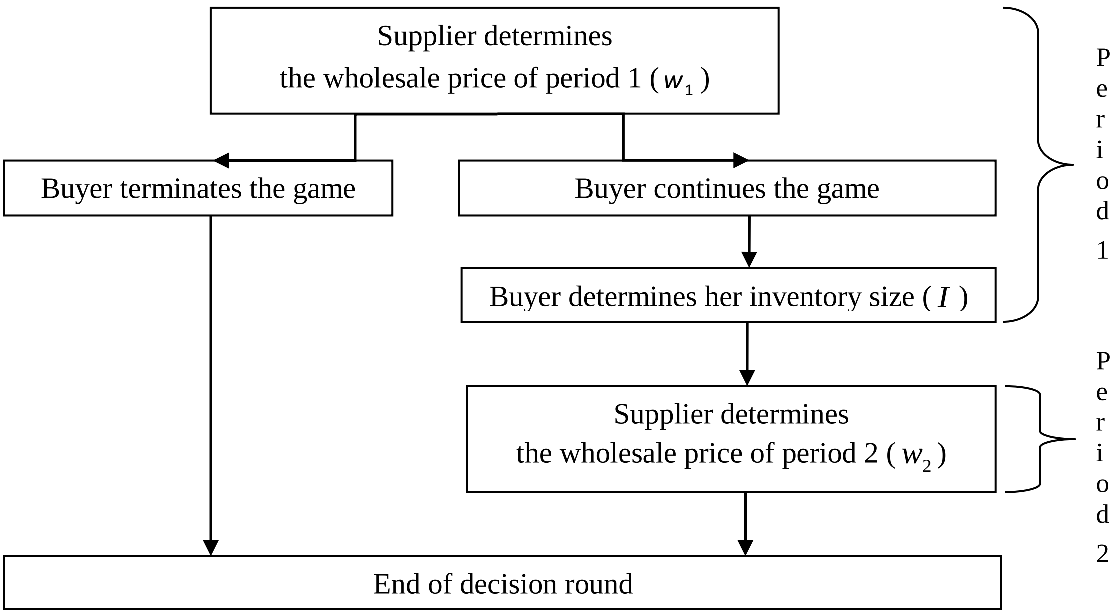

Figure 12: Sequence of Decisions

The suppliers may choose any wholesale price in the interval between 0 and 152. At the lower price bound the supplier earns nothing from selling his units and at the upper price bound buyers have no incentive to purchase any goods from the supplier.

The buyers decide to build up an inventory in the range between 0 and 38 units. The optimal response function of the buyer in (3.40) shows that even if both the wholesale price of the first-period and the inventory holding cost would be zero, quantities greater than 38 cannot be optimal. Hence, choosing values outside of the permitted intervals cannot be reasonable.

To further facilitate decision making, the subjects were provided with a profit calculator and a payoff table as decision support. The profit calculator displays the profits of both players for any combination of decision variables. The subjects could also use the payoff tables that displayed the profits for some integer value combinations of the decision variables. The subjects were informed that the tables do not contain payoff information for all possible values of the decision parameters and only serve as a guide, giving an overview of the payoff space. The instructions also point out that the onscreen profit calculator can be used to look up profits for any feasible combination of decision parameters. An English translation of the instructions for the treatment with low inventory holding costs and the payoff tables for both the low and the high cost treatment are contained in Appendix A.

4.3 Behavioral Hypotheses

The game implements a variant of ultimatum bargaining (Güth et al. (1982)), because the buyers can terminate the relation after receiving the supplier's offer in the first-period. This results in a risk of contract failure for the supplier as observed in many other behavioral supply chain studies (Keser and Paleologo (2004), Katok and Pavlov (2013), Katok et al. (2014)). The experimental literature of supply chains generally finds that the risk of contract failure can be alleviated by shifting payoffs from the proposer to the receiver. Hence, it can be conjectured that suppliers in HC and LC may reduce their first-period wholesale prices in order to reduce the risk of contract failure. In fact, the parameterization seems to make contract rejects more likely in HC than in LC, because in HC rejecting the contract is the only option buyers have to punish suppliers. Note that in the second period suppliers no longer have a risk of contract failure, because the experimental design only allows the payoff maximizing response of the buyers in the second period (i.e. buyers are given no rejection option in the second period; see: Figure 12). Hence, in line with Katok and Pavlov (2013) it is conjectured that due to the absence of contract failure risk suppliers will choose payoff maximizing second period wholesale prices. Finally, it is conjectured that the buyers in both HC and LC choose payoff maximizing inventory levels, because as second movers in the game they face no risk of contract failure. The conjectures regarding contract failure are summarized in the following behavioral hypothesis 1:

Behavioral Hypothesis 1 ("Contract failure risk"):

(1a) In HC and LC, the period 1 wholesale price is smaller than the equilibrium wholesale price.

(1b) In HC, the difference between observed and equilibrium period 1 wholesale prices is greater than in LC.

(1c) In HC and LC, the period 2 wholesale price is equal to the best response.

(1d) In HC and LC, the strategic inventories are equal to the best response levels.

The behavioral literature provides quite a bit of evidence on the effects of fairness in bilateral interactions. In both the LC and HC treatment, a supplier's concern for fairness leads to the choice of lower average wholesale prices. Such a decrease would dampen the effect of double marginalization and shift profits from the supplier to the buyer.

Note, that shifting payoffs to the buyer generally leads to a more equal distribution of profits, since the buyer earns considerably less in both treatments.

Since it is conjectured that lower first-period wholesale prices may either be caused by the risk of contract failure (as in hypothesis 1) or by fairness concerns (as described in hypothesis 2 below), the question is if differences in observations can be identified that support the one or the other hypothesis. A differentiation between the two hypotheses can be achieved by examining the difference between the wholesale prices in HC and LC. As stated in hypothesis 1b, if behavior is influenced by the risk of contract failure, a lower first-period wholesale prices in HC compared to LC should be observed. Such a difference is not expected, if behavior is guided by fairness concerns. In this case, first and second period wholesale prices that decrease the payoff differences between suppliers and buyers to the same extent in both treatments should be observed in the experiment.

The buyers' possibility of showing a concern for fairness is different in HC than in LC. In LC, a concern for fairness translates to lower levels of strategic inventory as long as the buyer's marginal loss is smaller than the supplier's marginal loss (see Appendix B). Going beyond this point would harm the buyer more than the supplier and, thus, increase payoff inequality. In contrast to the buyer in LC, it is expected that the buyers in HC are unable to reduce payoff inequality by reducing the strategic inventory, since they holds no inventory in equilibrium. The expected effects of fairness concerns on the interplay in the supply chains are summarized in hypotheses 2:

Hypothesis 2 ("Fairness Concerns"):

(2a) In HC and LC, the average wholesale price of both periods is smaller than the equilibrium wholesale price.

(2b) In HC and LC, the observed strategic inventory levels minimize the payoff difference between buyers and suppliers.

4.4 Results of the Laboratory Experiment

Table 4 displays the theoretical predictions of the strategic inventory model next to the observed mean and median values of the experiment for both treatments. The values for the individual and supply chain profits are shown excluding and including the cases, in which the game was terminated and both players earned zero (these values are displayed in brackets). Since decisions on the inventory size and the second period wholesale price are only made when the game is not terminated, only data from non-terminated games are contained in these aggregate values. In the low cost treatment (LC), 34 games of 360 were aborted, i.e. about 9%. The rate of termination in the high cost treatment (HC) was only about 4% (14 of 360 games were terminated), i.e. less than half the termination rate in LC.

Table 4: Theoretical Analysis vs. Experimental Results

	Low Cost Treatment (LC)			High Cost Treatment (HC)		
	Equilibrium	Median	Mean	Equilibrium	Median	Mean
Wholesale Price Period 1	80	70	70.11	76	76	73.91
Inventory	10	10	8.98	0	0	1.37
Wholesale Price Period 2	56	56	58.58	76	76	71.34
Profit Supplier	3,024	2,888 (2,884)	2,870.40 (2,599.30)	2,888	2,887.75 (2,885.38)	2,848.13 (2,729.46)
Profit Buyer	1,520	1,759.25 (1,744.63)	1,803.60 (1,633.33)	1,444	1,444 (1,444)	1,533 (1,470.08)
Profit Supply Chain	4,544	4,731.50 (4,686)	4,674 (4,232)	4,332	4,332 (4,332)	4,381 (4,199.54)

The observed mean and median values of the laboratory experiment are extremely close to the equilibrium predictions for both treatments. In fact, the empirical medians and the theoretical predictions are identical except for the first-period wholesale price and the corresponding profits in the LC treatment, in which the adoption of strategic inventories was observed to be almost perfectly in the range of values theoretically predicted. In the HC treatment, in which the holding costs are too high to allow for the adoption of a strategic inventory, almost no inventories are observed (the median of observed values is zero and the mean is just slightly greater than one). Furthermore, just as predicted by the game theoretic model, a substantial deviation between the median first and second period wholesale prices is observed in LC, but no difference between the two wholesale prices in HC. Overall, it seems that the strategic interaction that is incorporated in the game theoretic analysis of the strategic inventory game is an almost perfect predictor of observed behavior.

The only noteworthy deviation of the observed behavior from the game theoretic predictions is connected to a first-period wholesale price in LC that is significantly lower than the equilibrium price. Below, a more detailed analysis of the data is provided. The results are presented in four parts: the supplier's decision on the first-period wholesale price, the buyer's decision on the strategic inventory size, the supplier's decision on the second period wholesale price, and finally the resulting individual and overall supply chain profits. The statistical analyses are based on the independent observations (i.e. every observation is the median of 45 games).

4.4.1 Supplier's Period 1 Wholesale Price

Figure 13 displays the development of the median first-period wholesale prices over the 15 decision rounds for both treatments. In both treatments, the median first-period wholesale price starts about 8 or 9 points below the theoretical benchmark. While the median in HC quickly moves up to and then maintains at the equilibrium prediction by round 4, the observed first-period wholesale prices in LC tend to drop over time. Moreover, it is moving significantly further away from equilibrium towards the end of the experiment.[29] Hence, a clear difference between the behavior of the suppliers in HC and LC concerning the first-period wholesale prices is observed.

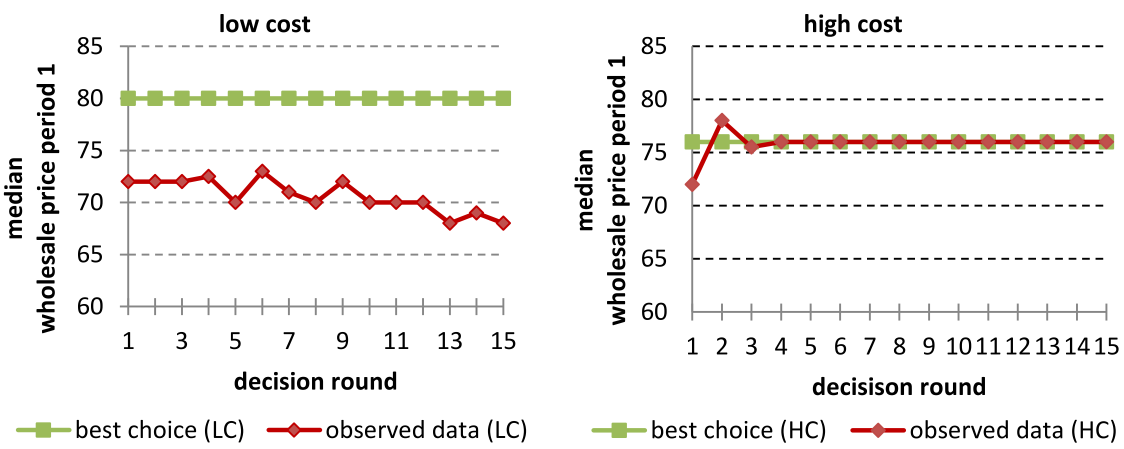

Figure 13: Development of Period 1 Wholesale Prices

[29] Comparing the observed values in the first five to those of the last five rounds using a sign test, the error probability is $p = 0.063$, two-tailed. Also, a significant negative correlation between the first-period wholesale prices and the decision round using Spearman's rank correlation measure ($r = -0.188$, $p < 0.001$) is found.

Since the lower than predicted first-period wholesale price in LC (i.e. lower than the dynamic equilibrium price and lower than the static outcome price, sign test, $p < 0.001$ and $p = 0.008$, two-tailed, correspondingly) only holds for LC, but not for HC, hypothesis 1a that states that the risk of contract failure leads to below equilibrium prices in both treatments cannot be supported. Also, no support for hypothesis 1b predicting lower first-period wholesale prices in HC than in LC can be found. Hence, the results show no clear evidence that suppliers' behavior is strongly affected by the risk of contract failure.

The other hypothesis that was derived from previous behavioral literature on supply chain interaction relates to fairness concerns. At first sight, it seems that the observed first-period wholesale prices in LC which are well below the equilibrium support hypothesis 2a. In fact, the data proves that payoff is shifted from the suppliers to the buyers via these low first-period prices in LC. This observation, however, is not sufficient to establish that the suppliers' behavior is driven by fairness concerns. If this were the case, lower than equilibrium first-period wholesale prices in HC (see hypothesis 2a) should also be observed. But, as depict in Figure 13 (right panel), this is not the case. Hence, there is also no clear evidence that suppliers' behavior is mainly affected by fairness concerns. The behavioral effects in supply chains with strategic inventories seem to go beyond fairness concerns and contract failure risk and, thus, beyond the concepts that are so far reported in the literature.

4.4.2 Buyer's Strategic Inventory Decision

On neither of the treatments a significant difference between the observed strategic inventory sizes and the corresponding equilibrium predictions is observed. Note, however, that while the equilibrium inventory size happens to be an empirical best response to the observed median first-period wholesale prices in HC, it is not an empirical best response to the much lower observed first-period wholesale prices in LC.

Table 5 displays the equilibrium, the empirical best response, and observed strategic inventory sizes for both treatments. The buyers' empirical best response to the suppliers' first-period wholesale prices in HC are very close to the equilibrium prediction. The best response to the median first-period wholesale price in HC is to adopt no strategic inventory (as in equilibrium), and the best response to the average first-period wholesale price in HC is to adopt a strategic inventory of size 1. For the buyers' inventory decision a median inventory size of zero and a mean inventory size of

1.37 is observed. For these values, no statistical differences between observed inventory sizes and the equilibrium predictions or the empirical best responses (sign test, p = 1, two-tailed) can be found. It seems that buyers in HC make strategic inventory decisions that are almost perfectly in line with the non-cooperative, payoff maximizing equilibrium of the game. Note, however, that buyers in HC have no leeway to reduce payoff inequality by reducing the level of strategic inventories, because inventory choices below zero are not feasible and inventory choices above zero increase inequality. Thus, observed behavior in HC neither contradicts hypothesis 1d nor 2b.

Table 5: Inventory Choices and Best Responses

Treatment	Equilibrium	Empirical Best Response		Observed Data	
		to Median	to Mean	Median	Mean
LC	10	13.67	13.33	10	8.98
HC	0	0	1	0	1.37

In the LC treatment, the observed inventory size is significantly larger than zero (sign test, p = 0.016, two-tailed). Hence, as predicted by theory, inventory is only utilized if the holding cost is sufficiently low. However, in contrast to the observation of best response inventory choices in HC, the observed inventory choices in LC cannot be considered as best responses to the observed first-period wholesale prices. Figure 14 displays the median best responses and the median observed inventory sizes in LC (left panel) and HC (right panel). It is evident that inventory choices are almost perfectly in line with best response in HC, but well below the best responses in LC.[30] On average, the chosen inventory size in LC is about 27 percent smaller than the best response. This difference is significant (sign test, p = 0.008, two-tailed). This lower than best response inventory choices contradicts hypothesis 1d, but strongly supports hypothesis 2b.

[30] Note that the observed median inventory size is always either an integer or exactly halfway between two integers. This makes the median slightly more volatile than the mean and explains the sudden, substantial drop we observe in the inventory size in round 6 of LC.

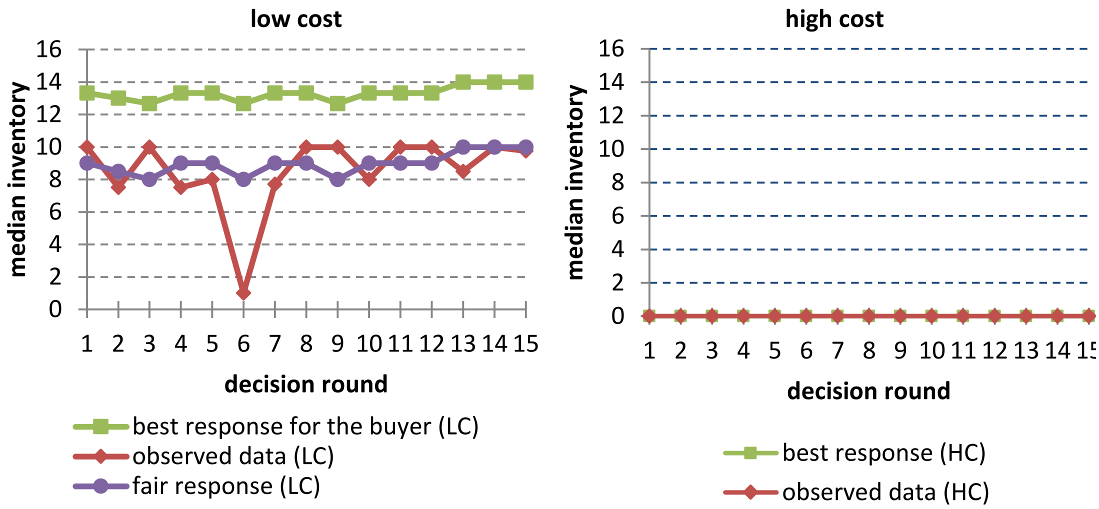

Figure 14: Development of Inventory Quantities

Figure 14 displays the median of the inventory size choices in LC that would minimize the payoff difference between buyers and suppliers ("fair response"). The derivation of the buyer's fair response inventory quantity is given in Appendix B. It seems that instead of choosing profit maximizing inventory sizes, buyers are choosing inventory sizes that equalize the profits of both supply chain partners as much as possible. As mentioned above, inventory choices in LC are significantly smaller than the best responses, but not significantly different from the "fair response" (sign test, p = 1, two-tailed).

4.4.3 Supplier's Period 2 Wholesale Price

In the last decision stage of the game, the supplier sets the second period wholesale price. Table 6 shows the equilibrium values for the second period wholesale prices, the empirical best responses and the observed data.

Table 6: Period 2 Wholesale Prices and Best Responses

Treatment	Equilibrium	Empirical Best Response		Observed Data	
		to Median	to Mean	Median	Mean
LC	56	56	58.04	56	58.58
HC	76	76	73.26	76	71.34

Neither a significant difference between the empirical best responses and the observed wholesale prices in the LC treatment (sign test, p = 1.000, two-tailed) nor in the HC treatment (sign test, p = 0.500, two-tailed) is observed. Hence, giving a best response to the inventory choice of the buyer seems to be the stable behavior of suppliers in the second period. The fact that the buyers' inventory choices are strategically affecting the wholesale prices of suppliers, as predicted in the analysis by Anand et al. (2008), is further supported by a correlation analysis. Calculating Spearman's rank correlation coefficients separately for both treatments, shows that the second period wholesale prices are significantly and negatively correlated to the inventory sizes both in LC ($r = -0.748$, $p < 0.001$) and in HC ($r = -0.496$, $p < 0.001$). Hence, in both treatments the suppliers take the inventory size of their buyer into account, choosing lower second period wholesale prices, the more inventory the buyer has acquired.

The observation of second period wholesale prices that are best responses to the buyers' inventory choice strongly support hypothesis 1c (i.e., in the absence of contract failure risk suppliers make profit maximizing contract offers). Yet, for testing the fairness hypothesis 2a, the second period wholesale prices in combination with the first-period wholesale prices need to be evaluated. Since in both treatments second period wholesale prices are observed which are fully in line with the theoretical predictions and since the first-period wholesale prices deviate from the theoretical predictions only in LC, it is not surprising that the observed average wholesale prices in HC are indistinguishable from the theoretical benchmarks (sign test, $p = 0.25$, two-tailed), while in LC the observed average wholesale prices are significantly smaller than the theoretical benchmarks (sign test, $p = 0.008$, two-tailed). However, as mentioned in section 4.3, if supplier's fairness concerns drive behavior, lower average wholesale prices would be expected in both treatments. Thus, hypothesis 2a needs to be rejected.

4.4.4 Supply Chain Performance and the Distribution of Payoffs

Table 7 shows an overview of the equilibrium and observed payoffs in both treatments. In HC, no difference between equilibrium and observed profits is found. In LC, however, there is clear evidence that suppliers have lower payoffs and that buyers have significantly higher payoffs than in equilibrium (sign test, $p = 0.008$, two-tailed). On the one hand, this implies less inequality in payoffs than in equilibrium. On the other hand, since the observed positive payoff difference for buyers is greater than the observed negative payoff difference for suppliers, the overall supply chain performance is significantly higher than in equilibrium (sign test, $p = 0.008$, two-tailed). Hence, summarized the behavioral effect of strategic inventories on supply chain performance is both efficiency and fairness enhancing.

Table 7: Comparison of the Profits of both Treatments

	Equilibrium		LC		HC	
	LC	HC	Median	Mean	Median	Mean
Profit Supplier	3,024 (66.55%)	2,888 (66.67%)	2,888	2,870.4 (61.41%)	2,887.75	2,848.13 (64.01%)
Profit Buyer	1,520 (33.45%)	1,444 (33.33%)	1,759.25	1,803.6 (38.59%)	1,444	1,533 (34.99%)
Profit Supply Chain	4,544	4,332	4,731.5	4,674	4,332	4,381.13

4.5 Discussion and Implications

Based on the standard model by Anand et al. (2008) that consists of a serial supply chain with two periods and price-sensitive demand, the first experimental test of the effect of strategic inventories on supply chain performance was conducted. In theory, if wholesale price contracts are used and holding costs are sufficiently low, building up a strategic inventory allows the buyer not only to increase her profit share, but also to enhance the overall supply chain performance by inducing a differentiated pricing behavior of the supplier. Verifying the predicted effects of strategic inventories in the field is extremely difficult, because supply chain interaction is generally embedded in a complex relationship that is simultaneously affected by numerous stochastic and strategic variables. The multiple confounds (i.e. parallel causal relationships) make the separation and identification of the effects of strategic inventories on prices and performance almost impossible in field data.

Using carefully devised controls and variations in the experiment to filter out all other causes and effects, the pure effect of strategic inventories can be observed. The laboratory results show a positive effect of strategic inventories on supply chain performance that qualitatively is perfectly in line with the theoretical results and that quantitatively goes even beyond the equilibrium prediction. As predicted theoretically no strategic inventories in the case of prohibitively high holding cost (HC treatment) is observed. Supply chain performance in this setting is neither enhanced nor impaired by the possibility of building up inventories. The case, in which the holding cost is sufficiently low (LC treatment), shows an extensive adoption of strategic inventories, leading to a strong enhancement of supply chain performance. In fact, the observed supply chain performance is even superior to the game theoretically expected enhancement, when the cost of holding inventory is low. It is shown that this enhancement of supply chain performance cannot be uniquely attributed to fairness preferences or perception of contract failure risk, because lower than predicted average wholesale prices are only seen in LC but not in HC. Two characteristics that explain why average wholesale prices are even lower than predicted in LC are identified.

The first striking difference in behavior across treatments is due to buyer empowerment (i.e. due to the fact that low cost buyers have a range of feasible inequality-reducing inventory choice alternatives that high cost buyers do not have). It turns out that low cost buyers frequently choose inventory sizes that are not payoff maximizing, but reduce the payoff inequality within the supply chain. It is possible that this buyer empowerment also influences the suppliers' perception of contract failure risk leading to lower than predicted average wholesale prices.

The second key difference between treatments is the additional distributional flexibility that results from the adoption of strategic inventories. It is shown that the seller's willingness to reduce the wholesale price in period 1 has clear limits. While many suppliers are willing to contribute the part of their payoffs exceeding the payoff that they would have achieved in the static solution (i.e. without a strategic inventory), hardly any suppliers are observed, who are willing to share their payoffs beyond this point. Hence, it seems that payoff in the static solution is a decisive benchmark, a focal point, for the suppliers' fairness concerns. A simple explanation why the static solution may be a natural focal point of the game is that it is the best outcome that can be enforced by suppliers (i.e. it is the maximin outcome). In fact, the existence of this focal point also explains why almost no sharing by the suppliers is seen in the high cost treatment. The highest achievable payoff for suppliers in that treatment is equal to the payoff in the static solution (i.e. at the level of the focal point). Securing payoffs at the focal point level, obviously, leaves no financial leeway for other-regarding behavior in the high cost treatment.

In sum, the theoretically predicted strategic interaction mostly dominates behavior in the conducted multi-period game. However, when the supply chain partners manage to cooperate and to generate a surplus, they tend to divide the surplus in a way that equalizes payoffs as also previously observed in the literature. The interesting new behavioral aspect, that is observed here, is that fairness only matters when both parties can actively contribute to generating a surplus. In the setting of the conducted experiment, the buyers can only contribute to the surplus, when their holding cost are low, allowing them to create and vary the size of their strategic inventories. Without the distributional flexibility that provides buyers with the strategic option to contribute to the joint surplus (i.e. without *buyer empowerment*) it is observed that suppliers see no reason to equalize payoffs in our setting.

The findings of the presented laboratory study have several implications for supply chain management. First, the findings suggest that when holding costs are reasonably low, inventories may (at least partially) be adopted for strategic reasons, both enhancing the supply chain performance and empowering the buyer. In other words, the results give strong empirical support to the theoretical findings of Anand et al (2008). Second, the results suggest that there may be behavioral effects that top off the purely strategic effect. Seeking a more equitable payoff distribution in the supply chain, the empowered buyers may harm the supply chain performance by choosing suboptimal small inventories. But this negative effect of buyer empowerment on supply chain performance is generally offset by the positive effect of the low first-period prices. Third, the results highlight that the positive effects on supply chain performance can only be achieved with some flexibility concerning the distribution of profits. There is evidence that the extent of profit sharing may strongly depend on focal points that emerge from the interaction situation and induce upper bounds for the willingness to share. Obviously, such focal points may be based on historical, legal, or cultural details of the interaction environment. The conducted study shows that they may also be based on strategic features of the interaction (e.g. a maximin outside option).

5 Strategic Inventory in Supply Chains with Horizontal Competition

In the standard model, the supply chain consists of only one supplier who sells goods to a single buyer. Hence, both supply chain members possess a high degree of monopoly power as they are only confronted with vertical competition. In real life scenarios, however, the supply chain structure often is more complex as described in the standard model, as supply chain members also compete horizontally against other suppliers or buyers. Horizontal competition within supply chains and the respective contract design for the single-period model have been examined extensively in literature. Some of the recent publications include Ingene and Parry (1995) who examine horizontal competition between multiple buyers that are not necessarily identical regarding their demand and cost function. They prove that a simple two-part tariff is unable to coordinate the supply chain if the supplier is forced to treat buyers identically (i.e. the supplier is not allowed to discriminate the buyers by offering differing, individual contracts). Only by using a menu of two-part tariffs the first-best solution can be achieved. However, the supplier does not always prefer such a menu over a non-coordinating contract. Hence, the first-best solution is not always achievable.

Bernstein and Federgruen (2003) extend the literature by considering the replenishment strategies of buyers within a supply chain with one supplier and multiple competing retailers. Their analysis features both price (Bertrand) and quantity (Cournot) competition between the buyers under linear and nonlinear pricing schemes. Moreover, they prove that the channel can be coordinated with nonlinear pricing schemes. Jain et al. (2001) consider a supply chain with a single supplier and multiple buyers under stochastic demand. In their setup, the buyers observe signals about the size of the demand. As these signals are private information the supplier implements advanced pricing mechanisms that nearly achieve channel coordination.

However, the described research does not take the effects of strategic inventory into account that may arise in multi-period interaction under vertical competition. Literature that combines both horizontal and vertical competition under multiple-period interaction is still rare. Expanding the standard model of Anand et al. (2008), the study of Desai et al. (2010) includes an analysis of a one supplier, two buyers supply chain. For the relevant case of competition between buyers their linear demand function with a parameter for competition intensity (based on the quadratic utility function of Shubik and Levitian (1980)) implements a Bertrand (i.e. price) competition.

In contrast to Desai et al. (2010) the underlying study of this chapter investigates selling quantity (Cournot) competition like it is also done by Viswanathan and Jang (2009) as well as by Viswanathan and Jang (2011).[31] Furthermore, this study generalizes prior research by considering an arbitrary number of buyers (Figure 15). The latter feature allows to analyze, to which extend strategic inventory plays a role if the supplier faces competing buyers and how the impact of strategic inventory is affected by reducing market power of the buyers via increasing their number.

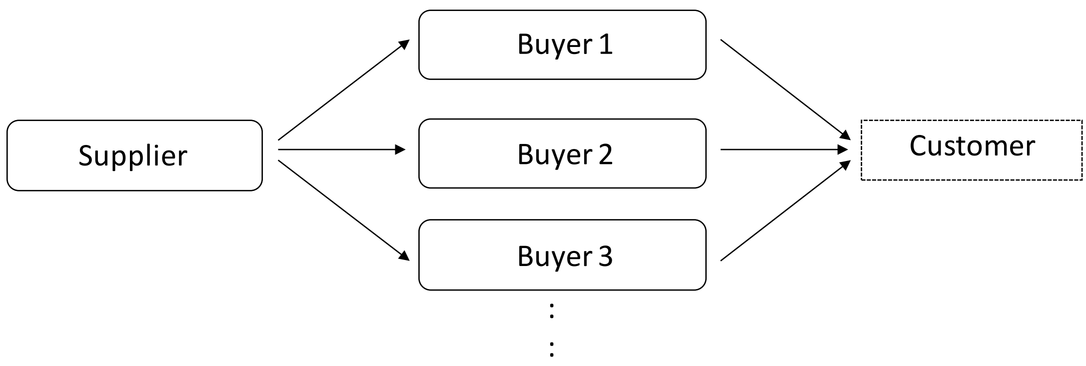

Figure 15: Horizontal Competition in Supply Chains

5.1 Model Description and Analysis

The model with horizontal competition ($e = HCM$) is based on the original two-period model of Anand et al. (2008). But, instead of selling to only one buyer, the supplier now has multiple identical buyers ($j,k = 1,..,N$) that can purchase items from him. These buyers then sell these goods on the same market (Figure 15). The supplier cannot discriminate between the different buyers and sets a universal wholesale price in each period (w_t^{HCM}). Each buyer (j), in turn, has to decide about his purchase ($Q_{j,t}^{HCM}$) and selling quantity ($q_{j,t}^{HCM}$). The price on the external market is described by a linear, deterministic demand function and depends on the sum of the overall sold units ($p_t^{HCM} = a - b \cdot \sum_{j=1}^{N} q_{j,t}^{HCM}$). By purchasing more items than selling, each buyer has the option to build up an inventory ($I_j^{HCM} = Q_{j,1}^{HCM} - q_{j,1}^{HCM}$) that she can use in the second period ($q_{j,2}^{HCM} = I_j^{HCM} + Q_{j,2}^{HCM}$). Holding inventory causes identical cost of h^{HCM} per unit for each

[31] The studies of Viswanathan and Jang (2009/ 2011) are a special case of this study with only two buyers. However, both of their (diverse) results cannot be confirmed by the presented study as inserting (N=2) into Table 8 and Table 10 shows.

buyer. At the end of the second period, unsold units have a salvage value of zero. Hence, the buyers will sell all of their goods at the end of period two. Also, just as in the standard model the production cost of the supplier as well as the handling costs of supplier and buyer are normalized to zero. Moreover, both supplier and buyers possess perfect information. Like in the standard model, still none of the classical reasons (see chapter 2) to build up inventory exist.

Therefore, the supplier's profit function is:

$$\begin{aligned}\pi_S^{HCM}\left(w_1^{HCM}, w_2^{HCM}\right) &= w_1^{HCM}\cdot\sum_{j=1}^{N}Q_{j,1}^{HCM} + w_2^{HCM}\cdot\sum_{j=1}^{N}Q_{j,2}^{HCM} \\ &= w_1^{HC}\cdot\left(\sum_{j=1}^{N}q_{j,1}^{HCM} + \sum_{j=1}^{N}I_j^{HCM}\right) + w_2^{HCM}\cdot\left(\sum_{j=1}^{N}q_{j,2}^{HCM} - \sum_{j=1}^{N}I_j^{HCM}\right)\end{aligned} \tag{5.1}$$

while the profit function of a single buyer (k) is described by:

$$\begin{aligned}\pi_{B,k}^{HCM}\left(q_{k,1}^{HCM}, I_k^{HCM}, q_{k,2}^{HCM}\right) &= p_1^{HCM}\left(\sum_{j=1}^{N}q_{j,1}^{HCM}\right)\cdot q_{k,1}^{HCM} - w_1^{HCM}\cdot Q_{k,1}^{HCM} - h^{HCM}\cdot\left(Q_{k,1}^{HCM} - q_{k,1}^{HCM}\right) \\ &\quad + p_2^{HCM}\left(\sum_{j=1}^{N}q_{j,2}^{HCM}\right)\cdot q_{k,2}^{HCM} - w_2^{HCM}\cdot Q_{k,2}^{HCM} \\ &= \left(a - b\cdot\sum_{j=1}^{N}q_{j,1}^{HCM}\right)\cdot q_{k,1}^{HCM} - w_1^{HCM}\cdot\left(q_{k,1}^{HCM} + I_k^{HCM}\right) - h^{HCM}\cdot I_k^{HCM} \\ &\quad + \left(a - b\cdot\sum_{j=1}^{N}q_{j,2}^{HCM}\right)\cdot q_{k,2}^{HCM} - w_2^{HCM}\cdot\left(q_{k,2}^{HCM} - I_k^{HCM}\right)\end{aligned} \tag{5.2}$$

Hence, the supply chain profit is:

$$\begin{aligned}\pi_{SC}^{HCM}\left(q_{k,1}^{HCM}, I_k^{HCM}, q_{k,2}^{HCM}\right) &= \pi_S^{HCM} + \sum_{k=1}^{N}\pi_{B,j}^{HCM} \\ &= \sum_{k=1}^{N}\left(a - b\cdot\sum_{j=1}^{N}q_{j,1}^{HCM}\right)\cdot q_{k,1}^{HCM} - h^{HCM}\cdot\sum_{k=1}^{N}I_k^{HCM} \\ &\quad + \sum_{k=1}^{N}\left(a - b\cdot\sum_{j=1}^{N}q_{j,2}^{HCM}\right)\cdot q_{k,2}^{HCM}\end{aligned} \tag{5.3}$$

The solution under individual optimization is conducted by backwards induction, the optimization starts with the selling quantity decision $q_{k,2}^{HCM}$ of any buyer k in the second period:

$$\frac{\partial \pi_{B,k}^{HCM}\left(q_{k,1}^{HCM}, I_k^{HCM}, q_{k,2}^{HCM}\right)}{\partial q_{k,2}^{HCM}} = a - b\left(\sum_{\substack{j=1\\j\neq k}}^{N} q_{j,2}^{HCM} + q_{k,2}^{HCM}\right) - bq_{k,2}^{HCM} - w_2^{HCM} = 0$$

$$q_{k,2}^{HCM} = \frac{a - b\left(\sum_{\substack{j=1\\j\neq k}}^{N} q_{j,2}^{HCM}\right) - w_2^{HCM}}{2b} \tag{5.4}$$

As every buyer is identical (i.e. $q_{j,2}^{HCM} = q_{B,2}^{HCM}\ \forall j$ with $q_{B,2}^{HCM}$ as single-buyer selling quantity) it holds that $\sum_{\substack{j=1\\j\neq k}}^{N} q_{j,2}^{HCM} = (N-1)\cdot q_{B,2}^{HCM}$ so that the selling quantity results in:

$$q_{B,2}^{HCM}\left(w_2^{HCM}\right) = \max\left(0,\ \frac{a - w_2^{HCM}}{(N+1)b}\right) \tag{5.5}$$

Hence, the total sold quantity in the second period is:

$$N\cdot q_{B,2}^{HCM}\left(w_2^{HCM}\right) = \sum_{j=1}^{N} q_{j,1}^{HCM}\left(w_1^{HCM}\right) = \frac{N}{(N+1)}\cdot\frac{a - w_2^{HCM}}{b} \tag{5.6}$$

The supplier integrates the buyers' reaction function for the overall selling quantity (5.6) into his profit function (5.1):

$$\pi_S^{HCM}\left(w_1^{HCM}, w_2^{HCM}\right) = w_1^{HCM}\cdot\left(\sum_{j=1}^{N} q_{j,1}^{HCM} + \sum_{j=1}^{N} I_j^{HCM}\right) + w_2^{HCM}\cdot\left(\frac{N}{(N+1)}\frac{a - w_2^{HCM}}{b} - \sum_{j=1}^{N} I_j^{HCM}\right) \tag{5.7}$$

Then, he determines the optimal wholesale price of the second period:

$$\frac{\partial \pi_S^{HCM}\left(w_1^{HCM}, w_2^{HCM}\right)}{\partial w_2^{HCM}} = \frac{N}{(N+1)}\cdot\frac{a - 2w_2^{HCM}}{b} - \sum_{j=1}^{N} I_j^{HCM} = 0 \tag{5.8}$$

$$w_2^{HCM}\left(\sum_{j=1}^{N} I_j^{HCM}\right) = \max\left(0,\ \frac{a - \left(\frac{N+1}{N}\right) b\cdot\sum_{j=1}^{N} I_j^{HCM}}{2}\right) \tag{5.9}$$

Hence, the buyers can influence the wholesale price setting of the supplier under horizontal competition. The impact of the inventory increases with an increase in buyers as $\lim_{N\to\infty}\frac{N+1}{N} = 1$. However, to obtain the absolute effect of an increase of horizontal competition also changes of the inventory size due to the increased competition have to be considered.

Next, considering their selling quantities from (5.5) the buyers anticipate the supplier's price setting (5.9) in their individual profit function:

$$\pi_{B,k}^{HCM}\left(q_{k,1}^{HCM}, I_k^{HCM}\right) = \left(a - b\cdot\sum_{j=1}^{N} q_{j,1}^{HCM}\right)\cdot q_{k,1}^{HCM} - w_1^{HCM}\cdot\left(q_{k,1}^{HCM} + I_k^{HCM}\right) - h^{HCM}\cdot I_k^{HCM}$$
$$+\left(a - \frac{N}{(N+1)}\cdot\frac{a + \left(\frac{N+1}{N}\right)b\cdot\sum_{j=1}^{N} I_j^{HCM}}{2}\right)\cdot\frac{a + \left(\frac{N+1}{N}\right)b\cdot\sum_{j=1}^{N} I_j^{HCM}}{2(N+1)b} \tag{5.10}$$
$$-\frac{a - \left(\frac{N+1}{N}\right)b\cdot\sum_{j=1}^{N} I_j^{HCM}}{2}\cdot\left(\frac{a + \left(\frac{N+1}{N}\right)b\cdot\sum_{j=1}^{N} I_j^{HCM}}{2(N+1)b} - I_k^{HCM}\right)$$

$$= \left(a - b\cdot\left(\sum_{\substack{j=1\\ j\neq k}}^{N} q_{j,1}^{HCM} + q_{k,1}^{HCM}\right)\right)\cdot q_{k,1}^{HCM} - w_1^{HCM}\cdot\left(q_{k,1}^{HCM} + I_k^{HCM}\right) - h^{HCM}\cdot I_k^{HCM}$$
$$+\left(a - \frac{N}{(N+1)}\cdot\frac{a + \left(\frac{N+1}{N}\right)b\cdot\left(\sum_{\substack{j=1\\ j\neq k}}^{N} I_j^{HCM} + I_k^{HCM}\right)}{2}\right)\cdot\frac{a + \left(\frac{N+1}{N}\right)b\cdot\left(\sum_{\substack{j=1\\ j\neq k}}^{N} I_j^{HCM} + I_k^{HCM}\right)}{2(N+1)b}$$
$$-\frac{a - \left(\frac{N+1}{N}\right)b\cdot\left(\sum_{\substack{j=1\\ j\neq k}}^{N} I_j^{HCM} + I_k^{HCM}\right)}{2}\cdot\left(\frac{a + \left(\frac{N+1}{N}\right)b\cdot\left(\sum_{\substack{j=1\\ j\neq k}}^{N} I_j^{HCM} + I_k^{HCM}\right)}{2(N+1)b} - I_k^{HCM}\right)$$

Each buyer k determines her optimal selling quantity as well as the optimal inventory size:

<u>selling quantity:</u>

$$\frac{\partial\pi_{B,k}^{HCM}\left(q_{k,1}^{HCM}, I_k^{HCM}\right)}{\partial q_{k,1}^{HCM}} = -bq_{k,1}^{HCM} + a - b\left(\sum_{\substack{j=1\\ j\neq k}}^{N} q_{j,1}^{HCM} + q_{k,1}^{HCM}\right) - w_1^{HCM} = 0 \tag{5.11}$$

$$q_{k,1}^{HCM}\left(w_1^{HCM}\right)=\max\left(0,\ \frac{a-\left(N-1\right)b\cdot q_{B,1}^{HCM}-w_1^{HCM}}{2b}\right) \tag{5.12}$$

<u>inventory size:</u>

$$\begin{aligned}\frac{\partial\pi_{B,k}^{HCM}\left(q_{k,1}^{HCM},I_k^{HCM}\right)}{\partial I_k^{HCM}}=&\frac{\left(N^3+N^2+N\right)a-2\left(N^3+N^2\right)\left(h^{HCM}+w_1^{HCM}\right)}{2N^2\left(N+1\right)}\\&+\frac{-\left(\sum_{\substack{j=1\\j\neq k}}^{N}I_j^{HCM}+I_k^{HCM}\right)bN^3-bN^3I_k^{HCM}-2\left(\sum_{\substack{j=1\\j\neq k}}^{N}I_j^{HCM}+I_k^{HCM}\right)bN^2}{2N^2\left(N+1\right)}\\&+\frac{-2bN^2I_k^{HCM}-bNI_k^{HCM}+\left(\sum_{\substack{j=1\\j\neq k}}^{N}I_j^{HCM}+I_k^{HCM}\right)b}{2N^2\left(N+1\right)}=0\end{aligned} \tag{5.13}$$

$$I_k^{HCM}\left(w_1^{HCM}\right)=\max\left(\begin{array}{c}0,\ \dfrac{\left(N^3+N^2+N\right)a-2\left(N^3+N^2\right)\left(h^{HCM}+w_1^{HCM}\right)}{\left(N+1\right)\left(2N^2+2N-1\right)b}\\[2ex]-\dfrac{\left(N^3+2N^2-1\right)b\cdot\sum_{\substack{j=1\\j\neq k}}^{N}I_j^{HCM}}{\left(N+1\right)\left(2N^2+2N-1\right)b}\end{array}\right) \tag{5.14}$$

As the buyers are identical, (5.12) and (5.14) can be generalized to:

$$q_{j,1}^{HCM}\left(w_1^{HCM}\right)=\max\left(0,\ \frac{a-w_1^{HCM}}{\left(N+1\right)b}\right) \tag{5.15}$$

with:

$$\sum_{j=1}^{N}q_{j,1}^{HCM}\left(w_1^{HCM}\right)=\max\left(0,\ \frac{N}{\left(N+1\right)}\frac{a-w_1^{HCM}}{b}\right) \tag{5.16}$$

and:

$$I_j^{HCM}\left(w_1^{HCM}\right)=\max\left(0,\ \frac{\left(N^2+N+1\right)a}{N\left(N+1\right)\left(N+2\right)b}-\frac{2\left(w_1^{HCM}+h^{HCM}\right)}{\left(N+2\right)b}\right) \tag{5.17}$$

with:

$$\sum_{j=1}^{N} I_j^{HCM}\left(w_1^{HCM}\right)=\max\left(0,\ \frac{\left(N^2+N+1\right)}{\left(N^2+3N+2\right)}\cdot\frac{a}{b}-\frac{N}{(N+2)}\cdot\frac{2\left(w_1^{HCM}+h^{HCM}\right)}{b}\right) \quad (5.18)$$

Like in the standard model, the supplier decides with his first-period wholesale price choice whether the buyer will build up inventory or not. According to (5.17) a buyer will only build up inventory if

$$w_1^{HCM}<\frac{\left(N^2+N+1\right)a-2N(N+1)h^{HCM}}{2N(N+1)}\ . \quad (5.19)$$

Case 1: dynamic solution (inventory is built up in equilibrium)

The supplier integrates his second period price decision from (5.9) and the buyers' first (5.16), (5.18) and second (5.6) period reaction functions into his profit function (5.1):

$$\begin{aligned}\pi_S^{HCM}\left(w_1^{HCM}\right)=&\frac{\left(N^2-2N+1\right)a^2-4\left(2N^4+8N^3+7N^2\right)\cdot\left(w_1^{HCM}\right)^2}{4N(N+2)^2(N+1)b}\\&+\frac{4\left(2N^4+8N^3+7N^2+1N\right)a\cdot w_1^{HCM}-8\left(N^3+N^2\right)h^{HCM}\cdot w_1^{HCM}}{4N(N+2)^2(N+1)b}\\&+\frac{+4\left(N^4+2N^3+N^2\right)\cdot\left(h^{HCM}\right)^2+4\left(N^3-N\right)a\cdot h^{HCM}}{4N(N+2)^2(N+1)b}\end{aligned} \quad (5.20)$$

He then determines the optimal first-period wholesale price:

$$\frac{\partial\pi_S^{HCM}\left(w_1^{HCM}\right)}{\partial w_1^{HCM}}=\frac{\left(2N^3+8N^2+7N\right)a-2\left(2N^3+8N^2+7N\right)w_1^{HCM}-2\left(N^2+N\right)h^{HCM}}{(N+2)^2(N+1)b}=0 \quad (5.21)$$

$$w_1^{HCM}=\frac{(N+1)\left(2N^2+6N+1\right)}{2N\left(2N^2+8N+7\right)}\cdot a-\frac{N+1}{2N^2+8N+7}\cdot h^{HCM} \quad (5.22)$$

For the dynamic solution (5.19) must be fulfilled in order to be feasible. Inserting the optimal first-period wholesale price (5.22) into the buyers' inventory respond function (5.17) shows that inventory will be built up as long as:

$$h^{HCM}<\frac{a}{2N(N+1)} \quad (5.23)$$

A summary of the solution with strategic inventory (dynamic solution) can be found in Table 8 where the values of all variables are described as functions of the number of buyers.[32]

In the case of only one buyer ($N=1$), the maximum holdings cost under which strategic inventory got utilized was quite high ($h^{SM} < a/4$ or $\frac{h^{SM}}{w_1^{SM}} < \frac{h^{SM^{\max}}}{w_1^{SM}\left(h^{SM^{\max}}\right)} = \frac{a/4}{a/2} = 50\%$). As (5.23) shows, the upper boundary for the holding cost shrinks, if the number of buyers in the supply chain increases. Figure 16 shows the development of the maximum holding cost rate for a build-up of strategic inventory:

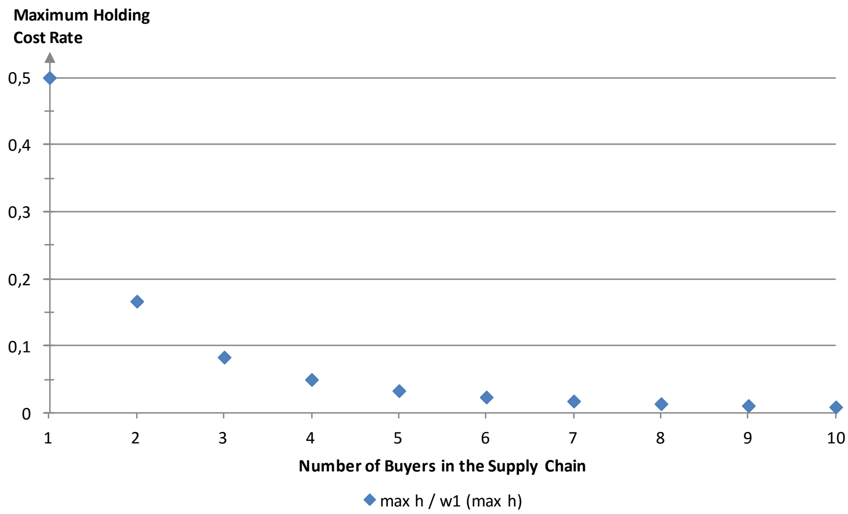

Figure 16: Maximum Holding Cost Rate for Utilization of Strategic Inventory

Starting at a relatively high level the upper bound for the holding costs decreases by two-thirds of the previous value as soon as a second buyer is added to the supply chain. Already with only 4 buyers in the supply chain the holding cost rate has to be below 5%, in order to lead to a build-up of strategic inventory. At the same time, even if holding costs are constant, the total amount of strategic inventory also decreases with an increase of buyers (see Table 8). Hence, both the occurrence of strategic inventory and its impact on supply chain performance decrease with an increase of horizontal competition. Generally, with an increase of the number of buyers, they continue to lose their monopoly power within the supply chain. Hence, the buyers surcharge on the supplier's wholesale price and their inventory size will decrease under an increasing

[32] As the standard model by Anand et al (2009) is the special case of $N=1$ within the horizontal competition model, the results of Table 1 can be reproduced from Table 8 and Table 9 by setting $N=1$.

number of buyers. As a result, the solution converges towards the first-best solution with the supplier earning the entire supply chain profit.

Table 8: Dynamic Solution under Horizontal Competition (Decision Variables)

	Dynamic Solution: $h^{HCM} < \frac{a}{2N(N+1)}$	
	Supplier/ Single Buyer j	Sum/ Global
w_1^{HCM}	$\frac{(2N^3+8N^2+7N+1)a-(2N^2+2N)h^{HCM}}{2N(2N^2+8N+7)}$	-
$q_{j,1}^{HCM}$	$\frac{(2N^3+8N^2+7N-1)a+(2N^2+2N)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$	$\frac{(2N^4+8N^3+7N^2-N)a+(2N^3+2N^2)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$
$Q_{j,1}^{HCM}$	$\frac{(2N^3+8N^2+11N+5)a-(8N^3+18N^2+10N)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$	$\frac{(2N^4+8N^3+11N^2+5N)a-(8N^4+18N^3+10N^2)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$
p_1^{HCM}	$\frac{(2N^3+12N^2+23N+15)a-(2N^2+2N)h^{HCM}}{2\cdot(N+1)(2N^2+8N+7)}$	-
I_j^{HCM}	$\frac{(4N+6)a-(8N^3+20N^2+12N)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$	$\frac{(4N^2+6N)a-(8N^4+20N^3+12N^2)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$
w_2^{HCM}	$\frac{(2N^3+8N^2+5N-3)a+(4N^3+10N^2+6N)h^{HCM}}{2N(2N^2+8N+7)}$	-
$q_{j,2}^{HCM}$	$\frac{(2N^3+8N^2+9N+3)a-(4N^3+10N^2+6N)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$	$\frac{(2N^4+8N^3+9N^2+3N)a-(4N^4+10N^3+6N^2)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$
$Q_{j,2}^{HCM}$	$\frac{(2N^3+8N^2+5N-3)a+(4N^3+10N^2+6N)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$	$\frac{(2N^4+8N^3+5N^2-3N)a+(4N^4+10N^3+6N^2)h^{HCM}}{2N(N+1)(2N^2+8N+7)b}$
p_2^{HCM}	$\frac{(2N^3+12N^2+21N+11)a+(4N^3+10N^2+6N)h^{HCM}}{2(N+1)(2N^2+8N+7)}$	-

Table 9: Dynamic Solution under Horizontal Competition (Profits)

	Dynamic Solution: $h^{HCM} < \frac{a}{2N(N+1)}$	
	Supplier/ Single Buyer j	Sum/ Global
$\pi_{B,j}^{HCM}$	$\frac{(4N^6+32N^5+96N^4+128N^3+63N^2-6N-7)a^2}{2N^2(2N^2+8N+7)^2(N+1)^2 b} - \frac{(8N^6+40N^5+72N^4+64N^3+38N^2+14N)ah^{HCM}}{2N^2(2N^2+8N+7)^2(N+1)^2 b} + \frac{(24N^6+120N^5+224N^4+184N^3+56N^2)(h^{HCM})^2}{2N^2(2N^2+8N+7)^2(N+1)^2 b}$	$\frac{(4N^6+32N^5+96N^4+128N^3+63N^2-6N-7)a^2}{2N(2N^2+8N+7)^2(N+1)^2 b} - \frac{(8N^6+40N^5+72N^4+64N^3+38N^2+14N)a\cdot h^{HCM}}{2N(2N^2+8N+7)^2(N+1)^2 b} + \frac{(24N^6+120N^5+224N^4+184N^3+56N^2)\cdot(h^{HCM})^2}{2N(2N^2+8N+7)^2(N+1)^2 b}$
π_{S}^{HCM}	$\frac{(2N^4+8N^3+7N^2+1)a^2}{2N(2N^2+8N+7)(N+1)b} - \frac{(4N^2+4N)a\cdot h^{HCM}}{2N(2N^2+8N+7)(N+1)b} + \frac{(4N^4+8N^3+4N^2)\cdot(h^{HCM})^2}{2N(2N^2+8N+7)(N+1)b}$	-
π_{SC}^{HCM}		$\frac{(4N^6+40N^5+156N^4+300N^3+291N^2+122N+9)a^2}{4(N+1)^2(2N^2+8N+7)^2 b} - \frac{(8N^5+48N^4+120N^3+161N^2+126N+42)a\cdot h^{HCM}}{4(N+1)^2(2N^2+8N+7)^2 b} + \frac{(8N^6+80N^5+268N^4+412n^3+300N^2+84N)\cdot(h^{HCM})^2}{4(N+1)^2(2N^2+8N+7)^2 b}$

<u>Case 2: static solution (no inventory is built up in equilibrium)</u>

If holding costs are prohibitively high, the buyer will not build up inventory in equilibrium. Hence, the two periods are decoupled and decisions will be identical in both periods (i.e. it is sufficient to analyze the solution for the one period model and apply the decisions to both periods). As the reaction functions of the second period have already been derived, the static solution can be simply obtained by inserting $I_j^{HC}=0$ into the suppliers' second period reaction function (5.9). Then, the obtained second period wholesale price can be inserted into the buyers' second period reaction function (5.5). As the decisions are identical in both periods, the first-period decisions can be obtained by copying the second period decisions. The complete static solution is summarized in Table 10:

Table 10: Static Solution under Horizontal Competition

	Static Solution $h \geq \frac{a}{2N(N+1)}$			Static Solution $h \geq \frac{a}{2N(N+1)}$	
	Supplier/ Single Buyer j	Sum/ Global		Supplier/ Single Buyer j	Sum/ Global
w_1^{HCM}	$\frac{a}{2}$	-	$q_{j,2}^{HCM}$	$\frac{a}{2(N+1)b}$	$\frac{N \cdot a}{2(N+1)b}$
$q_{j,1}^{HCM}$	$\frac{a}{2(N+1)b}$	$\frac{N \cdot a}{2(N+1)b}$	$Q_{j,2}^{HCM}$	$\frac{a}{2(N+1)b}$	$\frac{N \cdot a}{2(N+1)b}$
$Q_{j,1}^{HCM}$	$\frac{a}{2(N+1)b}$	$\frac{N \cdot a}{2(N+1)b}$	p_2^{HCM}	$\frac{(N+2)a}{2(N+1)}$	-
p_1^{HCM}	$\frac{(N+2)a}{2(N+1)}$	-	$\pi_{B,j}^{HCM}$	$\frac{a^2}{2(N+1)^2 b}$	$\frac{N \cdot a^2}{2(N+1)^2 b}$
I_j^{HCM}	0	0	π_S^{HCM}	$\frac{N \cdot a^2}{2(N+1)b}$	-
w_2^{HCM}	$\frac{a}{2}$		π_{SC}^{HCM}	-	$\frac{N(N+2)a^2}{2(N+1)^2 b}$

5.2 Impact of Strategic Inventory under Horizontal Competition

Whether the supply chain members as well as the entire supply chain benefit from the possibility to use strategic inventory depends on the one hand on the level of the holding cost (h^{HCM}) and on the other hand on the degree of horizontal competition (N). Anand et al (2008) have shown that with only one buyer in the supply chain, the supplier is always better off under the dynamic solution (i.e. the supplier will prefer the dynamic solution as long as holding costs are not prohibitively high). From the perspective of the buyer and the entire supply chain, profits are mostly higher under the dynamic solution. However, for very high holding costs there exists a small area in which profits are higher under the static solution (see Figure 9, Figure 10 and Figure 11).

This difference regarding the preference for one solution or the other (i.e. dynamic versus static) disappears as soon as horizontal competition exists in the supply chain. Still, the supplier is always better off under the dynamic solution, but as soon as a second buyer exists within the supply chain, a buyer also always prefers the dynamic solution.[33] Hence, under horizontal competition and wholesale price setting, all supply chain members and therefore the entire supply chain benefit from strategic inventory.

In Figure 17 the static solution gets compared to the best case scenario (zero holding cost) of the dynamic solution. Hence, Figure 17 shows the maximum advantage due to the utilization of strategic inventory.

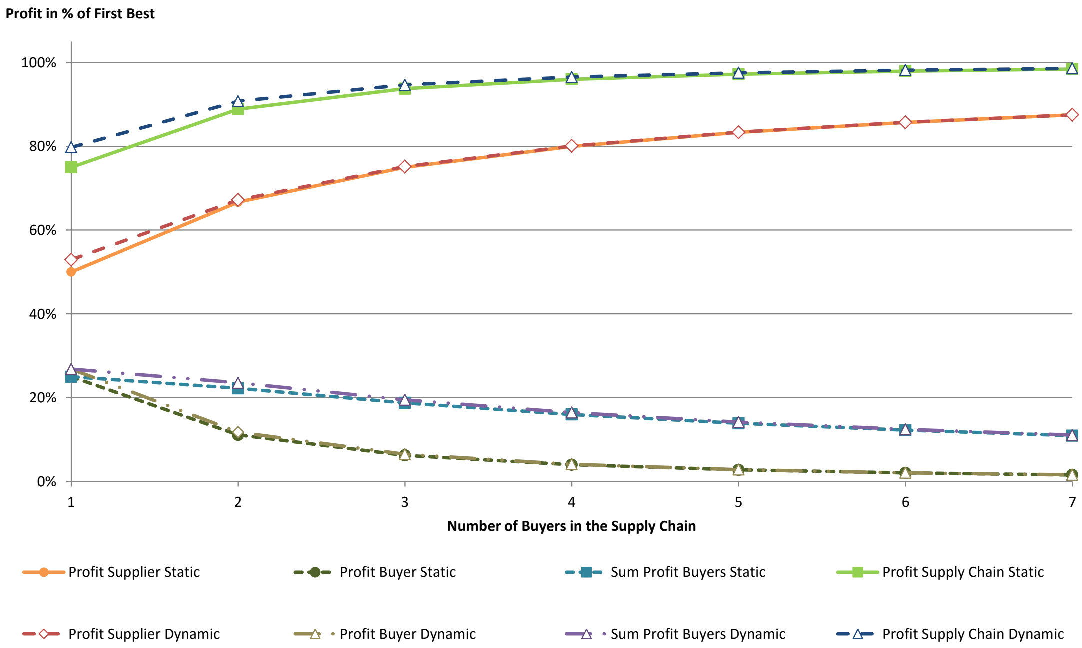

Figure 17: Profit Comparison Static vs. Dynamic under Horizontal Competition

It is straightforward to see that the benefit due to strategic inventory decreases, if horizontal competition increases. Already for the case of only four buyers the differences between static and dynamic solution are almost marginal. Hence, strategic inventory plays a significant role, if horizontal competition is weak (i.e. if the buyers have strong monopoly power). However, in order to evaluate this result it has to be kept in mind that the scope for improvements decreases, if more buyers enter the supply chain. In case of only one buyer, the supply chain profit of the static solution corresponds to 75% of the first-best solution. Hence, by moving from the static to the fist best solution the supply chain profit can be increased by one third. For the example of four suppliers, the static solution already reaches 96% of the first-best supply chain outcome and is, therefore, even higher than the dynamic solution in the case of only one buyer (79.76%). Hence, the improvement of the dynamic over the static solution should

[33] Comparisons between the static and dynamic solution for the buyer’s and supplier’s profits can be found in Appendix C.

also be measured considering the respective potential for improvement of the static solution (i.e. in comparison to the dead weight loss):

$$Relative\ Improvement(N) = \frac{\pi_{SC}^{HCM/Dynamic}(N) - \pi_{SC}^{HCM/Static}(N)}{\pi_{SC}^{FB} - \pi_{SC}^{HCM/Static}(N)} \quad (5.24)$$

The relative improvements in the best case scenario ($h^{HCM} = 0$) are given in Table 11. In the situation without horizontal competition the absolute difference between the efficiency level of the static and the dynamic solution is about 4.76%. This corresponds to a deadweight loss reduction of about 19%. For 6 buyers within the supply chain, the absolute difference is only about 0.21%. However, this still means a deadweight loss reduction of about 10.32%. This approach shows that the diminishing effect of strategic inventory is partly caused by the reduction of the double marginalization effect due to the increased horizontal competition. If only the reduction of dead weight lost is considered, the impact of strategic inventory still reduces, but at a much smaller rate.

Table 11: Deadweight Loss Reduction due to Strategic Inventory

Number of Buyers	1	2	3	4	5	6
Deadweight Loss of the Static Solution	25.00%	11.11%	6.25%	4.00%	2.78%	2.04%
Absolute Improvement of the Dynamic Solution	4.76%	1.86%	0.91%	0.51%	0.32%	0.21%
Relative Improvement	19.03%	16.75%	14.62%	12.87%	11.47%	10.32%

In summary, these results show that strategic inventory only plays a major role if the supply chain profit is strongly impaired by double marginalization, because of high monopoly power of the buyers. As soon as more buyers enter the supply chain, the double marginalization effect gets reduced. Therefore, on the one hand, the impact of strategic inventory gets lower and, on the other hand, the maximum value of holding costs, for which strategic inventory gets applied, reduces.

6 Strategic Inventory under Interest Rate Approaches

In the standard model of Anand et al. (2008) inventory holding costs are modeled as cost per unit and period. However, in practice a major fraction of the inventory holding cost depends on the value of the stored goods (e.g. opportunity cost due to locked capital, cost for insurance).[34] As long as the purchasing costs of the buyer (i.e. the wholesale price) are constant it does not make a difference whether holding costs are modeled as per unit or as value costs (i.e. they can be transferred into each other without impacting the solution).

However, in the dynamic case of the standard model, the supplier increases the first-period wholesale price (w_1^{SM}) in comparison to the static case. Hence, the buyer's first-period purchasing price per unit is not constant and, moreover, it directly depends on the holding cost parameter (h^{SM}) as $w_1^{SM} = \left(9a - 2h^{SM}\right)/17$. In practice, however, higher purchasing cost would likely cause higher holding costs, which is not considered in the standard model with its fixed holding cost parameter.

To consider price dependent inventory costs, two different approaches may be considered. In the inventory interest (IIR) model ($e = IIR$), the buyer's inventory costs are determined as a percentage of her purchasing costs by an interest rate parameter $\left(i^{IIR}\right)$. Hence, in contrast to the standard model (h^{SM}) the per unit holding costs of the buyer are no longer constant, but instead depend on the supplier's first-period wholesale price choice: $h^{IIR} = i^{IIR} \cdot w_1^{IIR}$.

While the IIR model only considers the opportunity cost of the capital that is locked due to inventory, the discounted cash flow approach extends the consideration of opportunity costs to all cash flows. The time value of money is considered by discounting future revenues. More specifically, in the discounted cash flow (DCF) model ($e = DCF$), all future payments are discounted by multiplying them with the factor $\left(\frac{1}{1+i^{DCF}}\right)^{t-1}$ where i^{DCF} stands for a general interest rate parameter. Hence, both supplier and buyer have a higher valuation for profits that are made in the first-period. As the interest rate of the DCF model only covers the part of holdings costs that are caused by the date of the realization of the cash flows, an additional per unit holding

[34] Nahmias (2009) pp. 204-205

costs rate (h^{DCF}) is used in the DCF model. This per unit holding costs rate covers the residual, out of pocket costs of holding inventory and consist of buyer's expenses for holding inventory that are not transfer payments towards the supplier (e.g. costs for insurances, warehouse operating costs). In the standard model, the increased first-period wholesale price caused the buyer to sell fewer goods in the first-period than in a single-period model. Thus, the buyer's profit of the first-period does not only get reduced by the investment in inventory, but additionally by the lower realization of profits due to selling activities. In the second period, however, the profit of the buyer is much higher than in the single-period model. This is caused by the fact that the selling quantity is higher due to a lower second period wholesale price. Moreover, the buyer can use her inventory, which has zero purchasing costs in the second period, to partly cover the customers demand. The supplier, in turn, also suffers from the lower first-period selling quantity. However, his first-period profit is higher than in a single-period model as the buyer invests into building up an inventory. In turn, his second period profit is lower than in the single-period model.

It is important to note that a mathematically equivalent[35] discounted cash flow approach was already conducted by Desai et al. (2010). In this study, the results of the DCF model get compared to the IIR model and the impact of the discount rate on the supplier's decisions gets examined in more detail.

6.1 Strategic Inventory under the Inventory Interest Rate Approach

In this model, the inventory holding costs are no longer independent of the purchasing cost. An increase of the first-period wholesale price now also leads to an increase of the holding costs. In comparison to the standard model, the supplier, therefore, also influences the buyer's holding cost with his first-period wholesale price decision.

Like in the standard model three possible solutions (first-best, static, dynamic) have to be considered:

The first-best solution serves as a benchmark. As holding inventory still causes a deadweight loss, no inventory is built up in the first-best solution. Hence, the change regarding the modelling of the holding cost parameter does not influence the first-best

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[35] Their ρ parameter corresponds to $\frac{1}{1+i^{DCF}}$ that is used in the following. Hence, the presented deduction of this section is mathematically identical to the prior analysis of Desai et al. (2010).

solution. Therefore, the first-best solution of the IIR model is identical to the solution of the standard model.

The static solution can be enforced by the supplier by using a commitment contract and automatically occurs if holding costs are prohibitively high. It is also identical to the solution of the standard model as per definition no inventory is built up. Hence, only the change of the dynamic solution and the maximum level of the interest rate that leads to the dynamic solution have to be investigated.

As the supplier does not carry any holding costs, his profit function remains the same as in the standard model (3.29):

$$\pi_S^{IIR}\left(w_1^{IIR}, w_2^{IIR}\right) = w_1^{IIR} \cdot \left(q_1^{IIR} + I^{IIR}\right) + w_2^{IIR} \cdot \left(q_2^{IIR} - I^{IIR}\right) \tag{6.1}$$

The buyers profit function changes regarding the inventory holding costs:

$$\begin{aligned}\pi_B^{IIR}\left(q_1^{IIR}, I^{IIR}, q_2^{IIR}\right) = &\left(a - b \cdot q_1^{IIR}\right) \cdot q_1^{IIR} - w_1^{IIR} \cdot \left(q_1^{IIR} + I^{IIR}\right) - i^{IIR} \cdot w_1^{IIR} \cdot I^{IIR} \\ &+ \left(a - b \cdot q_2^{IIR}\right) \cdot q_2^{IIR} - w_2^{IIR} \cdot \left(q_2^{IIR} - I^{IIR}\right).\end{aligned} \tag{6.2}$$

Also, the supply chain profit changes due to the change to price-dependent holding costs:

$$\pi_{SC}^{IIR}\left(q_1^{IIR}, I^{IIR}, q_2^{IIR}\right) = \left(a - b \cdot q_1^{IIR}\right) \cdot q_1^{IIR} - i^{IIR} \cdot w_1^{IIR} \cdot I^{IIR} + \left(a - b \cdot q_2^{IIR}\right) \cdot q_2^{IIR}. \tag{6.3}$$

6.1.1 Analysis of the Inventory Interest Rate Model

Again, the solution can be derived by backwards induction. First, the buyer's reaction function in period two has to be identified:

$$\frac{\partial \pi_B^{IIR}\left(q_1^{IIR}, I^{IIR}, q_2^{IIR}\right)}{\partial q_2^{IIR}} = a - 2bq_2^{IIR} - w_2^{IIR} = 0 \tag{6.4}$$

$$q_2^{IIR}\left(w_2^{IIR}\right) = \frac{a - w_2^{IIR}}{2b} \tag{6.5}$$

The supplier integrates the buyer's second period reaction function (6.5) into his profit function

$$\pi_S^{IIR}\left(w_1^{IIR}, w_2^{IIR}\right) = w_1^{IIR} \cdot \left(q_1^{IIR} + I^{IIR}\right) + w_2^{IIR} \cdot \left(\frac{a - w_2^{IIR}}{2b} - I^{IIR}\right) \tag{6.6}$$

and derives his optimal reaction function to the buyer's inventory size:

$$\frac{\partial \pi_S^{IIR}\left(w_1^{IIR}, w_2^{IIR}\right)}{\partial w_2^{IIR}} = \frac{a - w_2^{IIR}}{2b} - I^{IIR} - \frac{w_2^{IIR}}{2b} = 0 \tag{6.7}$$

$$w_2^{IIR}\left(I^{IIR}\right) = \frac{a}{2} - bI^{IIR} \tag{6.8}$$

Note, that both the supplier's and buyer's second period reaction functions are identical to the standard model. Hence, only if the buyer holds a strategic inventory size that differs from the size of the standard model a deviation from the solution of the standard model will occur.[36]

In the first-period, the buyer optimizes her profit function and, thereby, incorporates the supplier's second period wholesale price reaction function (6.8):

$$\begin{aligned} \pi_B^{IIR}\left(q_1^{IIR}, I^{IIR}\right) &= \left(a - b \cdot q_1^{IIR}\right) \cdot q_1^{IIR} - w_1^{IIR} \cdot \left(q_1^{IIR} + I^{IIR}\right) - i^{IIR} \cdot w_1^{IIR} \cdot I^{IIR} \\ &+ \left(\frac{3a}{4} - \frac{1}{2} bI^{IIR}\right) \cdot \left(\frac{a}{4b} + \frac{I^{IIR}}{2}\right) - \left(\frac{a}{2} - bI^{IIR}\right) \cdot \left(\frac{a}{4b} - \frac{I^{IIR}}{2}\right) \end{aligned} \tag{6.9}$$

First, she optimizes her selling quantity.

$$\frac{\partial \pi_B^{IIR}\left(q_1^{IIR}, I^{IIR}\right)}{\partial q_1^{IIR}} = a - 2bq_1^{IIR} - w_1^{IIR} = 0 \tag{6.10}$$

$$q_1^{IIR}\left(w_1^{IIR}\right) = \frac{a - w_1^{IIR}}{2b} \tag{6.11}$$

Again, this reaction function is identical to the function of the standard model. As long as the supplier sets the same price as in the standard model, the buyer will sell exactly the same quantity to the external market.

Second, the buyer derives the response function for the inventory size:

$$\frac{\partial \pi_B^{IIR}\left(q_1^{IIR}, I^{IIR}\right)}{\partial I^{IIR}} = \frac{3a - 4w_1^{IIR} - 4i^{IIR} \cdot w_1^{IIR} - 6b \cdot I^{IIR}}{4b} = 0 \tag{6.12}$$

$$I^{IIR}\left(w_1^{IIR}\right) = \frac{a}{2b} - \frac{2}{3b}\left(w_1^{IIR} + i^{IIR} \cdot w_1^{IIR}\right) = \frac{3a - 4 \cdot \left(1 + i^{IIR}\right) \cdot w_1^{IIR}}{6b} \tag{6.13}$$

[36] The second period results also show mathematically that the results of the static solution in the standard and the inventory interest rate model are identical, as the inventory size is per definition zero in the static solution.

Here, a first difference towards the reaction function of the standard model (3.40) occurs. There, the inventory holding costs are a constant factor that lowers the inventory size to the same degree but regardless of the actual level of the first-period wholesale price. In the IIR model, the interest rate determines how sensitive the buyer reacts to a change of the first-period wholesale price. In comparison to the standard model, an increase of the first-period wholesale price leads to a stronger reduction of the inventory size if $i^{IIR} = 0$. Further, setting $I^{IIR}\left(w_1^{IIR}\right) > 0$ in (6.13) shows that inventory is only built up if:

$$w_1^{IIR} < \frac{3a}{4 \cdot \left(1 + i^{IIR}\right)}. \tag{6.14}$$

Hence, for larger first-period wholesale prices that do not fulfill (6.14), the static solution will apply. A check regarding this upper bound will be conducted after the first-period wholesale price is found.

The supplier's profit function that includes the previous response functions is:

$$\begin{aligned}\pi_S^{IIR}\left(w_1^{IIR}\right) &= w_1^{IIR}\left(\frac{a - w_1^{IIR}}{2b} + \frac{3a - 4w_1^{IIR} - 4i^{IIR} \cdot w_1^{IIR}}{6b}\right) \\ &\quad + \left(\frac{2}{3} \cdot \left(1 + i^{IIR}\right) \cdot w_1^{IIR}\right)\left(\frac{a - \left(\frac{2}{3} \cdot \left(1 + i^{IIR}\right) \cdot w_1^{IIR}\right)}{2b} - \frac{3a - 4w_1^{IIR} - 4i^{IIR} \cdot w_1^{IIR}}{6b}\right) \\ &= \frac{\left(18a - 17w_1^{IIR} - 4i^{IIR}w_1^{IIR} + 4w_1^{IIR}\left(i^{IIR}\right)^2\right) \cdot w_1^{IIR}}{18b}.\end{aligned} \tag{6.15}$$

In the last step, the supplier derives the optimal first-period wholesale price:

$$\begin{aligned}\frac{\partial \pi_S^{IIR}\left(w_1^{IIR}\right)}{\partial w_1^{IIR}} &= \frac{18a - 17w_1^{IIR} - 4i^{IIR}w_1^{IIR} + 4w_1^{IIR}\left(i^{IIR}\right)^2}{18b} \\ &\quad + \frac{\left(-17 - 4i^{IIR} + 4\left(i^{IIR}\right)^2\right)w_1^{IIR}}{18b} = 0\end{aligned} \tag{6.16}$$

$$w_1^{IIR} = \frac{9a}{17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2} \tag{6.17}$$

By inserting the first-period wholesale price into the reaction functions, the dynamic solution can be derived. As mentioned above, the dynamic solution is only feasible, if condition (6.14) is fulfilled. Inserting (6.17) into (6.14) shows that the dynamic solution is feasible if $0 \le i^{IIR} < 0.5$.

Additionally, the supplier can always enforce the static solution by using a commitment contract. Hence, he will only implement the dynamic solution, if his profits are higher in comparison to the static solution. Comparing the results of the dynamic and static solution (Table 12) within the respective feasible range of the interest rate shows that the supplier always prefers the dynamic solution. Hence, as long as the interest holding cost rate is lower than 50% the supplier will implement the dynamic solution and strategic inventory is built up. Only for a higher holding cost rate the static solution applies. Moreover, a comparison of the respective profits shows that also the buyer and, therefore, the whole supply chain are always better off under the dynamic solution.[37] A summary of the IIR model is given by Table 12 and Table 13:

Table 12: Solution of the Inventory Interest Rate Model (Decision Variables)

	First-Best $e = FB$	Static $e = IIR$ $i^{IIR} \ge 0.5$	Dynamic $e = IIR$ $i^{IIR} < 0.5$
Wholesale Prices $\{ w_1^e , w_2^e \}$	-	$\left\{\frac{a}{2},\frac{a}{2}\right\}$	$\left\{\frac{9a}{17+4i^{IIR}-4\left(i^{IIR}\right)^2},\frac{6\cdot\left(1+i^{IIR}\right)\cdot a}{17+4i^{IIR}-4\left(i^{IIR}\right)^2}\right\}$
Purchase Quantities $\{ Q_1^e , Q_2^e \}$	$\left\{\frac{a}{2b},\frac{a}{2b}\right\}$	$\left\{\frac{a}{4b},\frac{a}{4b}\right\}$	$\left\{\frac{\left(13-4i^{IIR}-8\left(i^{IIR}\right)^2\right)a}{2\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b},\frac{3\cdot\left(1+i^{IIR}\right)a}{\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b}\right\}$
Inventory I^e	0	0	$\frac{\left(5-8i^{IIR}-4\left(i^{IIR}\right)^2\right)a}{2\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b}$
Sales Quantities $\{ q_1^e , q_2^e \}$	$\left\{\frac{a}{2b},\frac{a}{2b}\right\}$	$\left\{\frac{a}{4b},\frac{a}{4b}\right\}$	$\left\{\frac{2\cdot\left(2+i^{IIR}-\left(i^{IIR}\right)^2\right)a}{\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b},\frac{\left(11-2i^{IIR}-4\left(i^{IIR}\right)^2\right)a}{2\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b}\right\}$
Retail Prices $\{ p_1^e , p_2^e \}$	$\left\{\frac{a}{2},\frac{a}{2}\right\}$	$\left\{\frac{3a}{4},\frac{3a}{4}\right\}$	$\left\{\frac{\left(13+2i^{IIR}-2\left(i^{IIR}\right)^2\right)a}{17+4i^{IIR}-4\left(i^{IIR}\right)^2},\frac{\left(23+10i^{IIR}-4\left(i^{IIR}\right)^2\right)a}{2\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)}\right\}$

[37] See Appendix D for a mathematical comparisons of profits.

Table 13: Solution of the Inventory Interest Rate Model (Profits)

	First-Best $e = FB$	Static $e = IIR$ $i^{IIR} \geq 0.5$	Dynamic $e = IIR$ $i^{IIR} < 0.5$
	First-Best $e = FB$	Static $e = IIR$ $i^{IIR} \geq 0.5$	Dynamic $e = IIR$ $i^{IIR} < 0.5$
Profit Supplier π_S^e	-	$\frac{a^2}{4b}$	$\frac{9a^2}{2\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b}$
Profit Buyer π_B^e	-	$\frac{a^2}{8b}$	$\frac{\left(155+38i^{IIR}-60\left(i^{IIR}\right)^2+8\left(i^{IIR}\right)^3+32\left(i^{IIR}\right)^4\right)a^2}{4\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 b}$
Profit Supply Chain π_{SC}^e	$\frac{a^2}{2b}$	$\frac{3a^2}{8b}$	$\frac{\left(461+110i^{IIR}-132\left(i^{IIR}\right)^2+8\left(i^{IIR}\right)^3+32\left(i^{IIR}\right)^4\right)a^2}{4\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 b}$

6.1.2 Impact of the Holding Cost on the Strategic Inventory in the Inventory Interest Rate Model and Conclusions

To test how the actual size of the inventory interest rate affects the first-period wholesale price, the first order derivative has to be considered:

$$\frac{\partial w_1^{IIR}}{\partial i^{IIR}} = \frac{9a\cdot\left(-4+8i^{IIR}\right)}{\left(-17-4i^{IIR}+4\left(i^{IIR}\right)^2\right)^2} < 0 \qquad for \quad i < 0.5 \tag{6.18}$$

Hence, an increase of the interest rate leads to a reduction of the first-period wholesale price.

Moreover, the results of the SM and the IIR model can be directly compared to each other at the respective upper and lower bound of the dynamic solution. At the lower bound, no inventory holding costs exist ($i^{IIR} = 0$ and $h^{SM} = 0$) for both models. Hence, it is straightforward that in the absence of any holding cost, both models lead to an exactly identical solution. Formally, this can be shown by setting the holding cost parameters to zero in both models. For the first-period, for example, the wholesale price (standard model (3.43), IIR model (6.17)) results in an identical price of $9a/17$.

For the upper bound comparing the two models is also straightforward because if the holding cost reach the bound (i.e.: $i^{IIR} \geq 0.5$ or $h^{SM} \geq a/4$), the static solution will apply. Hence, as the static solution is identical in both models, there is also no difference between the models, if the respective holding costs parameters are above or exactly at the upper bound. However, it remains unclear, how the upper bound can be compared in terms of actual costs of holding inventory because of their difference in scale. To allow a comparison at the upper bound the inventory interest rate of the IIR model must be transferred into the per unit cost rate of the standard model (et vice versa). The respective per unit holding cost rate of the IIR model is $h^{IIR} = w^{IIR}\left(i^{IIR}\right) \cdot i^{IIR} = w^{IIR}\left(0.5\right) \cdot 0.5 = \frac{a}{4}$, while the respective inventory interest rate of the standard model is $i^{SM} = \frac{h^{SM}}{w^{SM}\left(h^{SM}\right)} = \frac{a/4}{w^{SM}\left(a/4\right)} = 0.5$. Therefore, the solutions of both models correspond to each other regarding the boundaries of the holding costs.

Now, the area within the boundaries is analyzed more specifically. As displayed by (6.13) the inventory interest rate influences how strongly the buyer reacts to price changes of the supplier. In the standard model (3.40), in contrast, the holding costs reduce the inventory size independently of the wholesale price. As a result, the supplier has to lower the first-period price stronger than in the standard model in which the holding costs are independent of the first-period wholesale price. Figure 18 shows the development of the first-period wholesale price under an increasing inventory interest rate under the parameters $a = 100$, $b = 1$ (These parameters will be used for all figures of the supply chain members decision variables and without causing a loss of generality[38]). Note, however that there is no possible way to exactly convert the inventory holding cost rate to a per unit cost rate that works in both directions and for all values (i.e. if the parameter is converted back the result would differ from the starting value).[39] However, this only creates a small disruption that is not visible by eye (see Appendix F for a comparison). Therefore, in the following the interest rate will be used as the baseline and the corresponding per unit holding cost rate of the standard

[38] See Appendix E.

[39] Multiplying the inventory interest rate by the corresponding first-period price of the IIR model delivers a per unit cost rate. However, if this per unit cost is used in the standard model to calculate the first-period wholesale price and then an interest rate is calculated with these values, it will differ from the starting interest rate.

model will be calculated by multiplying the inventory interest rate of the IIR model with the first-period wholesale price of the IIR model:

$$h^{SM} := i^{IIR} \cdot w_1^{IIR}\left(i^{IIR}\right) = i^{IIR} \cdot \frac{9a}{17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2} \tag{6.19}$$

Consequently, the first-period wholesale price of the standard model is obtained by $w_1^{SM}\left(h^{SM}\right) := w_1^{SM}\left(i^{IIR} \cdot w_1^{IIR}\left(i^{IIR}\right)\right)$.

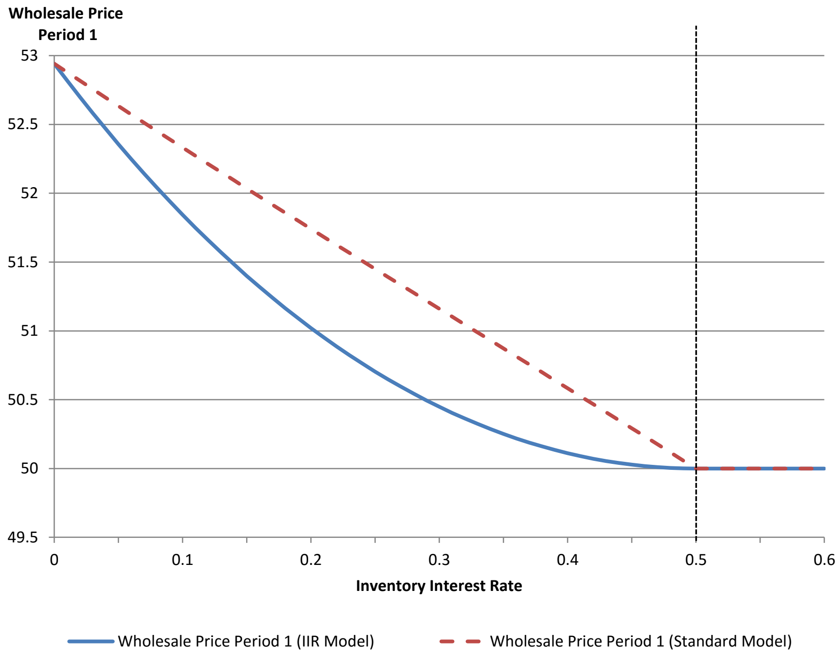

Figure 18: Development of First-period Wholesale Price in the IIR Model

The development of the first-period wholesale price shows that both models have the same starting and ending point. However, in the standard model the wholesale price decreases nearly linearly[40], while the curve between the two points is convex in the IIR model. As a result, the wholesale price in the IIR model is always smaller than in the standard model. This is caused by the more sensitive influence of a first-period wholesale price increase of the supplier on the inventory decision of the buyer (6.13). This means, if the inventory interest rate increases, the supplier starts with a higher decrease of the first-period wholesale price in the IIR model than in the standard model, because the level of the wholesale price also determines the level of the inventory cost.

[40] In contrast to the original SM model, the curve is no longer perfectly linear, because w_1^{SM} gets affected by the convex trend of w_1^{IIR} due to $h^{SM} = i^{IIR} \cdot w^{IIR}\left(i^{IIR}\right)$.

By setting lower wholesale prices the supplier countervails the per unit holding cost increase due to the higher inventory interest rate ($h^{IIR} = w^{IIR} \cdot i^{IIR}$).[41]

In summary, the buyer faces lower (or at least equal) first-period wholesale prices in the IIR model than in the standard model. As the reaction function of the first-period selling quantity is identical in both models ((3.38) and (6.11)), the lower first-period wholesale price results in a higher (or at least equal) selling quantity as described by Figure 19:

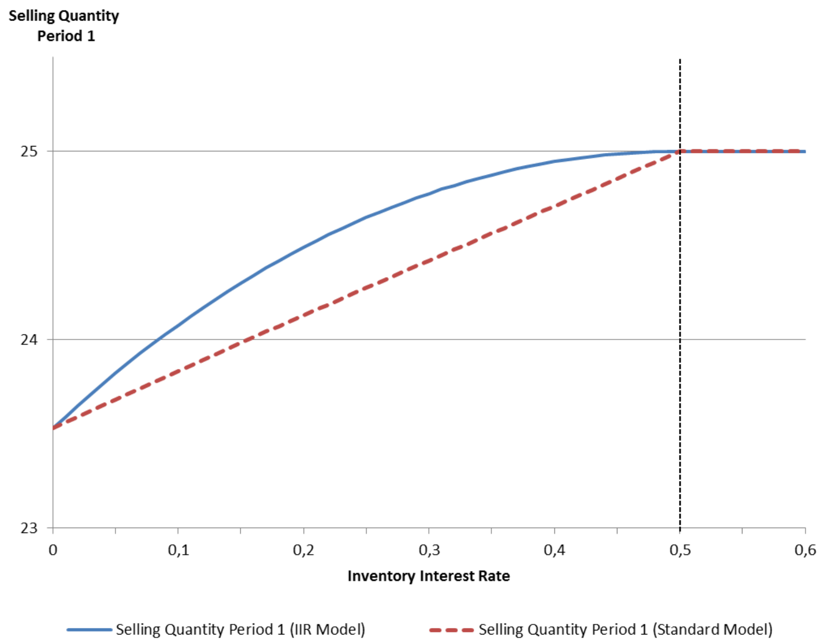

Figure 19: Development of the First-period Selling Quantity in the IIR Model

Hence, regarding the first-period, the double marginalization effect is weaker in the IIR model. However, it is still stronger than in the static solution, because the supplier raises the first-period wholesale price anticipating the buyer's increased first-period purchasing quantity due to her strategic inventory build-up.

In contrast to the selling quantity decision, the impact of the lower first-period wholesale price on the inventory size is more complicated. On one hand, a comparison of the reaction functions of the standard model (3.40) and the IIR model (6.13) shows that a lower wholesale price leads to an increase of the inventory size. Moreover, the inventory reducing compound of the standard model that depends on the per unit inventory holding cost rate (h^{SM}) does no longer exist in the IIR model, which also leads to higher inventory levels. On the other hand, the inventory interest rate in the IIR model enlarges the negative impact of the first-period wholesale price on the inventory

[41] Note that this can also be interpreted from another perspective: The higher the inventory holding cost rate is, the lower is the supplier's scope for an increase of the first-period wholesale price.

size. Hence, so far it can only be stated that the solutions of both SM and IIR model have the same minimum and maximum inventory size, as at these points (upper and lower bound of the dynamic solution) both solutions are identical.

A more detailed analysis of the effect of the holding costs parameter on the inventory size shows that in the standard model (see Table 2) an increase of the holding cost parameter (h^{SM}) causes a nearly linear decrease of the inventory size.[42] In the IIR model (Table 12), an increase of the holding cost parameter (i^{IIR}) leads to a progressively decreasing inventory size. Hence, under the previously described method to compare both solutions the inventory size is always larger in the IIR model (see Figure 20).

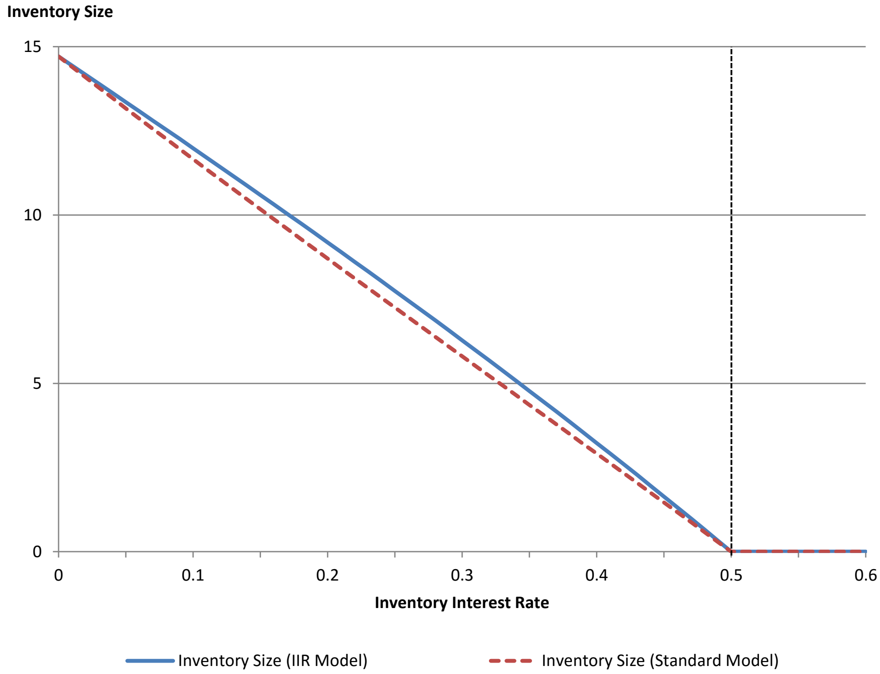

Figure 20: Development of the Inventory Size in the IIR Model

Hence, the impact of the inventory cost modification leads to a reduction of the first-period wholesale price, which in turn increases the inventory size. By combining the inventory size with the inventory costs parameter, the total holding costs of both models can be calculated.

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[42] Note, because h^{SM} gets replaced by $h^{SM} := i^{IIR} \cdot w_1^{IIR}$, an increase of i^{IIR} leads to a nearly linear decrease of the inventory size (I^{SM}).

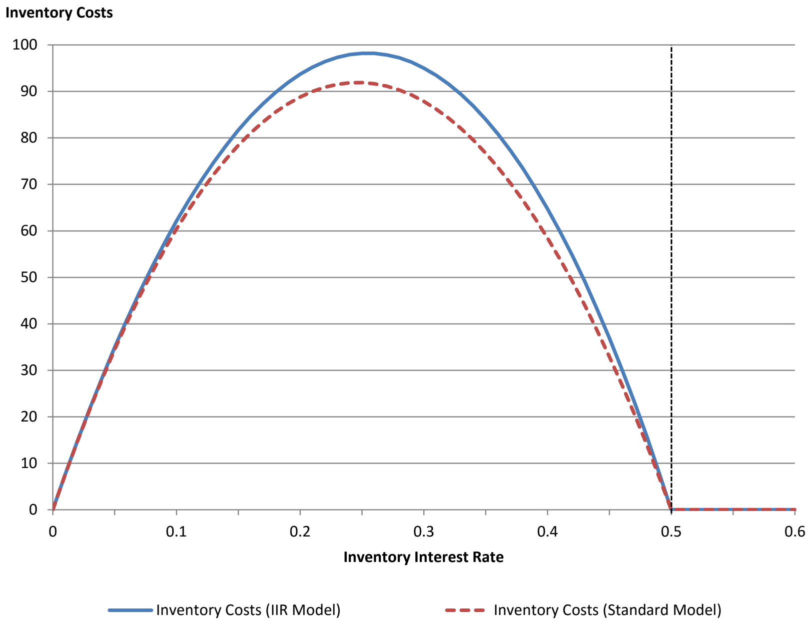

Figure 21: Development of Holding Cost in the IIR Model

As Figure 21 shows, the total holding costs in the IIR model $\left(\text{i.e.: } i^{IIR} \cdot w_1^{IIR} \cdot I^{IIR}\left(i^{IIR}\right)\right)$ are larger than in the standard model $\left(\text{i.e.: } h^{SM} \cdot I^{SM}\left(h^{SM}\right) \text{ with } h^{SM} := i^{IIR} \cdot w_1^{IIR}\right)$ due to the increased inventory size. Consequently, as the reaction functions of the second period are identical in both models, the larger inventory size in IIR will lead to a smaller second period wholesale price (Figure 22) and a larger selling quantity (Figure 23). Thus, in both periods, the double marginalization effect is weaker, if the per unit holding costs depends on the purchasing price (i.e. in the IIR model).

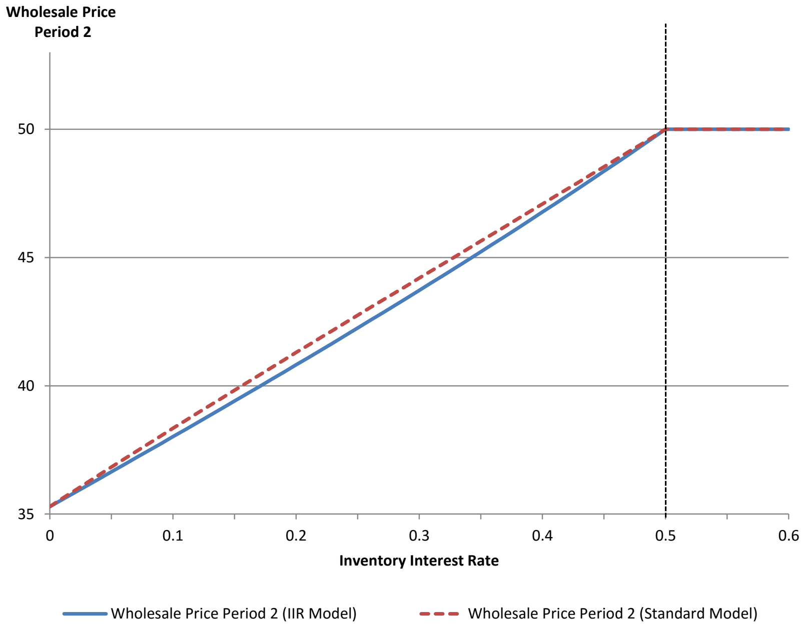

Figure 22: Development of the Second Period Wholesale Price in the IIR Model

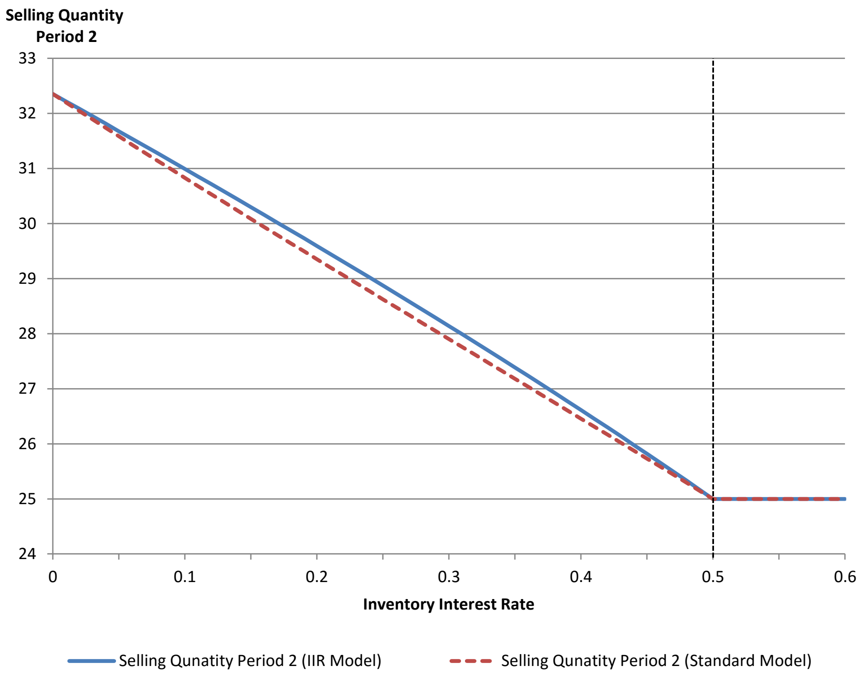

Figure 23: Development of the Second Period Selling Quantity in the IIR Model

Hence, regarding the profits there exist two contrary effects: the downside of increased total holding costs and the upside of the reduction of double marginalization. As already stated, both supplier and buyer always prefer the dynamic solution in the IIR model. In the standard model, in contrast, the buyer prefers the static solution, if holding costs are sufficiently high. This already shows that at least for high holding costs the buyer must be better off in the IIR model than in the standard model. The following more detailed analysis of the profits will show if this observation can be generalized to the entire range of inventory interest rates and to all supply chain members.

In comparison to the IIR model, the supplier has to set lower wholesale prices in both periods. In the first-period, the supplier is enforced to set a lower price than in the standard model, because the buyer reacts more sensitive to his price setting. In the second period, the supplier also sets a lower wholesale price, because the lower wholesale price incentivized the buyer to build up a larger strategic inventory. However due to the lower prices, the selling quantity goes up. As shown in Figure 24, both effects nearly neutralize each other regarding the profit of the supplier.[43]

[43] In fact, the supplier's profit is slightly lower in the IIR model in comparison to the standard model. See Appendix G for a mathematical comparison.

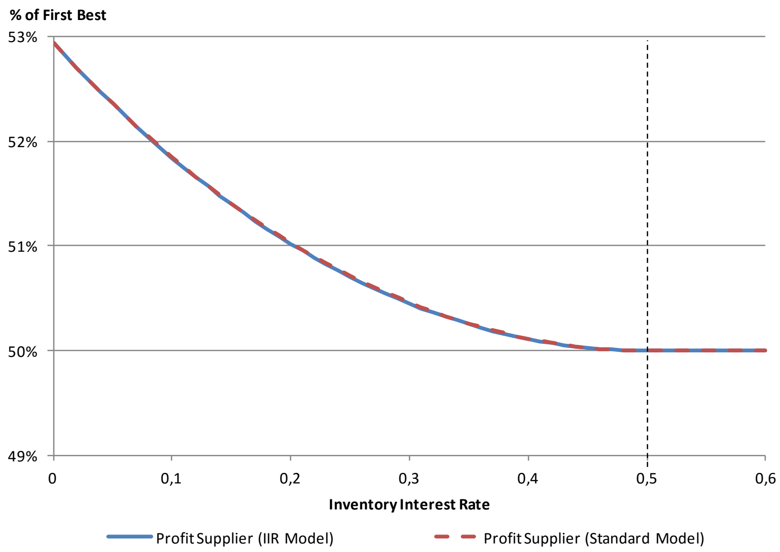

Figure 24: Comparison of the Supplier Profit in IIR and Standard Model[44]

Hence, from the supplier's perspective, the way inventory holding costs are modeled (i.e. as constant per unit holding costs or as an inventory interest rate) does not substantially impact the his profit. He prefers the dynamic over the static solution in both models and under similar inventory costs, almost no profit difference exists. However, the realization of the profit differs. Compared to the standard model, the comparable profit level is achieved with lower prices and a higher selling quantity.

In terms of the buyer's profit, however, several differences occur. Here, the profits are only identical at the boundaries of the dynamic solution. For the inventory cost values within the interval of the dynamic solution, the buyer's profit is always larger in the IIR model compared to the respective profit of the standard model (Figure 25 and Appendix G for a mathematical proof). Hence, the increase in total holding costs is outweighed by the positive effects of the lower wholesale prices, which leads to a weaker decrease of the buyer's profit under rising holding costs. Moreover, in contrast to the standard model, the buyer always prefers the dynamic over the static solution in the IIR model.

[44] While not visible for the naked eye, there is a small difference between both curves. However, the maximal difference is only about 0.0002% and depends on the direction of the translation of the inventory cost parameters. If the IIR model is used as the baseline (like in the figure above), the profit in the standard model is maximally 0.0002% larger (et vice versa).

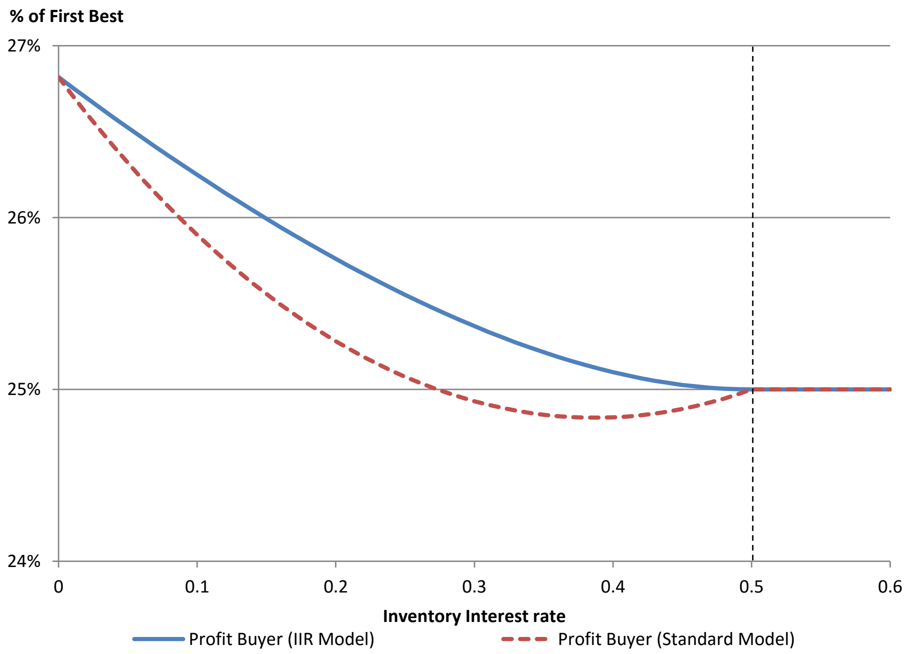

Figure 25: Comparison of the Buyer Profit in IIR and Standard Model

As the supplier's profit is approximately identical in both models and the buyer's profit of the dynamic solution is higher in the IIR model, the supply chain profit of the dynamic solution must also be higher in the case of purchasing price-dependent holding costs (see Figure 26 and Appendix G for a mathematical proof). Also, in the IIR model, the supply chain profit of the dynamic solution is always higher than the static solution (i.e. there is no interval in which the supply chain profit of the dynamic solution is smaller than the profit of the static solution like in the standard model).

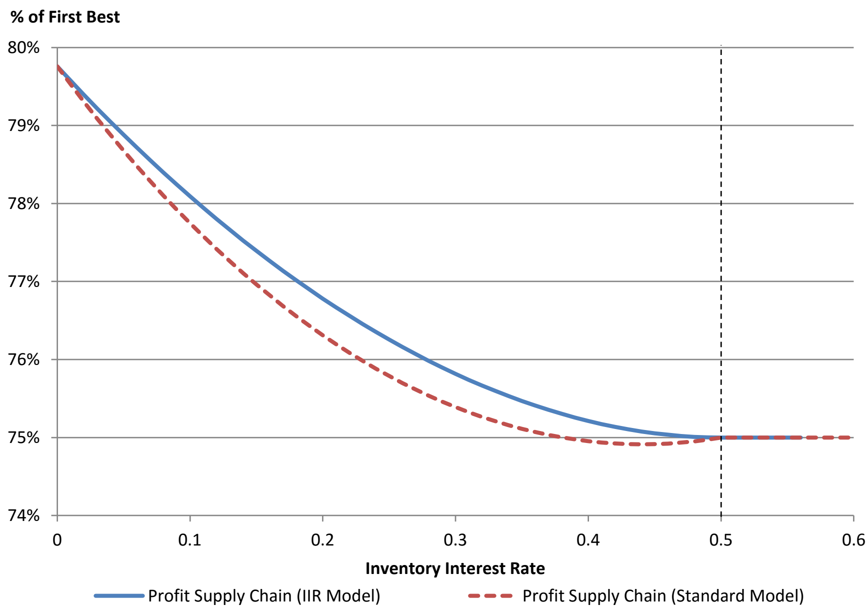

Figure 26: Comparison of the Supply Chain Profit in IIR and Standard Model

While the supply chain profit is larger in the IIR model, it is worthwhile to note that this difference is only caused by a higher profit of the buyer. Hence, a situation with purchase price-dependent inventory holding costs improves the buyer's situation regarding the distribution of the total profit.

In summary, the IIR model further highlights the impact of strategic inventory, as it also plays a pivotal role, if holding costs are interpreted as dependent from the purchasing costs (IIR model). Using an interest rate based holding cost leads to a higher inventory build-up than under the respective solution with per unit holding costs. Moreover, the interval with high holding cost, in which the profit levels were lower in the dynamic solution compared to the static solution, does no longer exist in the IIR model.

6.2 Strategic Inventory under the Discounted Cash Flow Approach

In the discounted cash flow (DCF) model ($e = DCF$), future cash flows are measured at their present value. Hence, supplier's and buyer's second period cash flows are discounted with an interest rate i^{DCF} . Hence, both supply chain members have a higher preference to realize profits in the first-period. Thus, the supplier might set a lower first-period wholesale price than in the standard model in order to encourage the buyer to choose a higher purchasing quantity and, thereby, to shift a larger fraction of his profits towards the first-period. However, the buyer will be less willing to build up inventory as an inventory build-up shifts costs from the second towards the first-period. These preferences of supplier and buyer get more pronounced if the interest rate increases. Hence, in the dynamic solution the role of strategic inventory might be of less importance in the DCF model than in the standard model, in which the buyer accepted to have a low profit margin in the first and a higher profit margin in the second period in comparison to the static solution.

Next to price dependent holding costs, the buyer might still have to carry additional out-of-pocket per unit holding cost (h^{DCF}) for her inventory. The extensions of the DCF model lead to the following profit functions, where $1/_{1+i^{DCF}}$ represents the discount factor:

$$\pi_S^{DCF}\left(w_1^{DCF}, w_2^{DCF}\right) = w_1^{DCF} \cdot \left(q_1^{DCF} + I^{DCF}\right) + \frac{w_2^{DCF} \cdot \left(q_2^{DCF} - I^{DCF}\right)}{1 + i^{DCF}} \tag{6.20}$$

$$\pi_B^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right) = \left(a - b\cdot q_1^{DCF}\right)\cdot q_1^{DCF} - w_1^{DCF}\left(q_1^{DCF} + I^{DCF}\right) - h^{DCF}\cdot I^{DCF} + \frac{\left(a - b\cdot q_2^{DCF}\right)\cdot q_2^{DCF} - w_2^{DCF}\left(q_2^{DCF} - I^{DCF}\right)}{1 + i^{DCF}} \tag{6.21}$$

$$\begin{aligned}\pi_{SC}^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right) &= \pi_S^{IIR}\left(w_1^{DCF}, w_2^{DCF}\right) + \pi_B^{IIR}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right)\\ &= \left(a - b\cdot q_1^{DCF}\right)\cdot q_1^{DCF} - h^{DCF}\cdot I^{DCF} + \frac{\left(a - b\cdot q_2^{DCF}\right)\cdot q_2^{DCF}}{1 + i^{DCF}}\end{aligned} \tag{6.22}$$

6.2.1 Solution of an Integrated Supply Chain (First-Best Solution)

In the first-best solution, only the aggregated supply chain profit (6.22) has to be considered. As in the standard model, no inventory is built up in the first-best solution because $\frac{\partial \pi_{SC}^{DCF}}{\partial I^{DCF}}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right) < 0$ for $h^{DCF} > 0$. The selling quantity of each period can be obtained easily as there is no longer an interaction between both periods:

$$\frac{\partial \pi_{SC}^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right)}{\partial q_1^{DCF}} = a - 2bq_1^{DCF} = 0 \tag{6.23}$$

$$\frac{\partial \pi_{SC}^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right)}{\partial q_2^{DCF}} = \frac{a - 2bq_2^{DCF}}{1 + i^{DCF}} = 0 \tag{6.24}$$

$$q_1^{DCF} = q_2^{DCF} = \frac{a}{2b} \tag{6.25}$$

Hence, the selling quantities are identical to the first-best selling quantity of the standard model. However, in contrast to the IIR model, the resulting first-best profits of the DCF model differ from the standard model as the fraction of profits that are realized in the second period are discounted. Inserting the selling quantities (6.25) into the profit function (6.22) results in:

$$\pi_{SC}^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right) = \frac{\left(2 + i^{DCF}\right)\cdot a^2}{4\cdot\left(1 + i^{DCF}\right)\cdot b} \tag{6.26}$$

Hence, only for $i^{DCF} = 0$ the supply chain profit is identical to the first-best supply chain profit of the standard model. With an increase in the interest rate (i^{DCF}) the supply chain profit decreases, because the second period profits is discounted. To analyze the effects of strategic inventory under individual optimization the following results have to be compared to the first-best solution of the DCF model (i.e. (6.26) serving as the new benchmark).

6.2.2 Individual Optimization of the Supply Chain Members

Again, the solution can be obtained by backward induction. First, the buyer's second period reaction function for the selling quantity is determined:

$$\frac{\partial \pi_B^{DCF}\left(q_1^{DCF}, I^{DCF}, q_2^{DCF}\right)}{\partial q_2^{DCF}} = \frac{a - 2bq_2^{DCF} - w_2^{DCF}}{1 + i^{DCF}} = 0 \tag{6.27}$$

$$q_2^{DCF}\left(w_2^{DCF}\right) = \frac{a - w_2^{DCF}}{2b} \tag{6.28}$$

The supplier integrates this reaction function into his profit function

$$\pi_S^{DCF}\left(w_1^{DCF}, w_2^{DCF}\right) = w_1^{DCF} \cdot \left(q_1^{DCF} + I^{DCF}\right) + \frac{w_2^{DCF} \cdot \left(\frac{a - w_2^{DCF}}{2b} - I^{DCF}\right)}{1 + i^{DCF}} \tag{6.29}$$

and determines his optimal reaction function for the second period wholesale price:

$$\frac{\partial \pi_S^{DCF}\left(w_1^{DCF}, w_2^{DCF}\right)}{\partial w_2^{DCF}} = \frac{\frac{a - w_2^{DCF}}{2b} - I^{DCF} - \frac{w_2^{DCF}}{2b}}{1 + i^{DCF}} = 0 \tag{6.30}$$

$$w_2^{DCF}\left(I^{DCF}\right) = \frac{a}{2} - bI^{DCF} \tag{6.31}$$

A comparison of the second period reaction functions of the DCF model (6.28), (6.31) with the reaction function of the inventory interest rate model (6.4), (6.8) shows that the introduction of the discount factor in the DCF model does not lead to a change in the second period reaction functions. Thus, only a difference coming from a change in the inventory size decision would cause different decisions in period two.

Integrating the second period reaction functions into the buyer's profit function gives:

$$\begin{aligned}\pi_B^{DCF}\left(q_1^{DCF}, I^{DCF}\right) &= \left(a - b \cdot q_1^{DCF}\right) \cdot q_1^{DCF} - w_1^{DCF} \cdot \left(q_1^{DCF} + I^{DCF}\right) - h^{DCF} \cdot I^{DCF} \\ &+ \frac{\left(\frac{3a}{4} - \frac{1}{2}bI^{DCF}\right) \cdot \left(\frac{a}{4b} + \frac{I^{DCF}}{2}\right) - \left(\frac{a}{2} - bI^{DCF}\right) \cdot \left(\frac{a}{4b} - \frac{I^{DCF}}{2}\right)}{1 + i^{DCF}}\end{aligned} \tag{6.32}$$

At this stage, the buyer has to determine both the reaction functions for his first-period selling quantity as well as the reaction function for the inventory size:

selling quantity:

$$\frac{\partial \pi_B^{DCF}\left(q_1^{DCF}, I^{DCF}\right)}{\partial q_1^{DCF}} = a - 2bq_1^{DCF} - w_1^{DCF} = 0 \tag{6.33}$$

$$q_1^{DCF}\left(w_1^{DCF}\right) = \frac{a - w_1^{DCF}}{2b} \tag{6.34}$$

inventory size:

$$\frac{\partial \pi_B^{DCF}\left(q_1^{DCF}, I^{DCF}\right)}{\partial I^{DCF}} = \frac{3a - 6bI^{DCF}}{4\left(1 + i^{DCF}\right)} - w_1^{DCF} - h^{DCF} = 0 \tag{6.35}$$

$$I^{DCF}\left(w_1^{DCF}\right) = \frac{3a - 4 \cdot \left(1 + i^{DCF}\right) \cdot \left(w_1^{DCF} + h^{DCF}\right)}{6b} \tag{6.36}$$

The comparison of the buyer's first-period reaction functions ((6.34), (6.36)) with the respective functions of the IIR model ((6.11), (6.13)) shows that the reaction functions for the selling quantity are identical. Moreover, the only difference in the inventory reaction function consists of the influence of the per unit holding cost rate (h^{DCF}) that was additionally integrated in the DCF model. If this term is left out in the comparison both functions would be identical (i.e. if h^{DCF} is set to zero, the functions become identical). Hence, up to this stage there exists no structural difference between the DCF and the IIR model.

According to (6.36) the buyer will only build up inventory, if

$$w_1^{DCF} < \frac{3a}{4 \cdot \left(1 + i^{DCF}\right)} - h^{DCF} \tag{6.37}$$

Hence, if condition (6.37) is not fulfilled, no strategic inventory is built up. In this case, the inventory (6.36) will be zero and periods are decoupled. Hence, wholesale price and selling quantity decisions will be identical in both periods. As before, such a solution without strategic inventory will be called static solution. The decisions can be determined by inserting $I^{DCF} = 0$ into the second period reaction functions ((6.31), (6.28)) and into the profit functions ((6.20), (6.21)). While the decisions of the static solution are identical to those of the standard model, the profits again differ because of the discounting of the second period. Under which conditions (6.37) is fulfilled will be determined after the first-period wholesale price is determined.

By integrating the buyer's reaction functions into the suppliers profit function, the relevant first-period profit function of the supplier can be calculated:

$$\pi_S^{DCF}\left(w_1^{DCF}\right)=\frac{18a\cdot w_1^{DCF}-\left(17+8i^{DCF}\right)\cdot\left(w_1^{DCF}\right)^2}{18b}+\frac{-4\left(1+i^{DCF}\right)w_1^{DCF}h^{DCF}+4\left(1+i^{DCF}\right)\cdot\left(h^{DCF}\right)^2}{18b} \tag{6.38}$$

The supplier then determines the optimal first-period wholesale price:

$$\frac{\partial\pi_S^{DCF}\left(w_1^{DCF}\right)}{\partial w_1^{DCF}}=\frac{9a-\left(17+8i^{DCF}\right)w_1^{DCF}-2\left(1+i^{DCF}\right)h^{DCF}}{9b}=0 \tag{6.39}$$

$$w_1^{DCF}\left(i^{DCF}\right)=\frac{9a-2\cdot\left(1+i^{DCF}\right)\cdot h^{DCF}}{17+8i^{DCF}} \tag{6.40}$$

Using this first-period wholesale price with the reaction functions and the profit functions delivers the results of the dynamic solution that are summarized alongside with the first-best and static solution in Table 14 and Table 15. However, like already mentioned this dynamic solution is only feasible if condition (6.37) is fulfilled. Inserting the first-period wholesale price (6.40) into that condition shows that the dynamic solution will only be feasible for the following combination of interest rate and per unit holding cost rate:

$$\begin{gathered}\left(5-4i^{DCF}\right)a-4\cdot\left(1+i^{DCF}\right)\cdot\left(5+2i^{DCF}\right)h^{DCF}>0\\ h^{DCF}<\frac{\left(5-4i^{DCF}\right)a}{4\cdot\left(1+i^{DCF}\right)\cdot\left(5+2i^{DCF}\right)}=\overline{h_D^{DCF}}\end{gathered} \tag{6.41}$$

Only, if (6.41) is fulfilled, the supplier is able to implement the dynamic solution. Else, the static solution applies because the combined cost of holding inventory (costs due to discounting and per unit holding cost) prohibit the buyer from building up inventory. An isolated analysis of the interest rate (i^{DCF}) and the per unit holding cost rate (h^{DCF}) in which the other parameter is set to zero shows that inventory is not built up if $h^{DCF}<\frac{a}{4}$ or $i^{DCF} > 1.25 \triangleq 125\%$. The upper bound for the unit holding cost rate is unsurprisingly identical to the standard model, as for $i^{DCF}=0$ the entire DCF model becomes identical to the standard model. The maximum interest rate however is surprisingly high in comparison to the IIR rate model where it was 0.5. Hence, regarding the feasibility of the dynamic solution, strategic inventory is possible for an even wider range of the interest rate than in the DCF model. It will be shown later on that this is caused by the suppliers increasing preference to shift profits into the first-period by lowering the first-period wholesale price as the interest rate increases.

However, in order to be the equilibrium solution the dynamic solution must not only be feasible but also be preferred over the static solution by the supplier. Else, the supplier would implement the static solution by committing on the second period price in the first-period. To test whether the supplier implements the dynamic solution and whether buyer and supply chain are better off under the dynamic solution, the profits of the dynamic solution have to be compared to the respective static profits.

First, the dynamic profit of the supplier gets compared to the static profit. The supplier will only implement the dynamic solution, if the corresponding profit is larger than the profit of the static solution:

$$\begin{aligned} \pi_S^{DCF,dynamic} &\overset{?}{>} \pi_S^{DCF,static} \\ \pi_S^{DCF,dynamic} - \pi_S^{DCF,static} &\overset{?}{>} 0 \end{aligned} \tag{6.42}$$

$$\frac{9a^2 - 4\left(1+i^{DCF}\right)ah^{DCF} + 4\left(2+3i^{DCF}+\left(i^{DCF}\right)^2\right)\left(h^{DCF}\right)^2}{2\cdot\left(17+8i^{DCF}\right)b} - \frac{\left(2+i^{DCF}\right)\cdot a^2}{8\cdot\left(1+i^{DCF}\right)\cdot b} \overset{?}{>} 0 \tag{6.43}$$

$$\begin{aligned} &\left(2+3i^{DCF}-8\left(i^{DCF}\right)^2\right)a^2 - \left(16+32i^{DCF}+16\left(i^{DCF}\right)^2\right)ah^{DCF} \\ &+\left(32+80i^{DCF}+64\left(i^{DCF}\right)^2+16\left(i^{DCF}\right)^3\right)\left(h^{DCF}\right)^2 \overset{?}{>} 0 \end{aligned} \tag{6.44}$$

Solving this for h^{DCF} shows that

$$\pi_S^{DCF,dynamic} > \pi_S^{DCF,static} \quad if \quad \begin{cases} 1.: & h^{DCF} < \dfrac{\left(2\cdot\left(1+i^{DCF}\right)-\sqrt{8\left(i^{DCF}\right)^3+17\left(i^{DCF}\right)^2}\right)a}{4\cdot\left(2+i^{DCF}\right)\cdot\left(1+i^{DCF}\right)} \\ 2.: & h^{DCF} > \dfrac{\left(2\cdot\left(1+i^{DCF}\right)+\sqrt{8\left(i^{DCF}\right)^3+17\left(i^{DCF}\right)^2}\right)a}{4\cdot\left(2+i^{DCF}\right)\cdot\left(1+i^{DCF}\right)} \end{cases} \tag{6.45}$$

As 2. of (6.45) violates the inventory condition (6.41) (i.e. inventory would be smaller than zero in the case of 2.), only 1. of (6.45) is relevant. Hence, the supplier favors and implements the dynamic solution if

$$0 \le h^{DCF} < \frac{\left(2\cdot\left(1+i^{DCF}\right)-\sqrt{8\left(i^{DCF}\right)^3+17\left(i^{DCF}\right)^2}\right)a}{4\cdot\left(2+i^{DCF}\right)\cdot\left(1+i^{DCF}\right)} =: \overline{h_S^{DCF}} \tag{6.46}$$

In contrast to the IIR model, there exists a combination of h^{DCF} and i^{DCF} under which the supplier could implement the dynamic solution but prefers implementing the static solution due to a higher profit (i.e. $\overline{h_S^{DCF}} < \overline{h_D^{DCF}}$).

In order to obtain a better comparison to the IIR model, the per unit holding costs (h^{DCF}) will be excluded in the following analysis (i.e.: $h^{DCF} = 0$) as the IIR model does not include constant per unit holding costs. Solving (6.46) for $h^{DCF} = 0$ delivers:

$$0 \le i^{DCF} < \frac{3+\sqrt{73}}{16} \approx 0.7215 \tag{6.47}$$

Hence, as long as the interest rate is lower than 72.15%, the supplier's profit is larger in the dynamic solution than in the static. For higher values, the static solution will be larger and, therefore, the supplier will implement this solution. To implement the static solution, the supplier is no longer bound to using a commitment contract as setting the static first-period wholesale price will prevent the buyer from building up inventory as soon as $i^{DCF} \ge 0$. A summary of the first-best, the static and the dynamic solution of the DCF model is given in Table 14 and Table 15.

Table 14: Summary of Decision Variables in the DCF Model[45]

	First-Best	Static $h^{DCF} \geq \overline{h_S^{DCF}}$	Dynamic $h^{DCF} < \overline{h_S^{DCF}}$
w_1^{DCF}	-	$\frac{a}{2}$	$\frac{9a - 2\cdot\left(1+i^{DCF}\right)h^{DCF}}{17+8i^{DCF}}$
Q_1^{DCF}	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{\left(13-4i^{DCF}\right)a - \left(18+26i^{DCF}+8\left(i^{DCF}\right)^2\right)h^{DCF}}{2\cdot\left(17+8i^{DCF}\right)b}$
q_1^{DCF}	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{\left(1+i^{DCF}\right)\cdot\left(4a+h^{DCF}\right)}{\left(17+8i^{DCF}\right)b}$
p_1^{DCF}	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{\left(13+4i^{DCF}\right)a - \left(1+i^{DCF}\right)h^{DCF}}{17+8i^{DCF}}$
I^{DCF}	0	0	$\frac{\left(5-4i^{DCF}\right)a - \left(20+28i^{DCF}+8\left(i^{DCF}\right)^2\right)h^{DCF}}{2\cdot\left(17+8i^{DCF}\right)b}$
w_2^{DCF}	-	$\frac{a}{2}$	$\frac{6\left(1+i^{DCF}\right)a + 2\cdot\left(5+7i^{DCF}+2\left(i^{DCF}\right)^2\right)h^{DCF}}{17+8i^{DCF}}$
Q_2^{DCF}	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{3\cdot\left(1+i^{DCF}\right)a + \left(5+7i^{DCF}+2\left(i^{DCF}\right)^2\right)h^{DCF}}{\left(17+8i^{DCF}\right)b}$
q_2^{DCF}	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{\left(11+2i^{DCF}\right)a - \left(10+14i^{DCF}+4\left(i^{DCF}\right)^2\right)h^{DCF}}{2\cdot\left(17+8i^{DCF}\right)b}$
p_2^{DCF}	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{\left(23+14i^{DCF}\right)a + \left(10+14i^{DCF}+4\left(i^{DCF}\right)^2\right)h^{DCF}}{2\cdot\left(17+8i^{DCF}\right)}$

[45] These results were first derived by Desai et al. (2010) and summarized in Table 1 of their study. Note that Desai et al. (2010) use a discount factor ρ. Inserting $\rho = 1/\left(1+i^{DCF}\right)$ into their results delivers the results as they are presented in Table 14 and Table 15.

Table 15: Summary of Profits in the DCF Model[46]

	First-Best	Static $h^{DCF} \geq \overline{h_S^{DCF}}$	Dynamic $h^{DCF} < \overline{h_S^{DCF}}$
π_S^{DCF}	-	$\frac{\left(2+i^{DCF}\right)\cdot a^2}{8\cdot\left(1+i^{DCF}\right)\cdot b}$	$\frac{9a^2-4\left(1+i^{DCF}\right)ah^{DCF}+4\left(2+3i^{DCF}+\left(i^{DCF}\right)^2\right)\left(h^{DCF}\right)^2}{2\cdot\left(17+8i^{DCF}\right)b}$
π_B^{DCF}	-	$\frac{\left(2+i^{DCF}\right)\cdot a^2}{16\cdot\left(1+i^{DCF}\right)\cdot b}$	$\frac{\left(155+230i^{DCF}+220\left(i^{DCF}\right)^2+64\left(i^{DCF}\right)^3\right)a^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} - \frac{\left(118-6i^{DCF}-204\left(i^{DCF}\right)^2-80\left(i^{DCF}\right)^3\right)ah^{DCF}}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} + \frac{\left(304+852i^{DCF}+840\left(i^{DCF}\right)^2+340\left(i^{DCF}\right)^3+48\left(i^{DCF}\right)^4\right)\left(h^{DCF}\right)^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b}$
π_{SC}^{DCF}	$\frac{(2+1)a^2}{4\cdot\left(1+i^{DCF}\right)b}$	$\frac{3\cdot\left(2+i^{DCF}\right)a^2}{16\cdot\left(1+i^{DCF}\right)b}$	$\frac{\left(461+680i^{DCF}+364\left(i^{DCF}\right)^2+64\left(i^{DCF}\right)^3\right)a^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} - \frac{\left(254+330i^{DCF}+60\left(i^{DCF}\right)^2-16\left(i^{DCF}\right)^3\right)ah^{DCF}}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} + \frac{\left(576+1660i^{DCF}+1704\left(i^{DCF}\right)^2+732\left(i^{DCF}\right)^3+112\left(i^{DCF}\right)^4\right)\left(h^{DCF}\right)^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b}$

Using the same method as before, it can also be determined for which combination of holding cost parameters the buyer prefers the dynamic solution.

$$\begin{aligned} \pi_B^{DCF,dynamic} &\overset{?}{>} \pi_B^{DCF,static} \\ \pi_B^{DCF,dynamic}-\pi_B^{DCF,static} &\overset{?}{>} 0 \end{aligned} \tag{6.48}$$

[46] See footnote 45.

$$\begin{bmatrix} \dfrac{\left(155+230i^{DCF}+220\left(i^{DCF}\right)^2-64\left(i^{DCF}\right)^3\right)a^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ -\dfrac{\left(118-6i^{DCF}-204\left(i^{DCF}\right)^2-80\left(i^{DCF}\right)^3\right)ah}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ +\dfrac{\left(304+852i^{DCF}+840\left(i^{DCF}\right)^2+340\left(i^{DCF}\right)^3+48\left(i^{DCF}\right)^4\right)h^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ -\dfrac{\left(2+\left(i^{DCF}\right)\right)\cdot a^2}{16\cdot\left(1+\left(i^{DCF}\right)\right)\cdot b} \end{bmatrix} \overset{?}{>} 0 \qquad (6.49)$$

$$\begin{bmatrix} \left(192\left(i^{DCF}\right)^3+480\left(i^{DCF}\right)^2+87i^{DCF}+42\right)a^2 \\ +\left(320\left(i^{DCF}\right)^3+816\left(i^{DCF}\right)^2+24i^{DCF}-472\right)ah^{DCF} \\ +\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)\left(h^{DCF}\right)^2 \end{bmatrix} \overset{?}{>} 0 \qquad (6.50)$$

It is straightforward that for $h^{DCF}=0$ condition (6.50) is always fulfilled and, therefore, that the supplier will prefer the dynamic over the static solution. For $h^{DCF}>0$ the analysis, however, is more complicated:

The left hand side of (6.50) can only be negative, if the term within the brackets in the second row is negative. Hence, it is straightforward that only for small values of i^{DCF}, the buyer might prefer the static over the dynamic solution. Solving (6.50) for h^{DCF} delivers:[47]

[47] See Appendix I.

$$\pi_B^{DCF,dynamic} > \pi_B^{DCF,static}$$

$$\textit{if} \;\; (1.):$$

$$h^{DCF} > \frac{\left(59-62i^{DCF}-40\left(i^{DCF}\right)^2\right)a}{4\cdot\left(76+61i^{DCF}+12\left(i^{DCF}\right)^2\right)\cdot\left(1+i^{DCF}\right)} + \frac{(Root)a}{4\cdot\left(76+61i^{DCF}+12\left(i^{DCF}\right)^2\right)\cdot\left(1+i^{DCF}\right)} \tag{6.51}$$

$$\textit{or if} \;\; (2.):$$

$$h^{DCF} < \frac{\left(59-62i^{DCF}-40\left(i^{DCF}\right)^2\right)a}{4\cdot\left(76+61i^{DCF}+12\left(i^{DCF}\right)^2\right)\cdot\left(1+i^{DCF}\right)} - \frac{(Root)a}{4\cdot\left(76+61i^{DCF}+12\left(i^{DCF}\right)^2\right)\cdot\left(1+i^{DCF}\right)}$$

$$\textit{with}: \quad Root := \sqrt{289-16490i^{DCF}-43167\left(i^{DCF}\right)^2-39956\left(i^{DCF}\right)^3-15872\left(i^{DCF}\right)^4-2304\left(i^{DCF}\right)^5}$$

For $i^{DCF} > 0.01678$ the value within the square root gets negative. Hence, only for a lower inventory interest rate a holding cost rate exists, under which the buyer does not prefer the dynamic solution.[48] In contrast to the supplier, both boundaries described in (6.51) are relevant. However, the area in which the static solution is better (i.e. one of the boundaries of (6.51) is not fulfilled) for the buyer is relatively small.

Comparing the supply chain profits shows that an area exists under which the supply chain profit of the static solution is larger than in the dynamic solution.[49] This was also observed for the SM but not for the IIR model (Figure 26).

6.2.3 The Impact of the Interest Rate on the Solution of the DCF Model

In the following, the per unit inventory costs are left out of the analysis (i.e.: $h^{DCF} = 0$) to provide a better understanding of the influence of the interest rate (i^{DCF}) on the parameters of the DCF model and to deliver a comparison to the IIR model that does not include the per unit holding cost rate. Again, the parameters $a = 100$ and $b = 1$ will

[48] Note, that the border for which the buyer prefers the static solution presented in the study of Desai et al. (2010) is incorrect because of a wrong interpretation by the authors. The border described by h_{rl} does not describe whether the buyer is better-off under the dynamic contract, but instead weather the dynamic contract is feasible. Hence, Desai et al. (2010) do not consider that there exists an area for the holding costs within the feasible range under which the buyer is worse-off under the dynamic contract. An example for the existence of such an area is already described by Anand et al. (2008) in their standard model for $i = 0$ and is generalized for $h \geq 0$ and $i \geq 0$ by (6.51) of this thesis.

[49] A mathematical proof and description is given in Appendix J.

be used for all figures in this section and the per unit holding cost parameter of the standard model will be derived from the IIR model in the same way as demonstrated in section 6.1 $\left(\text{i.e. } h^{SM} := i^{IIR} \cdot w_1^{IIR}\left(i^{IIR}\right)\right)$. Fixing parameters a and b is done without loss of generality for $h^{DCF} = 0$.[50]

In the first-period, the supplier determines the first-period wholesale price. In comparison to both the standard and the IIR model the wholesale price drops considerably stronger, if the interest rate increases (Figure 27). Already at $i^{DCF} = 12.5\%$ the first-period wholesale price of the DCF model equals the static wholesale price of the standard solution. Under a further increase of the interest rate, the first-period wholesale price even drops below the static first-period wholesale price of the standard model. Hence, in comparison to both the standard and the IIR model, a first-period wholesale price that is lower than the static price of the standard solution is possible. This is caused by the discounting of the second period: In order to pull the realization of profits into the first-period, the supplier lowers the first-period wholesale price. This action increases the buyer's benefit from an inventory build-up and, thus, partly compensates the decreased buyer's incentive caused by the higher holding costs (i^{DCF}).

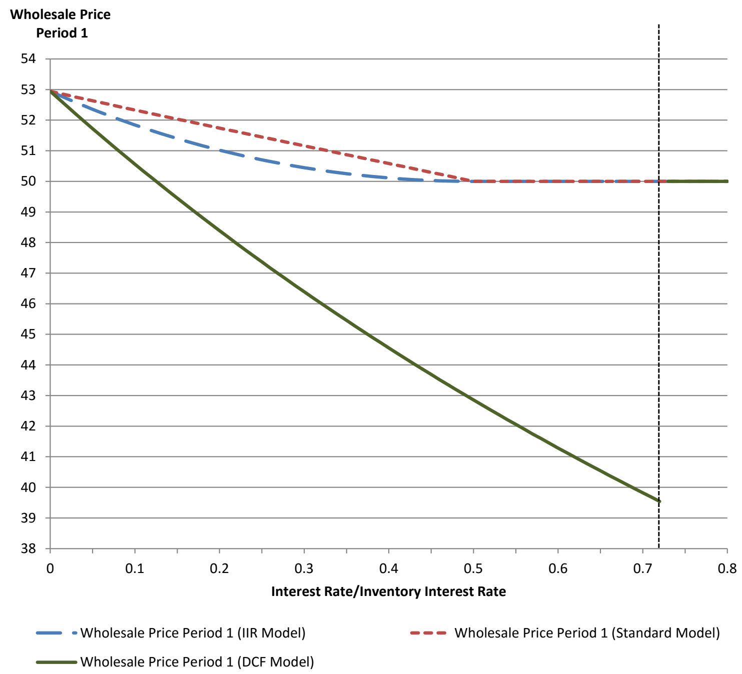

Figure 27: First-period Wholesale Price in the DCF Model

[50] See Appendix H.

As profits that are generated in the first-period do not get discounted, the level of the interest rate does not directly affect the first-period selling quantity, which only depends on the first-period wholesale price (6.34). However, as an increase of the interest rate decreases the first-period wholesale price, there exists an indirect link. Thus, the lower first-period wholesale price in the DCF model (Figure 27) causes a higher first-period selling quantity (Figure 28).

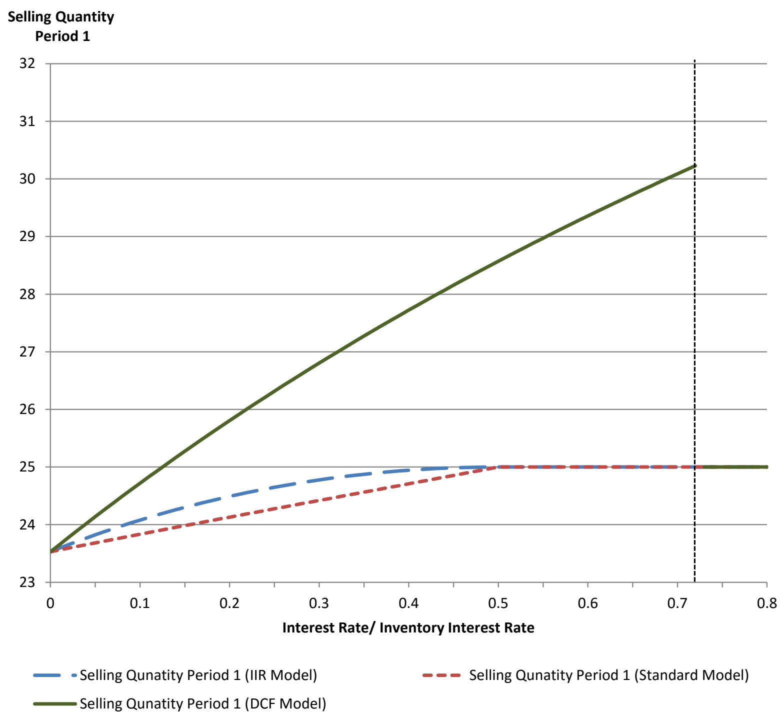

Figure 28: Development of First-period Selling Quantity in the DCF Model

In the DCF model, an increase of the interest rate (i^{DCF}) causes a stronger increase of the selling quantity than in the SM and IIR model. Both in the standard and in the IIR model, the first-period selling quantity cannot exceed the selling quantity of the static solution. Thus, the double marginalization effect is larger in period one, if strategic inventories are used (i.e. in the dynamic solution). Only due to a lower double marginalization effect in period two that is caused by the strategic inventory, an overall improvement regarding the profits gets possible (i.e. the profit improvement in the second period due to a lower double marginalization effect must outweigh both the loss due to the increased double marginalization effect of period one and the inventory holding cost). This property does no longer hold for the DCF model. Here, the static first-period selling quantity is already reached under the interest rate of $i^{DCF} = 12.5\%$ and quickly exceeded up to a maximum of $q_1^{DCF} \approx 30.24$ for $i^{DCF} \approx 72.15\%$. Thus, for an interest rate over $i^{DCF} = 12.5\%$, the double marginalization effect in period one is smaller compared to the standard and IIR model. However, the first-period selling quantity cannot reach the quantity of the first-best solution and, therefore, a loss due to the double marginalization effect persists in the first-period of DCF model.

While the effect of an interest rate increase was simple regarding the first-period selling quantity, the impact on the size of the strategic inventory is more complex. On the one hand, an increase of the interest rate increases the holding cost and, thus, reduces the incentive to build up strategic inventory. On the other hand, an increase of the interest rate also leads to a lower first-period wholesale price in comparison to the IIR model, because the supplier wants to increase the buyer's incentive to build up strategic inventory (i.e. pull his realization of the profits into the first-period).

The comparison of the inventory reaction function of the IIR (6.13) model and of the DCF model (6.36) showed that the functions are identical. Hence, under an identical interest rate (DCF model) and inventory interest rate (IIR model) an identical first-period wholesale price would lead to an identical size of the strategic inventory. However, as Figure 27 has shown, the first-period wholesale price is smaller in the DCF model compared to the IIR model, because the supplier aims to shift the realization of profits into the first-period. Consequently, in comparison to the IIR the buyer builds up a larger strategic inventory in the DCF model (Figure 29).

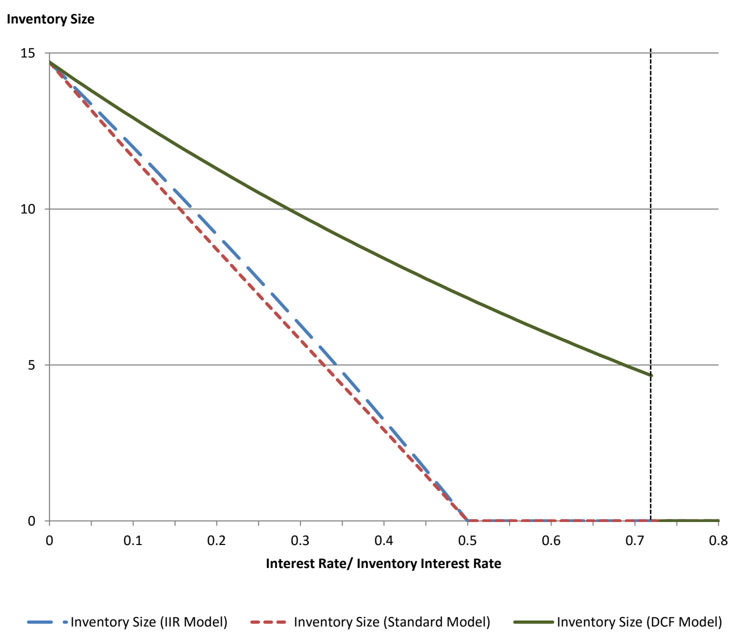

Figure 29: Development of the Inventory Size in the DCF Model

In both the IIR and the DCF model, the supplier lowers the first-period wholesale price, if the holding costs increase. However, in comparison to the standard and the IIR model increasing holding costs lead to a much higher first-period wholesale price reduction, because of the supplier's increased incentive to pull profits into the first-period in the DCF model. Consequently, the benefit from lower purchasing costs partly compensates the disadvantage of increased holding costs, which leads to a weaker decrease of the of strategic inventory size (Figure 29). Thus, the supplier successfully encourages the buyer to maintain a larger strategic inventory size in case of higher holding cost. However, while the supplier benefits from the earlier profit realization, a higher strategic inventory will also force him to set a lower second period wholesale price.

Additionally, the differences regarding the total holding cost have to be considered. In comparison to the IIR model, the purchasing cost (w_1^e) are lower in the DCF model for an identical interest rate (i^{DCF}) and inventory interest rate (i^{IIR}). Meanwhile, the size of the strategic inventory (I^{DCF}, I^{IIR}) is higher in the DCF model. Together, these two effects could either lead to lower or higher total holding costs ($i^e \cdot w_1^e \cdot q_1^e$).

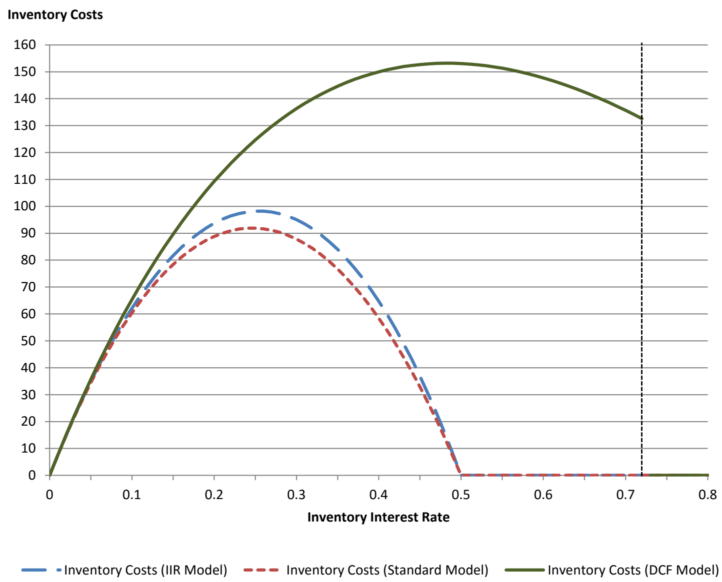

Figure 30: Total Holding Costs in the DCF Model

Despite the lower purchasing cost the increased amount of strategic inventory leads to higher total holding costs in comparison to both the standard and the IIR model. As Figure 30 shows, the difference is stronger for interest rates over $i^e \approx 0.25$, because the holding cost reaches its maximum in the standard and IIR model at this point and slowly decreases from this point on, while further increasing in the DCF model.

As already mentioned, a higher strategic inventory size of the buyer in the second period, forces the supplier to set a lower second period wholesale price as he only sets the price for the buyer's residual demand (Figure 31).

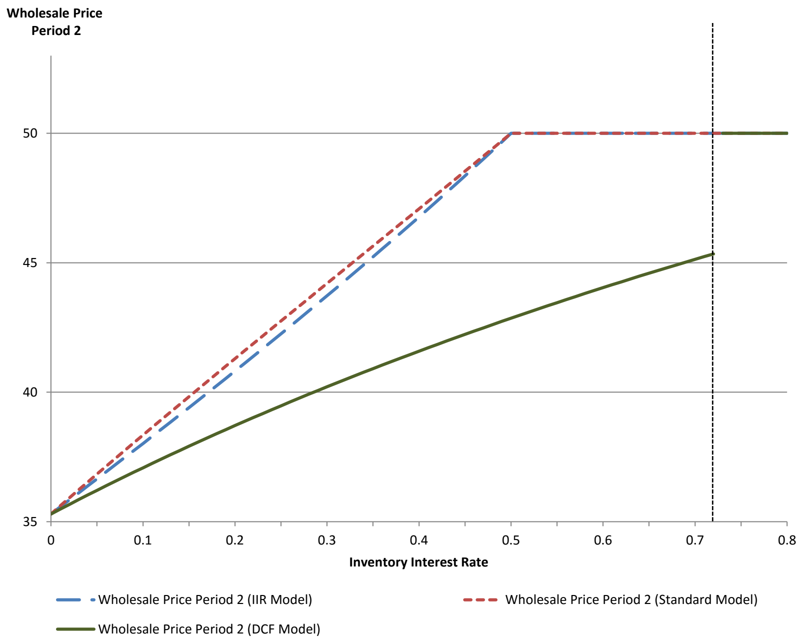

Figure 31: Second Period Wholesale Price in the DCF Model

A lower wholesale price generally drives the selling quantity up. Figure 32 shows that under increasing interest rate, the impact of strategic inventory is decreasing much slower in the DCF model. Hence, the double marginalization effect is again reduced to a larger extent in the DCF model than in the SM and the IIR model. Moreover, within the dynamic solution the second period wholesale price is no longer always smaller than the first-period wholesale price: Comparing the first and second period wholesale price (given in Table 14) shows that for $h^{DCF} = 0$ and $i^{DCF} > 0.5$ the supplier will set a lower first-period wholesale price in the first-period:

$$w_1^{DCF} \overset{?}{<} w_2^{DCF} \tag{6.52}$$

$$\frac{9a - 2\cdot\left(1+i^{DCF}\right)h^{DCF}}{17+8i^{DCF}} \overset{?}{<} \frac{6\cdot\left(1+i^{DCF}\right)a + 2\cdot\left(5+7i^{DCF}+2\left(i^{DCF}\right)^2\right)h^{DCF}}{17+8i^{DCF}} \tag{6.53}$$

For $h^{DCF} = 0$:[51]

$$9a \overset{?}{<} 6\left(1+i^{DCF}\right)a \tag{6.54}$$

[51] A general analysis with $h^{DCF} \geq 0$, $i^{DCF} \geq 0$ can be found on p. 93 of Desai et al (2010).

$$i^{DCF} > 0.5 \tag{6.55}$$

Hence, for $i^{DCF} > 0.5$ the effect that lowers the first-period wholesale price because the supplier wants to increase the buyer's incentive to build up inventory is stronger than the effect of the remaining strategic inventory that lowers the second period wholesale price.

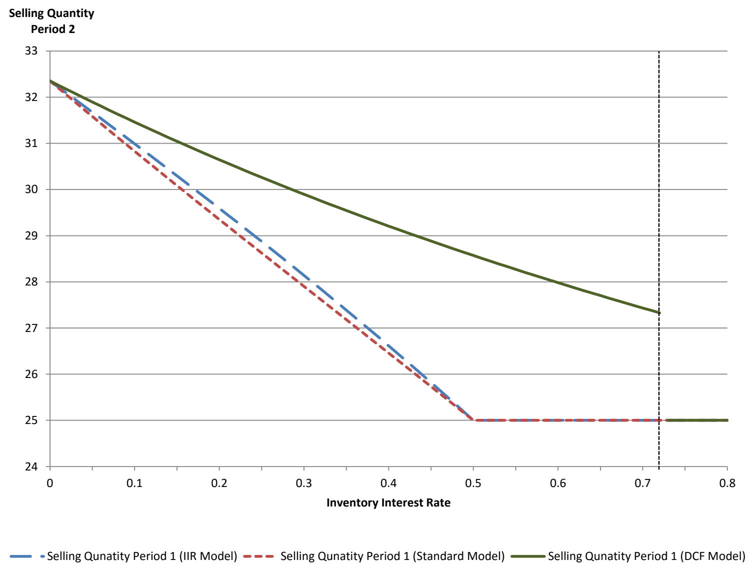

Figure 32: Second Period Selling Quantity in the DCF Model

The analysis of the supply chain member's decision variables has shown strong structural differences between the DCF model on the one hand and the SM and the IIR model on the other hand. These differences will likely also cause differences regarding the supply chain profit and its distribution among the supply chain members.

In the DCF model, the double marginalization effect was reduced in both periods. In the SM and IIR model, the double marginalization effect was only decreased in the second period, while it was even increased in the first-period. Moreover in the second period, the reduction of the double marginalization effect is more salient in the DCF model than in the standard and IIR model. This stronger reduction positively affects the supply chain profit of the DCF model. However, this enhancement must be set against the increased holding costs in the DCF model that are carried by the buyer (Figure 30).

Figure 34 shows that if the respective per unit holding costs (i.e. h^{SM}, h^{DCF}) and the (inventory) interest rates (i.e. i^{IIR}, i^{DCF}) are set to zero, all three models lead to an identical solution. In contrast to the standard and the IIR model, increasing interest rates lead to increasing supply chain efficiency (ratio of first-best profit). Hence, the profit enhancement due to the reduction of double marginalization outweighs the increase of the holding costs. The advantage regarding the supply chain profit increases, if the holding cost rate increases. Hence, in contrast to the standard model and as in the IIR model, there exists no area of interest rates, in which the supply chain profits are lowered due to the effects of strategic inventory.

Moreover, strategic inventory is used up unto an interest rate of $i^{DCF} \approx 72.15\%$, while it is only used up unto an interest rate of $i^{IIR} = 50\%$ in the IIR model.[52] Hence, the positive effects of strategic inventory persist for a larger area of holding cost.

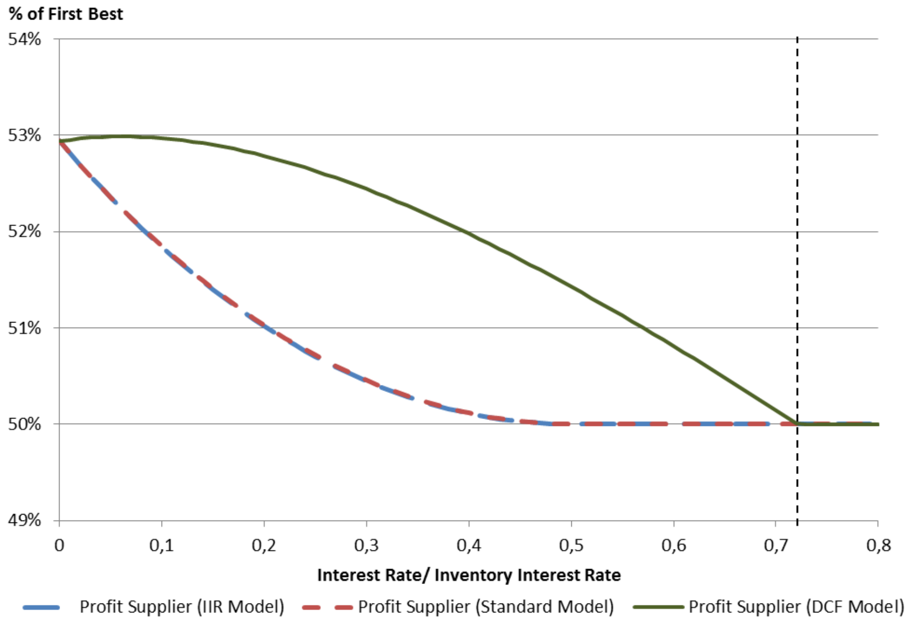

Figure 33: Comparison Supplier Profit in the DCF Model[53]

[52] In the standard model, strategic inventory is also only used up to a holding cost rate that corresponds to an interest rate of 50%.

[53] Profits are weighted with the corresponding first-best profit of the respective model (see footnote 54).

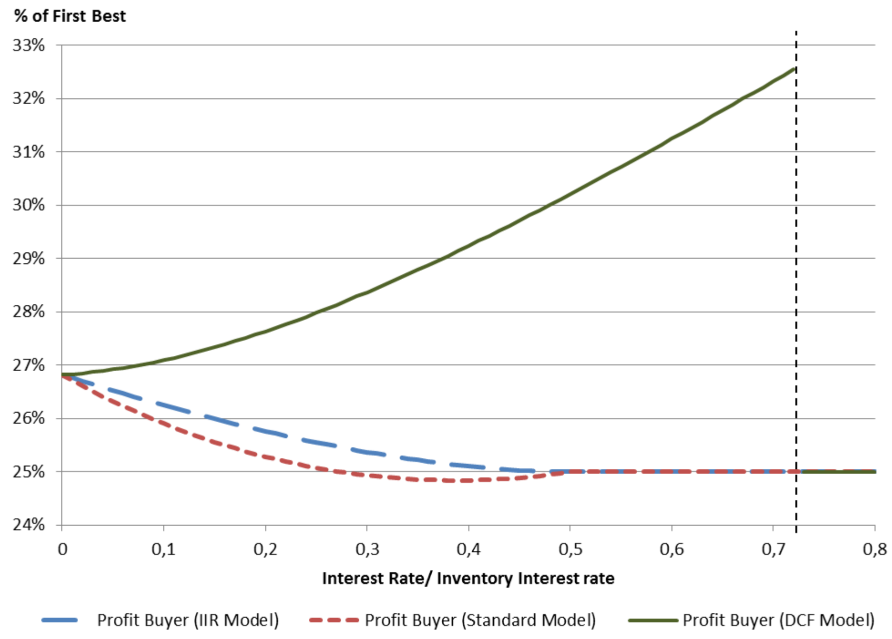

Figure 34: Comparison Buyer Profit in the DCF Model[54]

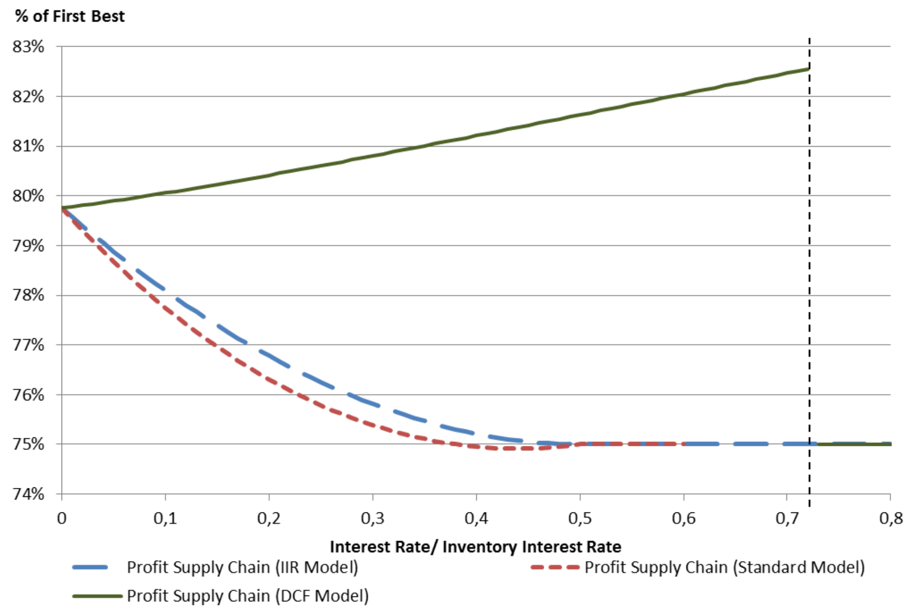

Figure 35: Comparison Supply Chain Profit in the DCF Model[55]

While the supply chain profit in the DCF model dominates the corresponding profits of the standard and the IIR model, it is unclear if both buyer and supplier benefit from the increase. The supplier tries to lessen the profit reduction caused by an increased interest rate by offering lower first-period wholesale prices to shift profits into the first-period.

[54] The discounting of the second period profits in the DCF model lowers the overall profits, if the interest rate increases. To achieve a proper comparison with the standard and the IIR model that do not contain this property, the profits are set in comparison to the corresponding first-best solution (i.e. the solution of the DCF model is weighted with the first-best solution of the DCF model given in Table 15).

[55] Profits are weighted with the corresponding first-best profit of the respective model (see footnote 54).

These lower prices also reduce the double marginalization effect. However, only the buyer benefits from the additional profits of the lowered double marginalization effect, while the supplier gives up a part of his profit. Last, the profit of the buyer gets decreased by the increased total holding cost.

The comparison of the solution of the DCF model with the standard and the IIR model (Figure 34) shows that the reduction of the double marginalization has the largest impact on the solution. Thus, the largest improvement towards the standard and IIR model can be seen on side of the buyer. The supplier’s profit, in turn, decreases under a rising interest rate. Thus, he can only partly compensate the reduction of his profit that is caused by an increased interest rate. As a result, the supplier prefers the static solution without strategic inventory, if the interest rate is high ($i^{DCF} \geq 72.15\%$). In this case, he will implement the static solution by setting the first-period wholesale price to the static value. By doing so, he can successfully prohibit the buyer from building up a strategic inventory, because of the high inventory holding cost due to the high interest rate.

6.3 Conclusion and Insights under Price-dependent Holding Costs

In summary, strategic inventory also plays a pivotal role in the DCF model. However, both in its utilization and its impact several differences towards the standard and the IIR model exist. In those models strategic inventory had two functions. The buyer builds up strategic inventory in the first-period, to lower the monopoly power of the supplier in the second period. The supplier, in turn, can implement an internal two-price block scheme for the second period by letting the buyer build up strategic inventory in the first-period. Hence, a part of the buyer’s demand is fulfilled by the units that have been purchased for the high first-period wholesale price and the supplier can price for the buyer’s residual demand with a low second period wholesale price. Because of these effects the supplier always benefits from the utilization of strategic inventory both in the standard and in the IIR model. The buyer also benefits from the usage of strategic inventory in both the standard (except for high holding cost) as well as in the IIR model, because the lower average wholesale price outweighs the increased holding costs.

In the DCF model, however, an additional utilization of strategic inventory arises. Here, the interest rate now affects both supply chain members as it determines how strongly they prefer generating profits in the first-period over generating profits in the second period. In case of the supplier, the interest rate determines the costs of selling units in the second period instead of already realizing those sales in the first-period (i.e. the higher the interest rate is the higher is the supplier's incentive to sell units in the first-period). As a result, the supplier has an incentive to lower the first-period wholesale price in order to encourage the buyer to build up a higher strategic inventory as this partly pulls the supplier's realization of profits into the first-period. This new utilization of strategic inventory in the DCF model massively impacts the model's solution.

The supplier has to balance the usage of strategic inventory between the function to implement a well-fitting two-price block scheme and the function to reduce the impact of the discounting of the second period profit. For low interest rates, the latter is less important and the supplier can focus on setting diverse prices between the first and the second period in order to implement a well-fitting two-pricing block scheme. For increasing interest rates, however, the supplier has to focus on reducing the effects of the discounting by lowering the first-period wholesale price. This usage is getting so dominating over the two-price block effect that for $i^{DCF} > 50\%$ the first-period wholesale price is even lower than the second period wholesale price.[56]

The additional pressure on the supplier due to the discounting has the following overall effects in comparison to the SM and the IIR model: The supplier has a stronger incentive to lower the first-period wholesale price to partly circumvent the discounting. The lower first-period wholesale price causes a higher first-period selling quantity and, therefore, decreases the double marginalization effect. Also, it increases the size of the strategic inventory. Despite the lower purchasing cost, the total inventory holding costs are higher due to the increased amount of strategic inventory. Additionally, the higher size of strategic inventory leads to a lower second period wholesale price setting of the supplier and, hence, an increased second period selling quantity. Consequently, the double marginalization effect is also decreased in the second period.

Overall, the profit enhancing effect (lower double marginalization) within the DCF model outweighs the negative effect (higher total holding costs) leading to a higher supply chain profit. Moreover, strategic inventory is utilized up to a much higher holding cost rate (i.e. $i^{DCF} < 72.15\%$ in the DCF model versus $i^{IIR} < 50\%$ in the IIR model). However, the buyer gains a higher fraction of the additional supply chain profit. The supplier, in turn, is only better off for low to medium interest rates (Figure 34). For high interest rates ($i^{DCF} > 72.15\%$) the supplier prefers the static solution and will implement it by setting the static first-period wholesale price.

[56] This, of course, still is a two-price block scheme, from which, however, only the buyer profits, while it reduces the profit of the supplier.

7 Strategic Inventories and the Role of Forwards

Both the theoretical as well as the behavioral analyses have shown that strategic inventory affects the solution of multi-period supply chain interactions. In case of a simple wholesale price contract, strategic inventory reduces the double marginalization effect. But, at the same time the inventory causes holding costs. In the standard model, the profit improvement due to double marginalization reduction in the dynamic solution outweighs the occurring holding costs for small to medium holding values $\left(h^{SM} < {}^{55a}\!/_{288} \approx 0.191a\right)$.[57] For $h^{SM} < {}^{21a}\!/_{152} \approx 0.138a$ both supplier's and buyer's profits exceed the respective profit of a solution without an utilization of strategic inventory (i.e. of the static solution).[58] Under IIR conditions, the utilization of strategic inventory always increases the profits of all supply chain members in comparison to the respective static solution. Thus, under a wholesale price contract the possibility to use strategic inventory is beneficial for both supply chain members in the majority of cases.

However, Anand et al. (2008) have shown that the supplier is no longer able to implement a dynamic vertical contract that simultaneously coordinates the supply chain and allows a free distribution of the supply chain profit between supplier and buyer if the buyer can utilize strategic inventory.

Hereby, achieving the first-best solution is prohibited by two reasons: First, the buyer is able to influence the price setting of the supplier by using strategic inventory. As a result, the supplier cannot determine wholesale prices that enable him to generate the maximum (i.e. first-best) supply chain profit and at the same time to extract the entire supply chain profit. Thus, he will determine wholesale prices that maximize his individual profit at the cost of overall supply chain profit. Second, as soon as the buyer builds up inventory, holding costs will occur. And, as the first-best solution does not contain any holding costs, a solution that includes such costs will always be inferior to the first-best solution.

These drawbacks of strategic inventory can only be removed by more sophisticated contracts. Anand et al. (2008) already proved in their fundamental paper that in the space of general commitment contracts, the supplier is able to achieve first-best profits (i.e. able to implement the first-best solution and to extract away all residual profits of

[57] See page 38 in section 3.

[58] See page 41 of section 3.

the buyer).[59] However, their commitment contract is a variant of a supplier who sells his firm to the buyer for the price of the sum of all coming first-best period profits. A less restricted contract variant to achieve first-best profits was shown by Arya and Mittendorf (2013). They also investigate a nonlinear pricing scheme (i.e. a quantity-contingent pricing scheme) that is supplemented by complex supplier-to-customer rebates. By implementing such a contract, the supplier can eliminate strategic inventory and achieve channel coordination.

Both the coordinating contracts of Anand et al. (2008) and of Arya and Mittendorf (2013) are problematic in terms of practical realization. In the commitment contract of Anand et al. (2008) the buyer must pay the sum of all future profits to the supplier. In Arya and Mittendorf (2013) a nonlinear pricing scheme is mixed with complex supplier-to-customer rebates. In this chapter, an easier way to circumvent the negative effects that are introduced by the buyer's option to use strategic inventory gets investigated. The presented model with forwards contracts ($i = FW$) aims on abolishing the physical form of strategic inventory. This target is identical to the commitment contract which was introduced by Anand et al. (2008). The striking difference is that in the model with forwards contracts, the buyer is still allowed to influence the supplier's prices via using non-physical stocks.

In contrast to the standard model of Anand et al. (2008), the forward contract model (FW) includes an additional option regarding the interaction within the supply chain: The supplier allows the buyer to place forwards (i.e. preorders) at a specific forward price that is determined by the supplier.[60] Hence, in each period, the supplier will set both the regular wholesale price of the actual period as well as the forward prices for preorders. And, the buyer has to decide both about her buying quantity as well as about her forward quantity.

In the forward contract model the time horizon remains at two periods (i.e. identical to the standard model). Accordingly, in the first-period, the supplier determines both the first-period wholesale price (w_1^{FW}) as well as the forward price (v^{FW}). The buyer, who faces a deterministic, price sensitive demand in each period ($p_t^{FW} = a - b \cdot q_t^{FW}$), then determines her buying (Q_1^{FW}) and her selling (q_1^{FW}) quantity. Still, unsold units from

[59] See Theorem 6 (p. 1802) of Anand et al (2008).

[60] While committing on supplying the buyer with goods to the condition of the forward price, the pricing scheme can only be interpreted as a commitment contract if the variable second period price is not smaller than the forward price. Table 16 shows that this is never the case.

period one will be carried over to the second period as inventory ($I^{FW} = Q_1^{FW} - q_1^{FW}$) and cause holding costs of h^{FW}. In contrast to the standard model, the buyer has the additional option to place preorders (O^{FW}) in the first-period. These preordered goods will be delivered in the second period at the cost of a unit price of v^{FW}. As the supplier produces preordered goods just in time, forwards will neither cause holding costs at the buyer's nor at the supplier's side. Further, no discrimination regarding the source of the goods that the buyer possesses at the start of the second period (I^{FW}, O^{FW}) is made. Hence, both preordered and stored goods (i.e. inventory) can be used absolutely identically in period two.

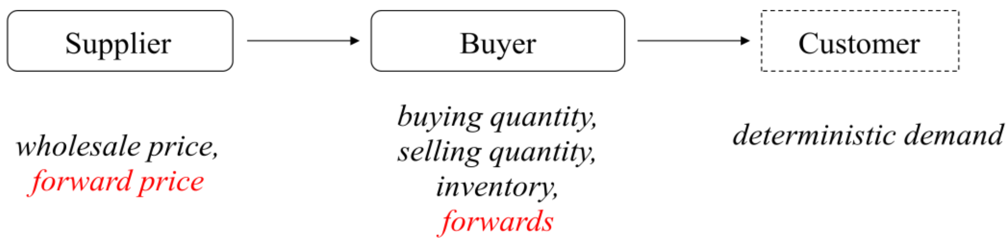

Figure 36: Decisions in the Strategic Inventory Model with Forward Buying

In the second period, the supplier only decides about his second period wholesale price. Determining a forwards price is unnecessary, because no further period is following and unused units have a salvage value of zero. The buyer also only has one decision left in the second period. Following the general structure of the derivation of the standard model, her selling quantity decision will be treated as the decision variable, while the other variables are calculated based on this decision (i.e.: $Q_2^{FW} = q_2^{FW} - \left(I^{FW} + O^{FW}\right)$ and $p_2^{FW} = a - b \cdot q_2^{FW}$). Moreover, both supplier and buyer possess perfect information. In summary, regarding the decisions the only difference between standard and the FW model are the forward price and forward quantity decisions in the first-period.

As outlined, the effect of inventory and forwards is identical in the second period as no discrimination between them is made. Likely, the buyer will be using the source that is less expensive for her. Hence, only if

$$v^{FW} \leq w_1^{FW} + h^{FW} \tag{7.1}$$

is fulfilled, the buyer will be willing to use preorders instead of inventory. However, whether or not condition (7.1) is fulfilled, is fully in control of the supplier.[61] The following analysis will show that the supplier has an incentive to let the buyer use forwards instead of inventory and how this affects the profit of the supply chain and its distribution among buyer and supplier.

7.1 First-Best and Static Solution of the Strategic Inventory Model with Forward Contracts

Before the dynamic solution (i.e. a solution that includes the utilization of strategic inventory and/ or preorders) of the FW model is derived, the corresponding benchmarks (first-best and static) will be determined. Again, the first-best and the static solution (i.e. solution with neither strategic inventory nor forwards) serve as benchmarks for the dynamic solution.

In the first-best solution, the supplier sets a wholesale price that corresponds to his marginal costs (i.e. $w_1^{FW} = w_2^{FW} = 0$). Hence, the supplier has no incentive to offer the buyer the option to buy forwards and no deviation from the standard model occur. In the static solution, strategic inventories as well as preorders are excluded by definition. Hence, the static solution is also unaffected by the extension to use preorders that was introduced in the FW model. As both the first-best and the static solution are not affected by the introduction of the option to use forwards, they remain identical to the respective solutions of the standard model. Hence, the standard and the FW model can be perfectly compared to each other. The benchmarks of the first-best and static solution are displayed in comparison to the dynamic solution of the FW model in Table 16 at the end of section 7.2.

In the following, the dynamic solution of the FW model will be investigated. This analysis also tests whether the supplier will implement the dynamic solution with strategic inventory and/ or forwards or if he will implement the static solution by using a commitment contract.

7.2 Dynamic Solution of the Forward Contract Model

As the supplier offers the option to preorder units at the forward price, both the supplier's as well as the buyer's profit function need to be modified. The supplier will

[61] The supplier can simply prohibit forwards by setting v^{FW} prohibitively high. Hence, the solution (and profits) of the standard model always serves as an outside option for the supplier.

also get the revenues that arise from the buyer's preorders ($v^{FW} \cdot O^{FW}$). Additionally, in the second period preordered goods will reduce the purchasing quantity of the buyer in the same manner as inventory does. The supplier's profit function in the FW model, therefore, is:

$$\pi_S^{FW} = w_1^{FW} \cdot \left(q_1^{FW} + I^{FW}\right) + v^{FW} \cdot O^{FW} + w_2^{FW} \cdot \left(q_2^{FW} - I^{FW} - O^{FW}\right) \tag{7.2}$$

The buyer, in turn, has to pay for the preordered goods, if she decides to utilize forwards. Also, forwards reduce the purchasing quantity in the second period that is needed to satisfy the selling quantity:

$$\begin{aligned}\pi_B^{FW} &= p_1^{FW} \cdot q_1^{FW} - w_1^{FW} \cdot \left(q_1^{FW} + I^{FW}\right) - v^{FW} \cdot O^{FW} - h^{FW} \cdot I^{FW} \\ &\quad + p_2^{FW} \cdot q_2^{FW} - w_2^{FW} \cdot \left(q_2^{FW} - I^{FW} - O^{FW}\right) \\ &= \left(a - b \cdot q_1^{FW}\right) \cdot q_1^{FW} - w_1^{FW} \cdot \left(q_1 + I^{FW}\right) - v^{FW} \cdot O^{FW} - h^{FW} \cdot I^{FW} \\ &\quad + \left(a - b \cdot q_2^{FW}\right) \cdot q_2^{FW} - w_2^{FW} \cdot \left(q_2^{FW} - I^{FW} - O^{FW}\right)\end{aligned} \tag{7.3}$$

Combining both the suppliers and buyers profit function gives the supply chain profit:

$$\pi_{SC}^{FW} = \left(a - b \cdot q_1^{FW}\right) \cdot q_1^{FW} - h^{FW} \cdot I^{FW} + \left(a - b \cdot q_2^{FW}\right) \cdot q_2^{FW} \tag{7.4}$$

Hence, the supply chain profit function remains unaffected by the introduction of forwards as they are just a further transfer payment between supplier and buyer without causing any deadweight loss.

As the solution of the FW model is also obtained by backwards induction, the optimization starts with the last decision (i.e. the second period selling quantity decision of the buyer) within the model:

$$\begin{aligned}\frac{\partial \pi_B^{FW}\left(q_1^{FW}, I^{FW}, O^{FW}, q_2^{FW}\right)}{\partial q_2^{FW}} &= a - 2bq_2^{FW} - w_2^{FW} = 0 \\ s.t.: q_2^{FW} &\geq I^{FW} + O^{FW}\end{aligned} \tag{7.5}$$

$$q_2^{FW}\left(w_2^{FW}\right) = \max\left\{I^{FW} + O^{FW}, \frac{a - w_2^{FW}}{2b}\right\} \tag{7.6}$$

Hence, the buyer will only purchase additional items in the second period, if

$$w_2^{FW} < a - 2b \cdot \left(I^{FW} + O^{FW}\right). \tag{7.7}$$

However, optimizing his second-period profit for given values of I^{FW} and O^{FW} the supplier has no incentive to set a prohibitively high second period wholesale price. It is assumed that condition (7.7) is fulfilled and (7.6) can be simplified to $q_2^{FW}\left(w_2^{FW}\right)=\frac{a-w_2^{FW}}{2b}$.

Inserting the buyer's second period response function (7.6) into the supplier's profit function (7.2) yields:

$$\pi_S^{FW} = w_1^{FW}\cdot\left(q_1^{FW}+I^{FW}\right)+v^{FW}\cdot O^{FW}+w_2^{FW}\cdot\left(\frac{a-w_2^{FW}}{2b}-I^{FW}-O^{FW}\right) \quad (7.8)$$

The supplier's second period response function, therefore, is:

$$\frac{\pi_S^{FW}\left(w_1^{FW},w_2^{FW}\right)}{\partial w_2^{FW}}=\frac{a-2\cdot w_2^{FW}}{2b}-I^{FW}-O^{FW}=0\,, \quad (7.9)$$

which results in the following response function:

$$w_2^{FW}\left(I^{FW},O^{FW}\right)=\max\left\{0,\tfrac{a}{2}-b\cdot\left(I^{FW}+O^{FW}\right)\right\}. \quad (7.10)$$

Inserting the right hand side of the supplier's second period reaction function (7.10) into the buyer's condition to buy additional units in the second period (7.7) shows that (7.7) is fulfilled if

$$I^{FW}+O^{FW}<\tfrac{a}{2b}\,, \quad (7.11)$$

which is also the supplier's condition to set positive second period wholesale prices (see (7.10)). As $\frac{a}{2b}$ is the buyer's first-best selling quantity (i.e. the quantity the buyer would purchase from the supplier and sell to her customers at a wholesale price of zero), it is straightforward that the supplier's second period wholesale price becomes irrelevant, if the buyer already owns $\frac{a}{2b}$ units through strategic inventory and/ or forwards. Moreover, it shows that, if the buyer has less items than $\frac{a}{2b}$ available, the supplier will always set the second period wholesale prize larger than zero and the buyer will always buy additional items in the second period.

Further, the supplier's second period reaction function (7.10) shows that both strategic inventory and forwards affect the supplier's second period wholesale price setting in the same way. Hence, forwards can be used in the exactly same strategic manner as strategic inventory.

Integrating the buyer's (7.6) and supplier's (7.10) second period reaction function into the buyer's profit function (7.3) gives the relevant profit function for the buyer's decisions about the first-period selling quantity (q_1^{FW}), the strategic inventory (I^{FW}) and the forwards (O^{FW}):

$$\pi_B^{FW} = \frac{a^2 + 12b \cdot a \cdot \left(I^{FW} + O^{FW}\right) + 16b \cdot a \cdot q_1^{FW} - 16b^2 \cdot \left(q_1^{FW}\right)^2 - 16b \cdot w_1^{FW} \cdot \left(q_1^{FW} + I^{FW}\right)}{16b} + \frac{-16b \cdot \left(v^{FW} \cdot O^{FW} + h^{FW} \cdot I^{FW}\right) - 12b^2 \cdot \left(\left(I^{FW}\right)^2 + \left(O^{FW}\right)^2\right) - 24b^2 \cdot I^{FW} \cdot O^{FW}}{16b} \tag{7.12}$$

Starting with the selling quantity optimization leads to:

$$\frac{\partial \pi_B^{FW}}{\partial q_1^{FW}} = a - 2bq_1^{FW} - w_1^{FW} = 0 \tag{7.13}$$

$$q_1^{FW}\left(w_1^{FW}\right) = \max\left\{0,\ \frac{a - w_1^{FW}}{2b}\right\} \tag{7.14}$$

Therefore, the buyer's first-period selling quantity reaction function is identical to the standard model. Still, the selling quantity is only dependent on the first-period wholesale price and, given w_1^{FW}, the decision is made independently from the decisions about the size of strategic inventory and forwards.

Next, the strategic inventory size gets determined:

$$\frac{\partial \pi_B^{FW}}{\partial I^{FW}} = \frac{3}{4}a - w_1^{FW} - h^{FW} - \frac{3}{2}b \cdot \left(I^{FW} + O^{FW}\right) = 0 \tag{7.15}$$

$$I^{FW}(w_1^{FW}, O^{FW}) = \max\left\{0, \frac{a}{2b} - \frac{2}{3b} \cdot \left(w_1^{FW} + h^{FW}\right) - O^{FW}\right\} \tag{7.16}$$

The buyer's strategic inventory reaction function can be divided into two-parts. The first part $\left(\frac{a}{2b} - \frac{2}{3b} \cdot \left(w_1^{FW} + h^{FW}\right)\right)$ is identical to the reaction function of the standard model. Still, a higher first-period wholesale price or higher holding cost reduces the size of the strategic inventory. New in the FW model, however, is the second part ($-O^{FW}$). Now, the size of the strategic inventory gets reduced by the amount of preordered items.

Hence, the first part of (7.16) defines a target level of strategic items (this target level of strategic items can be either fulfilled by inventory or by forwards) and only the amount that is not fulfilled by forwards will be met by strategic inventory.

Last, the optimal quantity of forwards gets optimized:

$$\frac{\partial \pi_B^{FW}}{\partial O^{FW}} = \frac{3}{4}a - v^{FW} - \frac{3}{2}b \cdot \left(I^{FW} + O^{FW}\right) \tag{7.17}$$

$$O^{FW}\left(v^{FW}, I^{FW}\right) = \max\left\{0, \frac{a}{2b} - \frac{2}{3b}v^{FW} - I^{FW}\right\} \tag{7.18}$$

This reaction function (7.18) is quite similar to the strategic inventory (7.16) reaction function. Comparing it piece by piece shows that the first part is almost identical to the inventory reaction function. Again, this can be interpreted as the target size of strategic items. Comparing this two target sizes shows that the target size of the strategic inventory is higher if $w_1^{FW} + h^{FW} < v^{FW}$ (et vice versa). Moreover, the amount of forwards gets reduced by the size of the chosen strategic inventory as described by the second part of (7.18) (i.e. $-I^{FW}$). This leads to the following intuitive solution: The buyer will build up her strategic item pool from the source with the higher target level and, therefore, only use the source with the cheaper overall purchasing costs ($w_1^{FW} + h^{FW}$ versus v^{FW}). The other source, in contrast, will be neglected:[62]

$$I^{FW}(w_1^{FW}) = \begin{cases} 0 & if \quad w_1^{FW} + h^{FW} \geq v^{FW} \\ \frac{a}{2b} - \frac{2}{3b} \cdot \left(w_1^{FW} + h^{FW}\right) & else \end{cases} \tag{7.19}$$

and:

$$O^{FW}(w_1^{FW}) = \begin{cases} 0 & if \quad w_1^{FW} + h^{FW} < v^{FW} \\ \frac{a}{2b} - \frac{2}{3b}v^{FW} & else \end{cases} \tag{7.20}$$

As the supplier decides about both w_1^{FW} and v^{FW}, he can also control which source the buyer will use. Hence, in the following, the two cases described by (7.19) and (7.20) will be analyzed individually:

[62] In case of indifference (i.e. $w_1^{FW} + h^{FW} = v^{FW}$), it is assumed that the buyer will only use forwards instead of strategic inventory as the holding costs of physical inventory would cause a deadweight lost from the perspective of the overall supply chain.

Case 1: Buyer only uses strategic inventory ($w_1^{FW}+h^{FW}<v^{FW}$):

In this simple case $I^{FW}(w_1^{FW})=\frac{a}{2b}-\frac{2}{3b}\cdot\left(w_1^{FW}+h^{FW}\right)$ and $O^{FW}=0$. Hence, this solution is identical to the solution of the standard model (Table 2).

Case 2: Buyer only uses forwards ($w_1^{FW}+h^{FW}\geq v^{FW}$):

In this case, forwards will replace strategic inventory (i.e. $O^{FW}(w_1^{FW})=\frac{a}{2b}-\frac{2}{3b}v^{FW}$ and $I^{FW}=0$).

The supplier has the following optimization problem:

$$\pi_S^{FW}\left(w_1^{FW},v_1^{FW}\right)=\frac{w_1^{FW}\cdot\left(a-w_1^{FW}\right)}{2b}-\frac{v^{FW}\cdot\left(4v^{FW}-3a\right)}{6b}+\frac{2}{3}\cdot v^{FW}\cdot\left(\frac{a-\frac{2}{3}v^{FW}}{2b}+\frac{4v^{FW}-3a}{6b}\right) \tag{7.21}$$

$$\text{s.t.}: w_1^{FW}+h^{FW}\geq v^{FW}$$

with the corresponding Lagrange function:

$$\begin{aligned}\pi_S^{FW}\left(w_1^{FW},v_1^{FW},u\right)=&\frac{w_1^{FW}\cdot\left(a-w_1^{FW}\right)}{2b}-\frac{v^{FW}\cdot\left(4v^{FW}-3a\right)}{6b}+\frac{2}{3}\cdot v^{FW}\cdot\left(\frac{a-\frac{2}{3}v^{FW}}{2b}+\frac{4v^{FW}-3a}{6b}\right)\\&+u\cdot\left(w_1^{FW}+h^{FW}-v^{FW}\right)\end{aligned} \tag{7.22}$$

The partial first-order derivatives are:

$$\frac{\partial\pi_S^{FW}}{\partial w_1^{FW}}=\frac{a}{2b}-\frac{w_1^{FW}}{b}+u \tag{7.23}$$

$$\frac{\partial\pi_S^{FW}}{\partial v^{FW}}=\frac{a}{2b}-\frac{8}{9b}\cdot v^{FW}-u \tag{7.24}$$

$$\frac{\partial\pi_S^{FW}}{\partial u}=w_1^{FW}+h^{FW}-v^{FW} \tag{7.25}$$

Setting the derivatives (7.23) and (7.24) equal to zero yields:

$$w_1^{FW}=\frac{1}{2}a+ub \tag{7.26}$$

$$v^{FW} = \frac{9}{16}a - u \cdot \frac{9}{8}b \tag{7.27}$$

To solve this problem the Karush-Kuhn-Tucker approach is applied:

Case 2.1: $u > 0$, $w_1^{FW} + h^{FW} - v^{FW} = 0$

In the first case, the restriction within the optimization problem (7.21) is binding. Hence, the supplier has to balance the first-period wholesale and the forward price, so that the forward price only exceeds the first-period wholesale price by the holding costs. The parameter u shows the extent of the supplier's profit increase if the restriction of (7.21) would be relaxed by one unit (i.e. if the buyer's holding cost would be one unit higher).

Inserting (7.26) and (7.27) into $w_1^{FW} + h^{FW} = v^{FW}$ gives:

$$\begin{aligned} w_1^{FW} + h^{FW} &= v^{FW} \\ \frac{1}{2}a + u \cdot b + h^{FW} &= \frac{9}{16} \cdot a - \frac{9}{8} b \cdot u \end{aligned}$$

$$u = \frac{a}{34b} - \frac{8}{17b} \cdot h^{FW} \tag{7.28}$$

By inserting (7.28) into (7.26) and (7.27) the first-period wholesale and the forward price can be calculated:

$$w_1^{FW} = \frac{9}{17} \cdot a - \frac{8}{17} \cdot h^{FW} \tag{7.29}$$

$$v^{FW} = \frac{9}{17} \cdot a + \frac{9}{17} \cdot h^{FW} \tag{7.30}$$

Examining the two first-period prices shows that the first-period wholesale price in (7.29) decreases and that the forward price in (7.30) increases if the holding cost (h^{FW}) increase, because the supplier has to balance the first-period wholesale and the forward price in order to ensure that the buyer uses forwards instead of strategic inventory (i.e. to fulfill $w_1^{FW} + h^{FW} - v^{FW} = 0$). Hence, he cannot set the first-period wholesale and the forward price independently from each other.

By inserting the two prices into the reaction functions the decision variables and the corresponding profits can be calculated. A summary of the results is given in Table 16.

Case 2.2: $u=0$, $w_1^{FW}+h^{FW}\geq v^{FW}$

In this case, the restriction of (7.21) is not binding. Hence, the supplier can set both the first-period wholesale and the forward price independently from each other and, therefore, to their respective optimal levels.

Inserting $u=0$ into (7.26) and (7.27) directly gives the first-period wholesale and the forward price:

$$w_1^{FW}=\frac{1}{2}a \tag{7.31}$$

$$v^{FW}=\frac{9}{16}a \tag{7.32}$$

The two prices in period one from (7.31) and (7.32) already deliver some interesting insights. In contrast to the standard model the first-period wholesale price is no longer dependent on the holding cost parameter. Instead, the supplier always uses the static price. Hence, the buyer's option to use strategic inventory does no longer influence the wholesale price setting in period one and the supplier can use the first-period wholesale price purely to optimize his income gained by revenue from items that the buyer will directly sell to her customers. Hence, he no longer has to use the first-period wholesale price as a part of the two pricing block implementation. This decoupling leads to a generally lower first-period wholesale price in Case 2.2 of the FW model. Consequently, the double marginalization effect of this solution would be lower than in the standard model, which would improve the overall supply chain profit.

Moreover, the forward price is always higher than the first-period wholesale price. Using the constraint $w_1^{FW}+h^{FW}\geq v^{FW}$ the minimum level of the holding cost parameter can be determined by calculating the difference between the two prices:[63]

$$\begin{aligned} &w_1^{FW}+h^{FW}\geq v^{FW} \\ &\underline{h}^{FW}=v^{FW}-w_1^{FW} \end{aligned}$$

$$\underline{h}^{FW}=\frac{9}{16}\cdot a-\frac{1}{2}a \tag{7.33}$$

$$h^{FW}\geq\frac{a}{16} \tag{7.34}$$

[63] The minimum level of the holding cost can also be found by inserting $u\leq 0$ into (7.28).

Hence, as soon as condition (7.34) is fulfilled, the supplier is able to implement Case 2.2. Otherwise, he can only implement Case 2.1. Table 16 summarizes the solution of Case 2.2.

Table 16: Solution of the Forward Contract Model

	First-Best	Standard Model		Forward Contract Model	
		Static $h^{SM} \geq a/4$	Dynamic $h^{SM} < a/4$	LFW $h^{FW} < a/16$	UFW $h^{FW} \geq a/16$
w_1^e	–	$\frac{a}{2}$	$\frac{9a-2h}{17}$	$\frac{9a-8h}{17}$	$\frac{a}{2}$
v^e	–	–	–	$\frac{9a+9h}{17}$	$\frac{9a}{16}$
q_1^e	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{4a+h}{17b}$	$\frac{4a+4h}{17b}$	$\frac{a}{4b}$
Q_1^e	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{13a-18h}{34b}$	$\frac{13a-4h}{34b}$	$\frac{3a}{8b}$
p_1^e	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{13a-h}{17}$	$\frac{13a-4h}{17}$	$\frac{3a}{4}$
I^e	0	0	$\frac{5a-20h}{34b}$	0	0
O^e	–	–	–	$\frac{5a-12h}{34b}$	$\frac{a}{8b}$
w_2^e	–	$\frac{a}{2}$	$\frac{6a+10h}{17}$	$\frac{6a+6h}{17}$	$\frac{3a}{8}$
q_2^e	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{11a-10h}{34b}$	$\frac{11a-6h}{34b}$	$\frac{5a}{16b}$
Q_2^e	$\frac{a}{2b}$	$\frac{a}{4b}$	$\frac{3a+5h}{17b}$	$\frac{3a+3h}{17b}$	$\frac{3a}{16b}$
p_2^e	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{23a+10h}{34}$	$\frac{234a+3h}{34}$	$\frac{11a}{16}$
π_S^e	–	$\frac{a^2}{4b}$	$\frac{9a^2-4ah+8h^2}{34b}$	$\frac{9a^2+ah-8h^2}{34b}$	$\frac{17a^2}{64b}$
π_B^e	–	$\frac{a^2}{8b}$	$\frac{155a^2-118ah+304h^2}{1156b}$	$\frac{155a^2+38ah+172h^2}{1156b}$	$\frac{35a^2}{256b}$
π_{SC}^e	$\frac{a^2}{2b}$	$\frac{3a^2}{8b}$	$\frac{461a^2-254ah+576h^2}{1156b}$	$\frac{461a^2+72ah-100h^2}{1156b}$	$\frac{103a^2}{256b}$

7.3 Comparisons of the Different Cases within the Forward Contract Model

Reconsidering that the supplier controls whether the buyer uses strategic inventory or forwards, a comparison regarding the profits within the different cases has to be made.

Case 1 can be achieved by setting $w_1^{FW} + h^{FW} < v^{FW}$. In this case, the buyer would only use strategic inventory (for $h^{FW} < \frac{a}{4}$) and the solution would be identical to the solution of the standard model. In Case 2, the buyer will use forwards instead of strategic inventory as $w_1^{FW} + h^{FW} \geq v^{FW}$. However, Case 2 is divided into two sub-cases depending on the level of the holding costs. Case 2.1 is relevant, if the holding costs are low (i.e. $h^{FW} < \frac{a}{16}$). In this case, the supplier has to balance the first-period wholesale and the forward price in order to assure that the buyer uses forwards instead of strategic inventory. This case will be denoted as limited forward solution (LFW). In Case 2.2, the supplier is no longer limited by $w_1^{FW} + h^{FW} \geq v^{FW}$, as the holding cost are high enough (i.e. $h \geq \frac{a}{16}$) that he is able to set both the first-period wholesale price as well as the forward price to his desired levels. Case 2.2 will, therefore, be denoted as unlimited forward solution (UFW).

While it is straightforward that the supplier will always prefer the unlimited over the limited case, it remains unclear whether he prefers the solution of the FW model over the solution of the standard model at all. Figure 37 displays which profit comparisons have to applied:

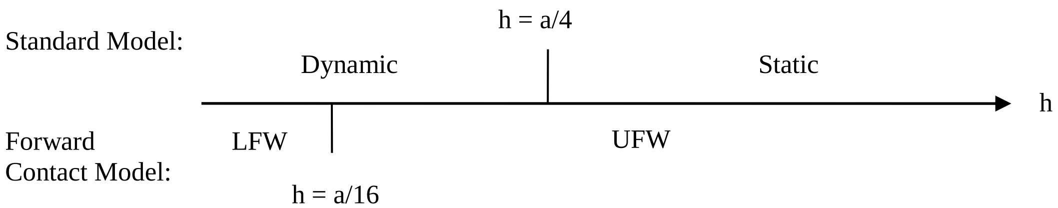

Figure 37: Comparisons of Standard and Forward Contract Model

For low holding cost ($h < \frac{a}{16}$), the solution of the limited forward contract model competes against the dynamic solution within the standard model. For the medium holding cost interval ($\frac{a}{16} \leq h < \frac{a}{4}$), the unlimited solution within the forward contract model gets available and competes against the dynamic solution of the standard model. Last, for high holding cost ($h \geq \frac{a}{4}$), the dynamic solution is no longer applicable. Hence, the unlimited solution of the FW model now competes against the static solution of the standard model.

Alongside with the comparison of the supplier profits that are necessary to distinguish which solution he will implement, the profits of the buyer and the supply chain will also be compared. This way it can be determined, how these profits are altered by the supplier's chosen alternative. Additionally, prior to the necessary comparisons, the two cases (LFW, UFW) within the FW model will also be compared in order to test, if the buyer and the supply chain also perform better in the UFW case.

7.3.1 Comparison within the Forward Contract Model

Within the forward contract model LWF and UFW need to be compared:

Supplier:

$$\mathrm{Pr}\,ofit\ Supplier\ LFW \overset{?}{\leq} \mathrm{Pr}\,ofit\ Supplier\ UFW$$

$$\frac{9a^2+ah-8h^2}{34b} \overset{?}{\leq} \frac{17a^2}{64b}$$

$$0 \overset{?}{\leq} \frac{(a-16h)^2}{1088b}$$

$$0 \overset{?}{\leq} (a-16h)^2 \qquad (7.35)$$

As already outlined above, (7.35) mathematically shows that the supplier always prefers the unlimited solution of the forward contract model. Therefore, he will implement the unlimited solution as soon as the holding costs are sufficiently high ($h \geq \frac{a}{16}$).

Buyer:

$$\mathrm{Pr}\,ofit\ Buyer\ LFW \overset{?}{\leq} \mathrm{Pr}\,ofit\ Buyer\ UFW$$

$$\frac{155a^2+38ah+172h^2}{1156b} \overset{?}{\leq} \frac{35a^2}{256b}$$

$$0 \overset{?}{\leq} \frac{195\cdot a^2-2432\cdot a\cdot h-11008\cdot h^2}{73984b}$$

$$0 \overset{?}{\leq} 195\cdot a^2-2432\cdot a\cdot h-11008\cdot h^2$$

$$0 \overset{?}{\leq} (195\cdot \mathrm{a}+688\cdot \mathrm{h})\cdot(\mathrm{a}-16\cdot \mathrm{h}) \qquad (7.36)$$

The buyer prefers the unlimited solution of the forward contract model over the limited solution if $h<\frac{a}{16}$. However, the unlimited solution cannot be implemented for $h<\frac{a}{16}$.[64] For $h>\frac{a}{16}$, the buyer then prefers the limited solution of the forward contract model. Hence, the buyers preference is contrary to that of the supplier and she will always be confronted with the less favorable case of the FW model. Whether the buyer is, nevertheless, better off in comparison to the standard model, will be examined in the following comparisons.

Supply Chain:

$$\begin{aligned}
\text{Profit Supply Chain LFW} &\overset{?}{\leq} \text{Profit Supply Chain UFW}\\
\frac{461\cdot a^2+72\cdot a\cdot h-100\cdot h^2}{1156b} &\overset{?}{\leq} \frac{103\cdot a^2}{256b}\\
0 &\overset{?}{\leq} \frac{263\cdot a^2-4608\cdot a\cdot h+6400\cdot h^2}{73984b}\\
0 &\overset{?}{\leq} 263\cdot a^2-4608\cdot a\cdot h+6400\cdot h^2\\
0 &\overset{?}{\leq} (263\cdot a-400\cdot h)\cdot(a-16\cdot h)
\end{aligned} \tag{7.37}$$

Condition (7.37) shows that for $h<\frac{a}{16}$ the supply chain profit would be higher, if UFW would apply. However, only LFW can be implemented under these holding costs. For $\frac{a}{16}<h<\frac{263}{400}a$ the profit would be larger under the limited solution. However, LFW is not chosen by the supplier. Lastly, for $h>\frac{263}{400}a$ the UFW solution outperforms the solution of LWF again.

[64] Note, that the unlimited solution cannot be implemented, because of the buyer's behavior: If the supplier would offer the prices of the unlimited solution, the buyer would build up inventory instead of ordering items at the forward price. To prohibit the utilization of inventory the supplier has to use the modified prices of the limited solution. As both supply chain members would prefer the unlimited solution, they could agree on implementing it using more sophisticated contracts.

7.3.2 Comparisons between Standard and Forward Contract Model

<u>Low Holding Costs:</u> LFW versus dynamic

The relevant interval of the holding costs is: $0 \leq \mathrm{h} < \frac{a}{16}$. For higher holding cost, the LWF solution gets replaced by the UFW solution.

<u>Supplier:</u>

$$\begin{aligned}
\Pr ofit\ Supplier\ LFW &\overset{?}{\geq} \Pr ofit\ Supplier\ Dynamic \\
\frac{9 \cdot a^2 + a \cdot h - 8 \cdot h^2}{34b} &\overset{?}{\geq} \frac{9 \cdot a^2 - 4 \cdot a \cdot h + 8 \cdot h^2}{34b} \\
\frac{5ah - 16h^2}{34b} &\overset{?}{\geq} 0
\end{aligned}$$

$$h \cdot (5a - 16h) \overset{?}{\geq} 0 \tag{7.38}$$

The supplier prefers the LWF solution over the dynamic solution if $h < \frac{5}{16}a$. Hence, within the relevant interval, he always prefers the solution with forwards instead of strategic inventory.

<u>Buyer:</u>

$$\begin{aligned}
\Pr ofit\ Buyer\ LFW &\overset{?}{\geq} \Pr ofit\ Buyer\ Dynamic \\
\frac{155 \cdot a^2 + 38 \cdot a \cdot h + 172 \cdot h^2}{1156b} &\overset{?}{\geq} \frac{155 \cdot a^2 - 118 \cdot a \cdot h + 304 \cdot h^2}{1156b} \\
\frac{39 \cdot a \cdot h - 33 \cdot h}{289b} &\overset{?}{\geq} 0
\end{aligned}$$

$$h \cdot (13a - 11h) \overset{?}{\geq} 0 \tag{7.39}$$

The buyer prefers the LWF solution over the dynamic solution of the standard model if $h < \frac{13}{11}a$. Thus, in the relevant interval the buyer is better off, if forwards are offered by the supplier.

Supply Chain:

$$\begin{aligned}
\text{Profit Supply Chain LFW} &\overset{?}{\geq} \text{Profit Supply Chain Dynamic}\\
\frac{461\cdot a^2+72\cdot a\cdot h-100\cdot h^2}{1156b} &\overset{?}{\geq} \frac{461\cdot a^2-254\cdot a\cdot h+576\cdot h^2}{1156\cdot b}\\
\frac{163\cdot a\cdot h-338\cdot h^2}{578b} &\overset{?}{\geq} 0
\end{aligned}$$

$$h\cdot(163a-338h) \overset{?}{\geq} 0 \tag{7.40}$$

It has already been shown that both supplier and buyer prefer the LWF solution over the dynamic solution. Consequently, the supply chain performance is also better under the LFW solution of the forward contract model in comparison to the dynamic solution of the standard model.

In summary, switching from the standard model towards the forward contract model is profit enhancing for the individual supply chain members and for the entire supply chain if holding costs are low ($h<\frac{a}{16}$).

Medium Holding Costs: UFW versus dynamic SM

Next, the solutions of the forward contract model and the standard model for the holding cost interval $\frac{a}{16}\leq h<\frac{a}{4}$ are conducted. For lower holding cost, the UFW solution would not be possible in the FW model and for higher holding cost the static solution would replace the dynamic solution in the standard model.

Supplier:

$$\begin{aligned}
\text{Profit Supplier UFW} &\overset{?}{\geq} \text{Profit Supplier Dynamic SM}\\
\frac{17a^2}{64b} &\overset{?}{\geq} \frac{9\cdot a^2-4\cdot a\cdot h+8\cdot h^2}{34b}\\
\frac{a^2+128\cdot a\cdot h-256\cdot h^2}{1088b} &\overset{?}{\geq} 0
\end{aligned}$$

$$a^2+128\cdot a\cdot h-256\cdot h^2 \overset{?}{\geq} 0 \tag{7.41}$$

The supplier prefers the unlimited advanced orders over the dynamic solution, if $h<\frac{4+\sqrt{17}}{16}a\approx 0.508a$. As the dynamic solution cannot be implemented, if $h>\frac{a}{4}$, the supplier always prefers the UFW solution of the FW model over the dynamic solution of the standard model.

Buyer:

$$\text{Profit Buyer UFW} \overset{?}{\geq} \text{Profit Buyer Dynamic SM}$$

$$\frac{35a^2}{256b} \overset{?}{\geq} \frac{155 \cdot a^2 - 118 \cdot a \cdot h + 304 \cdot h^2}{1156b}$$

$$\frac{195 \cdot a^2 + 7552 \cdot a \cdot h - 19456 \cdot h^2}{73984b} \overset{?}{\geq} 0$$

$$195 \cdot a^2 + 7552 \cdot a \cdot h - 19456 \cdot h^2 \overset{?}{\geq} 0 \tag{7.42}$$

The buyer is also better off under UFW, as she would only prefer the dynamic solution if $h > \left(\frac{59}{304} + \frac{17 \cdot \sqrt{61}}{608} \right) a \approx 0.412 \cdot a$, which is outside of the relevant interval (the dynamic solution would have already been replaced by the static solution under such high holding costs).

Supply Chain:

$$\text{Profit Supply Chain UFW} \overset{?}{\geq} \text{Profit Supply Chain Dynamic SM}$$

$$\frac{103 \cdot a^2}{256b} \overset{?}{\geq} \frac{461 \cdot a^2 - 254 \cdot a \cdot h + 576 \cdot h^2}{1156 \cdot b}$$

$$\frac{263 \cdot a^2 + 16256 \cdot a \cdot h - 36864 \cdot h^2}{73984b} \overset{?}{\geq} 0$$

$$(a + 64h) \cdot (263a - 576h) \overset{?}{\geq} 0 \tag{7.43}$$

Again, both supplier and buyer prefer the solution of the forward contract model over the solution of the standard model. Consequently, the supply chain must also perform better in the FW model compared to the standard model. This can also be seen mathematically (7.43) as the dynamic solution would only be better if $h > \frac{263}{576} a$. However, for such high holding costs it would already been replaced by the static solution.

In summary, the FW model always outperforms the standard model for all supply chain members and, therefore, also improves the supply chain performance for medium holding costs ($\frac{a}{16} \leq h < \frac{a}{4}$).

<u>High Holding Costs:</u> UFW versus static SM

In the last interval ($h \geq \frac{a}{4}$), the dynamic solution is no longer feasible as holding costs are too high. Therefore, the UFW solution of the FW model needs to be compared against the static solution of the standard model.

<u>Supplier:</u>

$$\text{Profit Supplier UFW} \overset{?}{\geq} \text{Profit Supplier Static SM}$$
$$\frac{17a^2}{64b} \overset{?}{\geq} \frac{a^2}{4 \cdot b}$$
$$\frac{a^2}{64b} \overset{?}{\geq} 0 \tag{7.44}$$

<u>Buyer:</u>

$$\text{Profit Buyer UFW} \overset{?}{\geq} \text{Profit Buyer Static SM}$$
$$\frac{35a^2}{256b} \overset{?}{\geq} \frac{a^2}{8b}$$
$$\frac{3a^2}{256b} \overset{?}{\geq} 0 \tag{7.45}$$

<u>Supply Chain:</u>

$$\text{Profit Supply Chain UFW} \overset{?}{\geq} \text{Profit Supply Chain Static SM}$$
$$\frac{103a^2}{256b} \overset{?}{\geq} \frac{3a^2}{8b}$$
$$\frac{7a^2}{256b} \overset{?}{\geq} 0 \tag{7.46}$$

The three comparisons ((7.44), (7.45) and (7.46)) prove that the unlimited solution of the forward contract model is always superior to the static solution.

Therefore, the solutions of the FW model (LFW and UFW) always perform better than their relevant counterparts of the standard model (dynamic and static). For low holding costs ($h < \frac{a}{16}$), the supplier can only implement the limited solution as he is forced to ensure that the buyer uses forwards instead of strategic inventory. Although the buyer would also prefer the unlimited solution, it only becomes available if holding costs are sufficiently high ($h \geq \frac{a}{16}$).

7.4 Detailed Analysis of Decisions and Profits within the Forward Contract Model

As shown in Table 16, strategic inventory gets successfully removed by forwards in the FW model. Further, the profit comparisons have shown that both buyer and supplier are always better off in the FW model. However, it remains unclear to which extend the elimination of holding cost is the driving factor for the improvement of the individual profits in the FW model. To analyze this, the profits of the standard model are modified with the holding costs. For the supplier, a fictional profit is calculated that shows his profit, if he would gain the entire improvement due to a holding cost removal ($\pi_S^{SM} + h^{SM} \cdot I^{SM}$). Figure 38 shows that the suppliers profit enhancement in the FW model exceeds the savings caused by the removal of the holding cost (all figures are normalized with $a = 100$ and $b = 1$). Hence, additional factors must exist that further improve his profit (i.e. either via an additional increase of supply chain profit or via extracting a part of the buyer's profit).

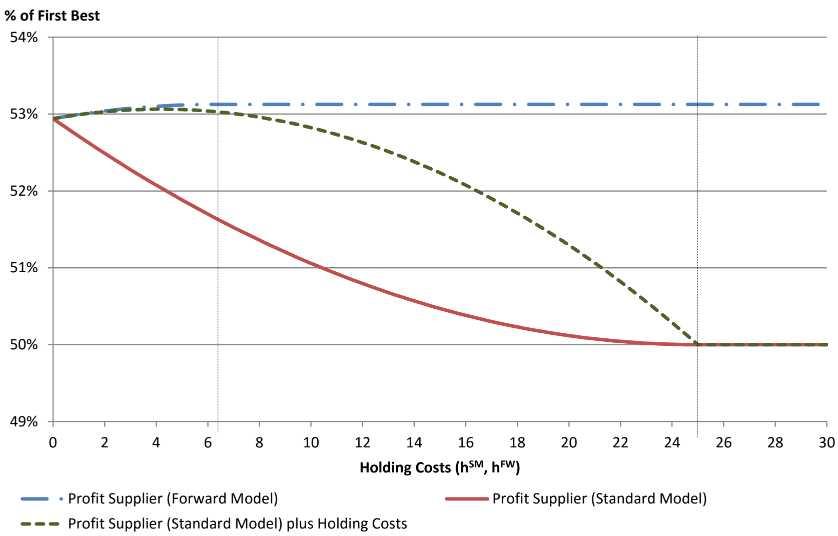

Figure 38: Profit Supplier in the Forward Contract Model

For the buyer, a similar approach is used. Here, the fictional profit is considered that would occur, if she no longer had to pay the holding costs ($\pi_B^{SM} + h^{SM} \cdot I^{SM}$). As shown in Figure 39, with the exception of a low holding cost parameter the improvement of the FW model also exceeds the improvement that can be expected due to the removal of holding costs.

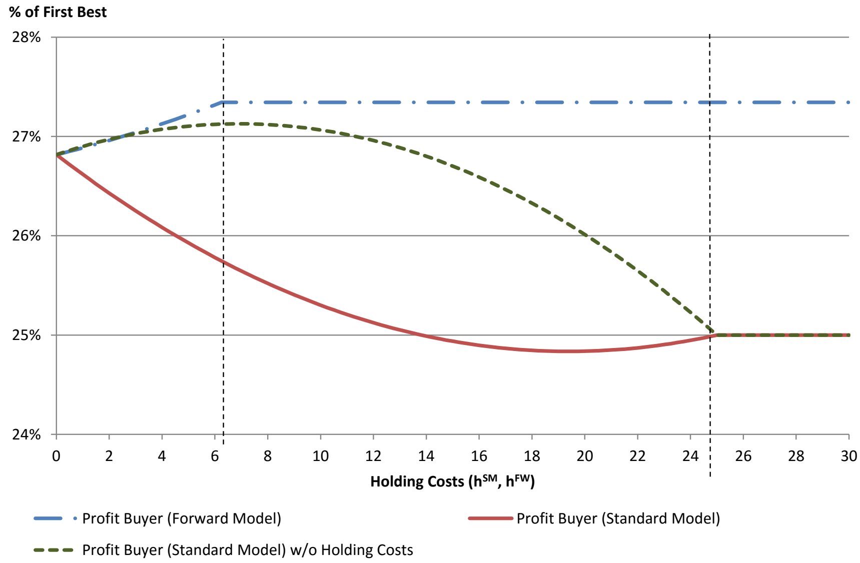

Figure 39: Profit Buyer in the Forward Contract Model

Hence, both the supplier and the buyer can improve their profit to a larger extent than given by the additional profit due to the holding cost removal. As the savings must also be distributed among supplier and buyer (i.e. they can only get a faction of the entire savings), the effect for the whole supply chain gets even more distinctive (Figure 40). Regarding the entire supply chain, the virtual benchmark is obtained by: $\pi_{SC}^{SM} + h^{SM} \cdot I^{SM}$.

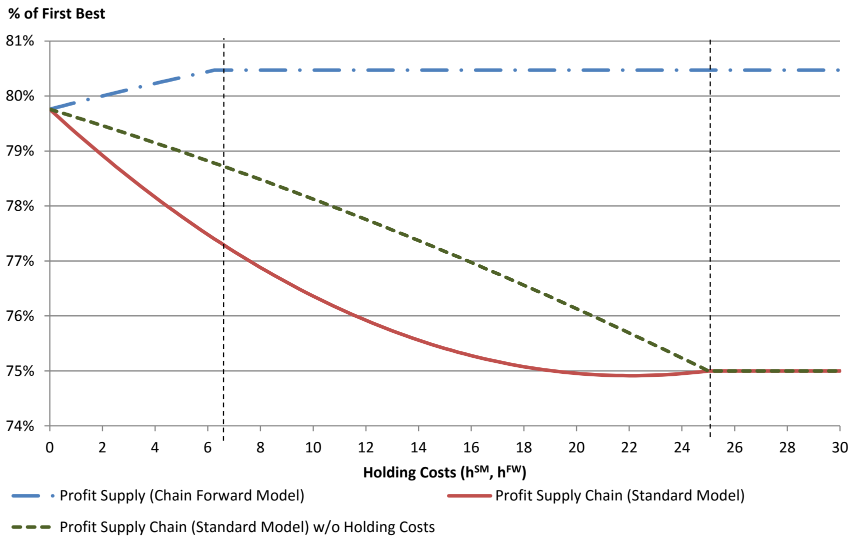

Figure 40: Profit Supply Chain in the Forward Contract Model

Even, if holding costs would be saved, the supply chain performance in the FW model is always above the supply chain performance of the standard model without holding costs. The FW model even always outperforms the best case scenario ($h^{SM} = 0$) of the standard model. A step by step comparison of the decision variables between standard and forward contract model will show, where this additional improvement arises.

In the following, the development of the decision variables under increasing holding cost will be examined in more detail. Again, the parameters $a = 100$ and $b = 1$ will be used for the graphical illustrations. The utilization of these parameters is without loss of generality.

In the first-period, two prices have to be compared (Figure 41). While the first-period wholesale price of the forward contract model can be directly compared to its counterpart of the standard model, there is no direct counterpart for the forward price. However, from the buyer's perspective the forward price determines the price to possess an item at the start of the second period. Hence, the SM counterpart can simply be obtained by adding the per unit holding cost to the first-period wholesale price ($w_1^{SM} + h^{SM}$).

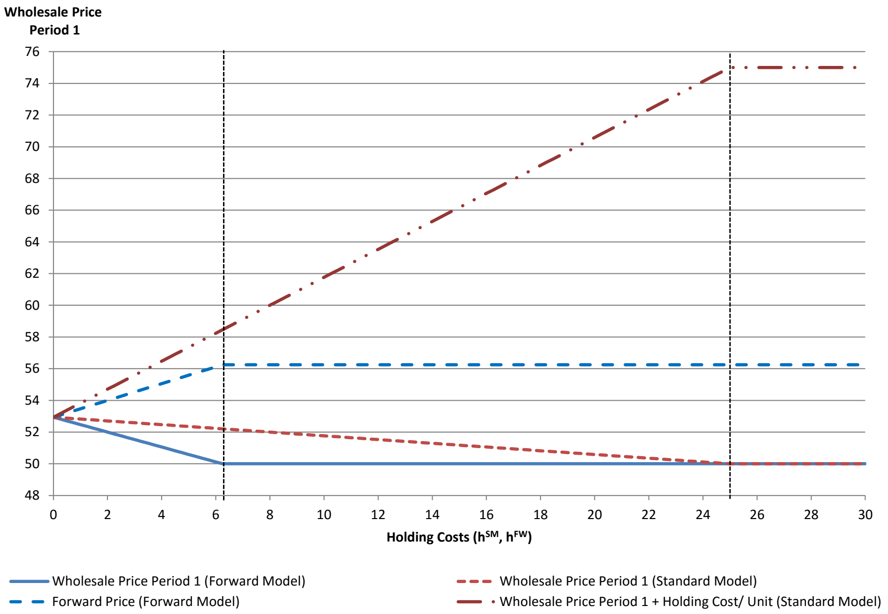

Figure 41: First-Period Prices Forward Contract Model

In the standard model, the first-period wholesale price had two functions. First, it determines to which price the buyer can purchase goods that are meant to be directly resold in the first-period. Second, it also determines to which costs she can build up inventory in the second period. Hence, the supplier had to balance his first-period wholesale price to fulfill both functions as best as possible. To control the buyer's strategic inventory build-up, he had to set a higher first-period wholesale price in the first-period. This, however, increases the double marginalization effect at the cost of supply chain performance.

In the unlimited solution of the FW model, the supplier can perfectly separate both functions of the first-period wholesale price. Thus, he can set the first-period wholesale price to his desired level maintaining the level of double marginalization identical to the static solution. Moreover, he is willing to let the buyer build up goods for the second period (here: forwards) as he gets a higher price for those items (w_1^{SM} versus v^{FW}). As a result, he will set the forward price to his profit maximizing level in the unlimited solution of the forward contract model.

In the limited case of the FW model, the supplier is forced to move away from his desired values that he can achieve in the unlimited solution in order to ensure that the buyer uses forwards instead of inventory. However, as he still is able to additionally charge the per unit holding costs for second period goods (forwards), he can partly separate the two functions of the first-period wholesale price. This again leads to a lower first-period wholesale price and, thus, to a lower level of double marginalization. Also, as he can charge the per unit holding costs within the limits defined by his forward price, he is more willing to let the buyer build up second period goods and charges a lower price for those goods. This price is even lower than in the standard model as the level of the per unit holding costs hinders him from reaching his desired level.

In summary, the first-period wholesale price is always lower in the forward contract model than in the standard model. Consequently, the buyer will sell more items to her customers (Figure 42). As a result, the double marginalization effect in the first-period is lower in the forward contract model leading to an additional profit enhancement in comparison to the standard model. Further, the supplier always extracts some of the profit enhancement due to the removal of holding costs by setting a forward price that is higher than the first-period wholesale price of the standard model (Figure 41).

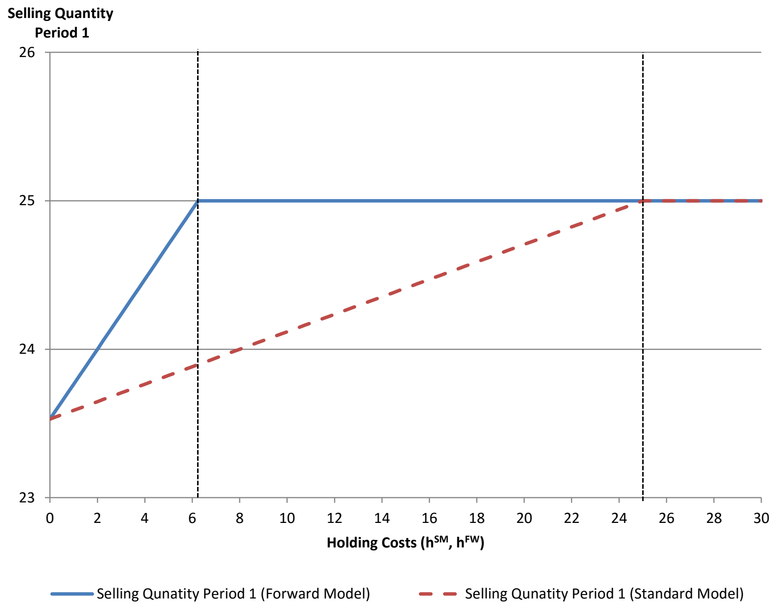

Figure 42: First-period Selling Quantity Forward Contract Model

Comparing the prices to build up items for the second period has already shown that these are also lower in the forward contract model. As shown by (7.19) differences between the amount of strategic inventory and forwards solely depend on differences between the prices to build up second period items. Consequently, the lower build-up price ($v^{FW} \leq w_1^{SM} + h^{SM}$) in the forward contract model causes a higher size of forwards in the FW model in comparison to the strategic inventory size in the standard model (Figure 43).

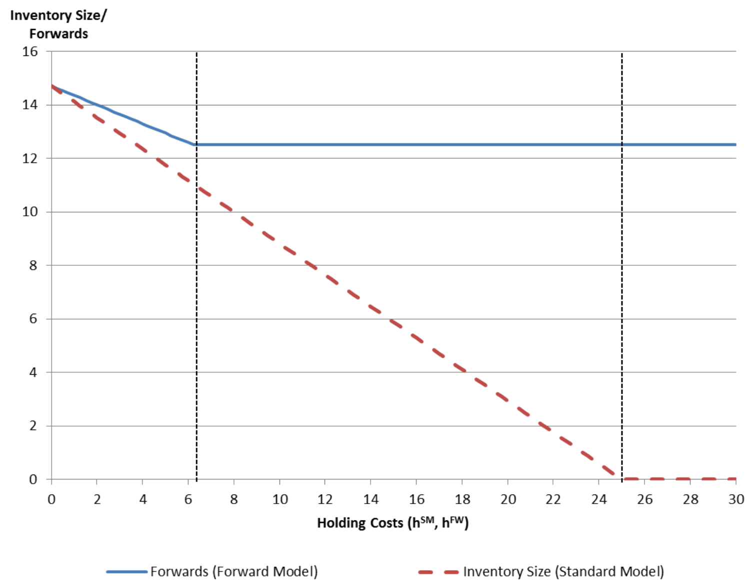

Figure 43: Forward Size in the Forward Contract Model

In the forward contract model the build-up of second period items is higher and maintains at a constant high level even if the holding costs increase. Thus, the impact of strategic items is always relevant in the forward contract model and does no longer get negligible, if the per unit holding cost rate are high. This is also reflected in the second period wholesale price of the supplier (Figure 44).

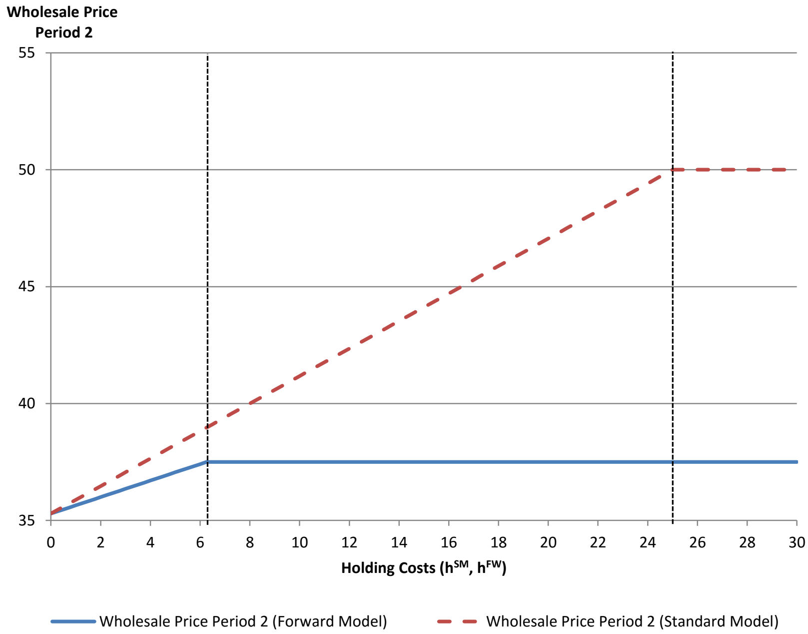

Figure 44: Second Period Wholesale Price in the Forward Contract Model

Due to the higher amount of items that the buyer possesses at the start of period two, the supplier will set a lower second period wholesale price as he only considers the buyer's residual demand of the second period. Due to the lower second period wholesale price, the buyer will again sell more items in comparison to the standard model (Figure 45). Hence, the double marginalization effect is even stronger reduced by the utilization of forwards than it was reduced by strategic inventory in the standard model leading to a further improvement of supply chain performance. The described reduction of double marginalization becomes even more pronounced because the forwards were purchased at a lower price than in the standard model.

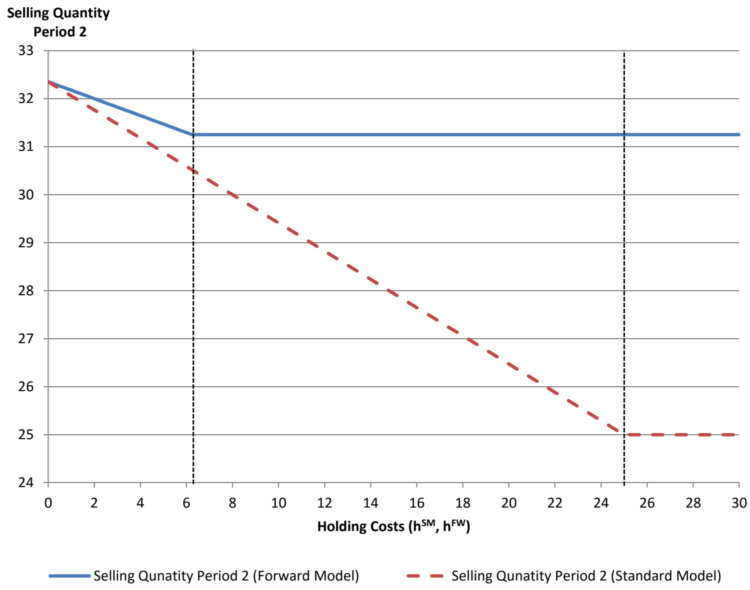

Figure 45: Second Period Selling Quantity in the Forward Contract Model

7.5 Conclusions on the Forward Contract Model

The analysis of the forward contract model has shown that strategic inventory can be completely removed from supply chains by allowing the buyer to order forwards from the supplier. These forwards replace strategic inventory and inherit its strategic function (i.e. enforcing the supplier to set a lower second period wholesale price). As forwards are not physically present at the end of period one, the holding costs that occurred through strategic inventory no longer occur.

Moreover, the detailed analysis has shown that the first-period wholesale price had to fulfill two functions in the standard model. On the one hand, it determines the price that the buyer has to pay for goods that he has to pay for items that she wants to directly resell to her customers. On the other hand, it determines to which costs the buyer can build up inventory. In the forward contract model, the supplier can better separate these functions, as he can set prices for both functions separately.

For low holding costs, the supplier must ensure that the buyer will use forwards instead of strategic inventory by keeping the difference between first-period wholesale price and forward price at the level of the holding costs. Offering the two different prices in the first-period also allows him to partly extract the savings due to the removal of the holding cost for the strategic inventory. However, as soon as the holding costs are high enough the supplier does no longer need to prohibit the buyer from using a strategic inventory instead of forwards. The supplier then can implement his desired prices as the buyer will prefer using forwards instead of building up strategic inventory.

Due to the improved separation regarding the two functions of the first-period wholesale price, the supplier is able to set a lower first-period wholesale price which will decrease the double marginalization effect in the first-period. And, also in the second period, the double marginalization effect gets reduced: As the supplier can partly extract the saved holding costs utilizing the forward price, he is more willing to let the buyer build up forwards than he was willing to let the buyer build up strategic inventory. This leads to a higher forward quantity in comparison to the strategic inventory size. As a result of the high size of forwards, the supplier will also charge a lower second period wholesale price. Consequently, the reduced double marginalization effect in both periods provide an additional boost of supply chain performance, so that the total profit enhancement of the forward contract model goes even beyond the improvement that was expected due to the saved holding costs. Both supplier and buyer are able to achieve a higher profit level in comparison to the standard model and, therefore, the supply chain performance is also closer to the first-best solution. The first-best solution, however, cannot be achieved as double marginalization is not fully removed via the forward contract.

Last, the impact of forwards is no longer limited by the level of the per unit holding costs. In the standard model, the impact of strategic inventory reduces, if holding costs increase, and even disappears, if the per unit holding costs are prohibitively high. In the forward contract model, instead, higher per unit holding costs even increase the impact on the supply chain decisions. As forwards impact the solution of the supply chain regardless of the actual level of the per unit holding costs and both supplier and buyer prefer the utilization of forwards over the utilization of strategic inventory, the impacts of forwards should be considered in multi-period supply chain interactions alongside with the impacts of strategic inventory.

8 Conclusions and Outlook

The research objective of this thesis is to contribute to the relative new research stream of strategic inventory in multi-period interaction in vertical supply chains. In Chapter 3 the profit loss that is caused by the double marginalization effect and the effectiveness of a coordinating contract was reviewed. Moreover, the standard model of Anand et al (2008) that features multi-period interaction within a vertical supply chain was presented. In the standard model that serves as a baseline for the theoretical extensions of this thesis, buyers build up strategic inventory to offset the monopoly power of the supplier in the second period. Hence, the utilization of strategic inventory clearly differs from the classical reasons to hold inventory that were briefly summarized in first section of chapter 2. By building up a strategic inventory the buyer is able to force the supplier to set a lower second period wholesale price. The supplier, in turn, anticipates the inventory build-up of the buyer and increases the first-period wholesale price. This way he can implement an internal two block scheme in the second period.

While the strategic interaction through inventory may improve supply chain performance in the case of a simple wholesale price contract for both supply chain members, it prevents the coordinating effect of more complex contracts at the same time. Hence, despite its profit enhancing effects strategic inventory is also a threat for the supply chain performance.

In chapter 4, a laboratory study was presented that is the first empirical validation on the effects of strategic inventories in supply chain management. The results deliver clear evidence for the behavioral relevance of strategic inventories and the efficiency enhancing effect that qualitatively is perfectly in line with the theoretical results and that quantitatively goes even beyond the equilibrium prediction. It is shown that this enhancement of supply chain performance cannot be uniquely attributed to fairness preferences of the suppliers or to the perception of contract failure risk. Instead, an empowerment of the buyers is identified as she is able to reduce the suppliers profit with her inventory size decision. This empowerment may also influence the suppliers' perception of contract failure risk leading to lower than predicted average wholesale prices. Moreover, it can be observed that the supplier is willing to forgo the additional profit that can be achieved by the adoption of strategic inventory. Hence, he is willing to reduce his first-period wholesale price as long as he remains at the profit level that he would have achieved if no strategic inventory would be utilized. Hardly any suppliers are observed, who are willing to share their payoffs beyond this point. Hence, this payoff seems to be a decisive benchmark, a focal point, for the suppliers' fairness concerns.

The findings of the presented laboratory study have several implications for supply chain management. First, the findings suggest that when holding costs are reasonably low, inventories may (at least partially) be adopted for strategic reasons, both enhancing the supply chain performance and empowering the buyer. In other words, the results give strong empirical support to the theoretical findings of Anand et al (2008). Second, the results suggest that there may be behavioral effects that top off the purely strategic effect. Seeking a more equitable payoff distribution in the supply chain, the empowered buyers may harm the supply chain performance by choosing suboptimally small inventories. But, this negative effect of buyer empowerment on supply chain performance is generally offset by the positive effect of the low first-period prices. Third, the results highlight that the positive effects on supply chain performance can only be achieved with some flexibility concerning the distribution of profits. There is evidence that the extent of profit sharing may strongly depend on focal points that emerge from the interaction situation and induce upper bounds for the willingness to share. Obviously, such focal points may be based on historical, legal, or cultural details of the interaction environment.

In chapter 5, horizontal competition was integrated into the model by allowing an arbitrary number of competing buyers. Hence, the impact of horizontal competition on the vertical competition between supplier and buyer(s) was analyzed. It is shown that the impact of strategic inventory reduces quickly if additional buyers exist within the supply chain. This is caused by the reduction of the monopoly power of the buyer, if the amount of buyers is increased. As a result, the double marginalization effect reduces strongly and, consequently, the room for improvement due to strategic inventory also decreases. Although, measuring the impact of strategic inventory by its ability to reduce the remaining inefficiency through the double marginalization effect gives a slightly better impression of the impact of strategic inventory, it can be summarized that strategic inventory only has a major impact on the supply chain solution, if the buyer's side possesses a high degree of monopoly power.

In the first section of chapter 6, the constant per unit holding cost rate of the standard model was replaced by a more appropriate approach with holding costs that depend on the purchasing price of the buyer: Instead of using a constant per unit holding cost rate, the holding costs are determined by an inventory interest rate that gets multiplied with the purchasing price. The results of this inventory interest rate model indicate no strong structural differences towards the standard model with constant per unit holding costs rates. However, as the supplier also determines the buyer’s holding cost by setting the first-period wholesale price, he will set this price lower in comparison to the standard model. As a result, the buyer will build up slightly larger strategic inventory sizes and the double marginalization effect is reduced stronger than in the standard model. As the buyer participates stronger from the arising additional profit enhancement, she also is always better off under the dynamic solution. In summary, the inventory interest model further highlights the impact of strategic inventory in multi-period vertical competition, as it also plays a pivotal role, if holding costs are accounted as dependent from the purchasing costs. As long as the inventory interest rate is not prohibitively high both supply chain members always prefer the dynamic solution and the impact of strategic inventory is slightly larger in the inventory interest rate model than in the standard model.

In the second section of chapter 6, the discounted cash flow approach of Desai et al. (2010) with both a discount factor for the second period and a constant per unit holding cost rate was examined and compared to the IIR model. In contrast to the previous variants of holding costs, the supplier gets also affected by this modification of the discounted cash flow model as his fraction of profits that is realized in the second period gets discounted as well. Hence, for the supplier the interest rate determines the costs of selling units in the second period instead of realizing those sales in the first-period (i.e. the higher the interest rate the higher is the supplier’s incentive to sell units in the first-period). As a result, the supplier has an incentive to lower the first-period wholesale price in order to encourage the buyer to build up a higher strategic inventory that partly pulls the supplier’s realization of profits into the first-period. This new utilization of strategic inventory in the discounted cash flow model massively impacts the model’s solution leading to structural differences in comparison to the standard and the inventory interest rate model.

In the discounted cash flow model, the supplier has to balance the usage of strategic inventory between the function to implement a well-fitting two-price block scheme and the function to reduce the impact of the discounting of the second period profit. For low interest rates, the latter is less important and the supplier can focus on setting diverse prices between the first and the second period in order to implement a well-fitting two-pricing block scheme across periods. For increasing interest rates, however, the supplier has to focus on reducing the effects of the discounting by lowering the first-period wholesale price. This usage is getting so dominating over the two-price block effect that for high interest rates the first-period wholesale price is even lower than the second period wholesale price. This low first-period wholesale price results in a higher strategic inventory build-up and, consequently, also in a lower second period wholesale price in comparison to the standard model. As mainly the buyer profits from the profit enhancing effect of a reduced double marginalization effect, the supplier does no longer always prefer the dynamic over the static solution. However, the interest rate at which he will implement the static solution is significantly higher than the maximum inventory interest rate of the inventory interest rate model under which the dynamic solution is feasible.

In chapter 7, a situation is modeled where the supplier possesses the opportunity to offer his buyer an option to preorder items in the first-period at a specific preorder price. These items than get delivered to the customer at the beginning of the second period. This described contract modification of the forward contract model aims on removing the holding cost of strategic inventory by offering a non-physical version of strategic items that does not cause holding costs while fulfilling the same function as inventory. The analysis of the forward contract model shows that indeed both supplier and buyer are always better off in the forward contract model and, therefore, strategic inventory gets successfully replaced by preorders. The relative performance of the forward contract model depends on the level of the per unit holding costs. However, in contrast to previous models an increase of the holding costs is beneficial for the supply chain performance:

For low holding costs, the supplier has to ensure that the buyer will use forwards instead of strategic inventory by keeping the difference between first-period wholesale price and preorder price at the level of the holding costs. Offering the two different prices in the first-period also allows him to partly extract the savings due to the removal of the holding cost for the strategic inventory. However, as soon as the holding costs are high enough the supplier does no longer need to prohibit the buyer from using a strategic inventory instead of preorders. The supplier then can implement his desired prizes as the buyer will prefer using forwards instead of building up strategic inventory. Hence, the forward contract model is also able to (partly) remove the negative effects regarding double marginalization that was caused by the increased first-period wholesale price in the standard model.

Finally, the results of this thesis also have some implications for future research, emphasizing that the role of strategic inventories should be considered both in non-cooperative supply chain modeling and in behavioral research.

Anand et al. (2008) have shown that strategic inventory is relevant for any convex and, moreover, under certain conditions also for concave demand functions. However, they have only considered demand functions in which the demand of the second period is independent of the first-period selling quantity. In practice, however, recent selling quantities might impact the future demand. Hereby, both a negative as well as a positive correlation between future demand and recent selling quantities might occur: Considering the life cycle of a product, a negative correlation might, for example, exist if the market gets saturated by recent selling quantities. In turn, a positive correlation, for example, is possible if recent selling quantities help to establish the product on the market. Both correlations might interfere with the buyer's willingness to build up strategic inventory and should, therefore, be examined carefully.

Moreover, in the current literature on strategic inventory, operative reasons to build up inventory (e.g. safety stock, cycle inventory, anticipation stock) are ruled out by design in order to investigate the pure effect of strategic inventory. However, in order to get valid evidence on the practical relevance of strategic inventory, the operational reasons to build up inventory need to be considered. For example, buyers might have an incentive to build up inventories both to hedge against uncertainties (safety stock) as well as through strategic considerations. In such a scenario, the buyers might be able to use their entire unsold goods as a strategic tool, while the supplier would anticipate this. Hence, the findings of Anand et al. (2008) cannot directly be transferred to such a (more realistic) scenario. Also, the existence of cycle inventory and anticipation stock (both on the side of the supplier and/ or the buyer) might interfere with the utilization of strategic inventory.

Another important open issue is the design of simple, dynamic contracts that can be used to implement the optimal supply chain performance in the presence of strategic inventories. So far, only relatively complex contracts have been developed (e.g. supplier-to-customer rebates). Moreover, these contracts have to be tested under scenarios with both strategic as well as operational reasons to build up inventory. It would be interesting to examine, under which conditions advanced contracts are strategically feasible and behaviorally robust.

Another direction for further research is to study the role of information asymmetries, especially regarding the inventory size in a setting with strategic inventories. It is not yet clear how the supplier can set optimal wholesale prices without the exact knowledge of the inventory size. More complex contracts, such as screening contracts, may be necessary to successfully deal with these information asymmetries. But, if the contracts become too complex, it may be more effective to rely on less sophisticated contractual agreements that take truth-telling and trust into account (see e.g. Charness and Dufwenberg (2011), Inderfurth et al. (2013)).

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Appendix:

Appendix A: Sample Instructions of the Experimental Study including the Corresponding Profit Tables

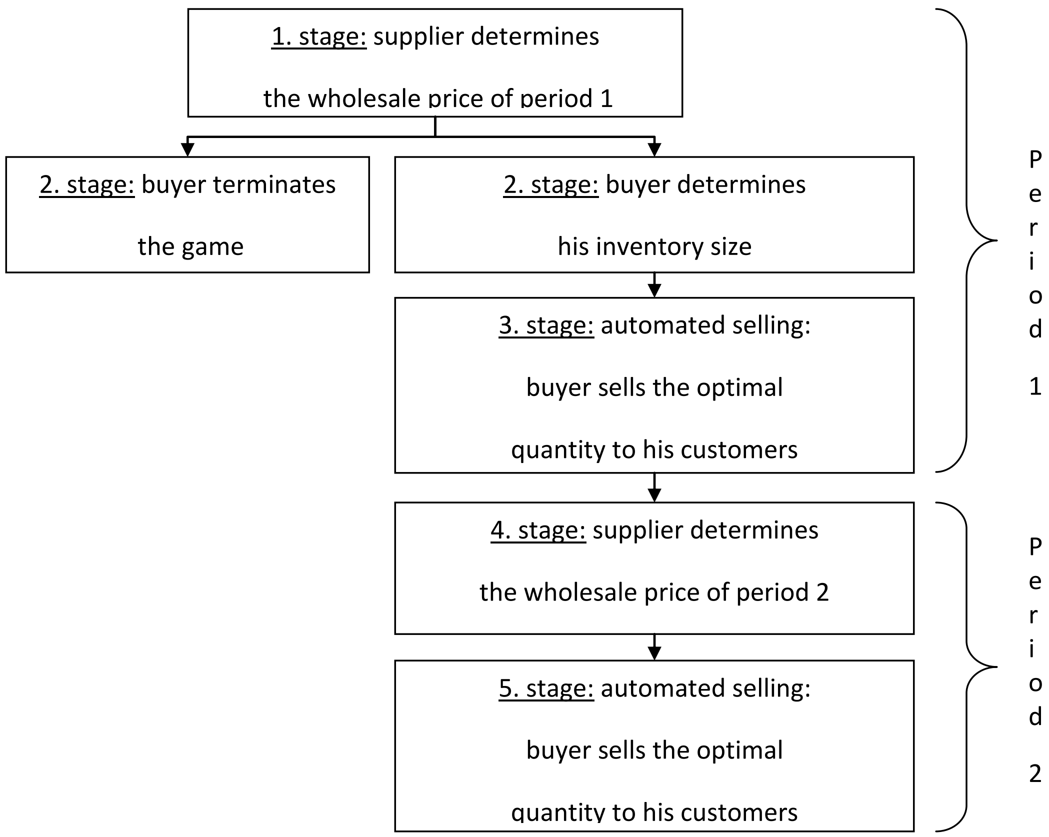
Instructions

Please read the instructions carefully and raise your hand, if you have a question.

In the experiment, in which you participate now, you can earn lab dollars (LD) that will be converted into money and paid to you at the end of the experiment. The amount of LD that you will earn in each of the decision rounds depends both on your decisions and the decisions of your co-player. Every decision you make in the experiment is anonymous.

Background:

You are in a vertical supply chain consisting of one supplier and one buyer. At the beginning of the game you will be assigned either the role of the supplier or of the buyer. This allocation will be maintained in all 15 rounds of the experiment. At the beginning of each round you will be randomly assigned to one player with the other role. Each round is divided into five stages (see figure):



Stage 1: Decision of the Supplier in Period 1

In the first decision stage of the experiment, the supplier sets the wholesale price at which the buyer may purchase goods in the first-period. For the wholesale price each value in the interval from 0 to 152 with a maximum precision of three digits after the decimal point is allowed. In the attached payout table the wholesale price of the first-period is displayed on the left side of each table. However, the table does not include all feasible prices but only the 0 and the numbers in the interval between 52 and 84 in steps of four. Thus, the payout table serves only as guidance. For the prices not listed, use the profit calculator provided.

	0	
	52	
wholesale price period 1	56	
	…	
	84	

Stage 2: Decision of the Buyer in Period 1

At the beginning of the second stage in period one the buyer is informed about the current wholesale price. Then, the buyer can decide whether to continue or to terminate the game. If the buyer terminates the game, the round ends immediately. If the buyer instead continues the game, he decides about his inventory size. Here, each value in the interval between 0 and 38 with a maximum precision of three digits after the decimal point is allowed.

For each unit purchased to build up inventory the wholesale price of the first-period needs to be paid to the supplier. Additionally, holding costs of 4 LD per item are caused. In the attached payout table the inventory size is displayed in the upper left corner. Again, not all possible inventory sizes but only those in the interval between 0 and 20 in steps of five are given. For other values the provided calculator can be used to obtain the corresponding profits.

wholesale price period 1	0	
	52	
	56	
	…	
	84	

Stage 3: Automated Vending of the Buyer in Period 1

If the game was not terminated, an automated vending for the first-period is operated after the buyer has decided about his inventory size. In this stage, the program calculates the optimal selling quantity of the buyer and sells them on the market. The optimal selling quantity only depends on the wholesale price of the first-period, but not on the inventory size of the buyer.

Stage 4: Decision Supplier in Period 2

If the game was not terminated, the buyer's inventory size will be communicated to the supplier and the supplier is prompted to set the wholesale price of the second period. For the wholesale price, each value in the interval from 0 to 152 with a maximum precision of three digits after the decimal point is allowed. In the payout table the price of the second period is displayed in the columns (top edge). Again, only the 0 and the values in the interval between 52 and 84 are given in steps of four. For further calculations the provided calculator can be used.

inventory		wholesale price period 2				
		0	52	56	…	84
wholesale price period 1	0					
	52					
	56					
	…					
	84					

Stage 5: Automated Vending of the Buyer in Period 2

If the game was not terminated, again an automated vending for the second period will be conducted after the supplier has set the wholesale price of the second period. Again, the exact quantity that maximizes the profit of the buyer will be sold. If the buyer built up inventory in the first-period, he can use these goods for the vending process and needs to buy fewer units from the buyer in the second period. Thus, the optimal selling quantity of the second period depends on both the wholesale price of the second period and the inventory size of the buyer. It should be noted that the buyer will not purchase further goods from the supplier in the second period, if the wholesale price of the second period is too high. In this case, the buyer would only sell his inventory to the external market.

Calculation of the Profits of each Round:

After all decisions have been made, the profits of the current round for both the supplier and the buyer will be displayed. If the trade was rejected, the profit would be zero LD for both players. In all other cases, the respective profits for the specified values of the wholesale prices and the inventory size correspond to the profits given by the payout table or the calculator.

Control Question:

The wholesale price of the first-period is 72 LD, the inventory size is 15 and the wholesale price of the second period is 60 LD. What are the profits of the supplier and the buyer for this round? Please use the payout table and write down your answers on the prepared paper. Please wait until we have checked your responses. A correct answering of this control question is required to participate in the experiment.

How will the payment be carried out?

Your payment (in Euros) matches the sum of your LD divided by 3000. This means that 30 LD correspond to exactly 1 euro cent. You will be paid at the end of the experiment. Please wait until we call your name.

Good luck.

Profit Table Supplier (LC and HC):

		Wholesale Price Period 2									
Inventory	0	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	1.300	1.344	1.380	1.408	1.428	1.440	1.444	1.440	1.428
	52	1.300	2.600	2.644	2.680	2.708	2.728	2.740	2.744	2.740	2.728
	56	1.344	2.644	2.688	2.724	2.752	2.772	2.784	2.788	2.784	2.772
	60	1.380	2.680	2.724	2.760	2.788	2.808	2.820	2.824	2.820	2.808
	64	1.408	2.708	2.752	2.788	2.816	2.836	2.848	2.852	2.848	2.836
	68	1.428	2.728	2.772	2.808	2.836	2.856	2.868	2.872	2.868	2.856
	72	1.440	2.740	2.784	2.820	2.848	2.868	2.880	2.884	2.880	2.868
	76	1.444	2.744	2.788	2.824	2.852	2.872	2.884	2.888	2.884	2.872
	80	1.440	2.740	2.784	2.820	2.848	2.868	2.880	2.884	2.880	2.868
	84	1.428	2.728	2.772	2.808	2.836	2.856	2.868	2.872	2.868	2.856

		Wholesale Price Period 2									
Inventory	5	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	1.040	1.064	1.080	1.088	1.088	1.080	1.064	1.040	1.008
	52	1.560	2.600	2.624	2.640	2.648	2.648	2.640	2.624	2.600	2.568
	56	1.624	2.664	2.688	2.704	2.712	2.712	2.704	2.688	2.664	2.632
	60	1.680	2.720	2.744	2.760	2.768	2.768	2.760	2.744	2.720	2.688
	64	1.728	2.768	2.792	2.808	2.816	2.816	2.808	2.792	2.768	2.736
	68	1.768	2.808	2.832	2.848	2.856	2.856	2.848	2.832	2.808	2.776
	72	1.800	2.840	2.864	2.880	2.888	2.888	2.880	2.864	2.840	2.808
	76	1.824	2.864	2.888	2.904	2.912	2.912	2.904	2.888	2.864	2.832
	80	1.840	2.880	2.904	2.920	2.928	2.928	2.920	2.904	2.880	2.848
	84	1.848	2.888	2.912	2.928	2.936	2.936	2.928	2.912	2.888	2.856

		Wholesale Price Period 2									
Inventory	10	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	780	784	780	768	748	720	684	640	588
	52	1.820	2.600	2.604	2.600	2.588	2.568	2.540	2.504	2.460	2.408
	56	1.904	2.684	2.688	2.684	2.672	2.652	2.624	2.588	2.544	2.492
	60	1.980	2.760	2.764	2.760	2.748	2.728	2.700	2.664	2.620	2.568
	64	2.048	2.828	2.832	2.828	2.816	2.796	2.768	2.732	2.688	2.636
	68	2.108	2.888	2.892	2.888	2.876	2.856	2.828	2.792	2.748	2.696
	72	2.160	2.940	2.944	2.940	2.928	2.908	2.880	2.844	2.800	2.748
	76	2.204	2.984	2.988	2.984	2.972	2.952	2.924	2.888	2.844	2.792
	80	2.240	3.020	3.024	3.020	3.008	2.988	2.960	2.924	2.880	2.828
	84	2.268	3.048	3.052	3.048	3.036	3.016	2.988	2.952	2.908	2.856

		Wholesale Price Period 2									
Inventory	15	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	520	504	480	448	408	360	304	240	168
	52	2.080	2.600	2.584	2.560	2.528	2.488	2.440	2.384	2.320	2.248
	56	2.184	2.704	2.688	2.664	2.632	2.592	2.544	2.488	2.424	2.352
	60	2.280	2.800	2.784	2.760	2.728	2.688	2.640	2.584	2.520	2.448
	64	2.368	2.888	2.872	2.848	2.816	2.776	2.728	2.672	2.608	2.536
	68	2.448	2.968	2.952	2.928	2.896	2.856	2.808	2.752	2.688	2.616
	72	2.520	3.040	3.024	3.000	2.968	2.928	2.880	2.824	2.760	2.688
	76	2.584	3.104	3.088	3.064	3.032	2.992	2.944	2.888	2.824	2.752
	80	2.640	3.160	3.144	3.120	3.088	3.048	3.000	2.944	2.880	2.808
	84	2.688	3.208	3.192	3.168	3.136	3.096	3.048	2.992	2.928	2.856

		Wholesale Price Period 2									
Inventory	20	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	260	224	180	128	68	0	0	0	0
	52	2.340	2.600	2.564	2.520	2.468	2.408	2.340	2.340	2.340	2.340
	56	2.464	2.724	2.688	2.644	2.592	2.532	2.464	2.464	2.464	2.464
	60	2.580	2.840	2.804	2.760	2.708	2.648	2.580	2.580	2.580	2.580
	64	2.688	2.948	2.912	2.868	2.816	2.756	2.688	2.688	2.688	2.688
	68	2.788	3.048	3.012	2.968	2.916	2.856	2.788	2.788	2.788	2.788
	72	2.880	3.140	3.104	3.060	3.008	2.948	2.880	2.880	2.880	2.880
	76	2.964	3.224	3.188	3.144	3.092	3.032	2.964	2.964	2.964	2.964
	80	3.040	3.300	3.264	3.220	3.168	3.108	3.040	3.040	3.040	3.040
	84	3.108	3.368	3.332	3.288	3.236	3.176	3.108	3.108	3.108	3.108

Profit Table Buyer (LC):

		Wholesale Price Period 2									
Inventory	0	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	5.776	4.138	4.040	3.946	3.856	3.770	3.688	3.610	3.536	3.466
	52	4.138	2.500	2.402	2.308	2.218	2.132	2.050	1.972	1.898	1.828
	56	4.040	2.402	2.304	2.210	2.120	2.034	1.952	1.874	1.800	1.730
	60	3.946	2.308	2.210	2.116	2.026	1.940	1.858	1.780	1.706	1.636
	64	3.856	2.218	2.120	2.026	1.936	1.850	1.768	1.690	1.616	1.546
	68	3.770	2.132	2.034	1.940	1.850	1.764	1.682	1.604	1.530	1.460
	72	3.688	2.050	1.952	1.858	1.768	1.682	1.600	1.522	1.448	1.378
	76	3.610	1.972	1.874	1.780	1.690	1.604	1.522	1.444	1.370	1.300
	80	3.536	1.898	1.800	1.706	1.616	1.530	1.448	1.370	1.296	1.226
	84	3.466	1.828	1.730	1.636	1.546	1.460	1.378	1.300	1.226	1.156

		Wholesale Price Period 2									
Inventory	5	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	5.756	4.378	4.300	4.226	4.156	4.090	4.028	3.970	3.916	3.866
	52	3.858	2.480	2.402	2.328	2.258	2.192	2.130	2.072	2.018	1.968
	56	3.740	2.362	2.284	2.210	2.140	2.074	2.012	1.954	1.900	1.850
	60	3.626	2.248	2.170	2.096	2.026	1.960	1.898	1.840	1.786	1.736
	64	3.516	2.138	2.060	1.986	1.916	1.850	1.788	1.730	1.676	1.626
	68	3.410	2.032	1.954	1.880	1.810	1.744	1.682	1.624	1.570	1.520
	72	3.308	1.930	1.852	1.778	1.708	1.642	1.580	1.522	1.468	1.418
	76	3.210	1.832	1.754	1.680	1.610	1.544	1.482	1.424	1.370	1.320
	80	3.116	1.738	1.660	1.586	1.516	1.450	1.388	1.330	1.276	1.226
	84	3.026	1.648	1.570	1.496	1.426	1.360	1.298	1.240	1.186	1.136

		Wholesale Price Period 2									
Inventory	10	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	5.736	4.618	4.560	4.506	4.456	4.410	4.368	4.330	4.296	4.266
	52	3.578	2.460	2.402	2.348	2.298	2.252	2.210	2.172	2.138	2.108
	56	3.440	2.322	2.264	2.210	2.160	2.114	2.072	2.034	2.000	1.970
	60	3.306	2.188	2.130	2.076	2.026	1.980	1.938	1.900	1.866	1.836
	64	3.176	2.058	2.000	1.946	1.896	1.850	1.808	1.770	1.736	1.706
	68	3.050	1.932	1.874	1.820	1.770	1.724	1.682	1.644	1.610	1.580
	72	2.928	1.810	1.752	1.698	1.648	1.602	1.560	1.522	1.488	1.458
	76	2.810	1.692	1.634	1.580	1.530	1.484	1.442	1.404	1.370	1.340
	80	2.696	1.578	1.520	1.466	1.416	1.370	1.328	1.290	1.256	1.226
	84	2.586	1.468	1.410	1.356	1.306	1.260	1.218	1.180	1.146	1.116

		Wholesale Price Period 2									
Inventory	15	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	5.716	4.858	4.820	4.786	4.756	4.730	4.708	4.690	4.676	4.666
	52	3.298	2.440	2.402	2.368	2.338	2.312	2.290	2.272	2.258	2.248
	56	3.140	2.282	2.244	2.210	2.180	2.154	2.132	2.114	2.100	2.090
	60	2.986	2.128	2.090	2.056	2.026	2.000	1.978	1.960	1.946	1.936
	64	2.836	1.978	1.940	1.906	1.876	1.850	1.828	1.810	1.796	1.786
	68	2.690	1.832	1.794	1.760	1.730	1.704	1.682	1.664	1.650	1.640
	72	2.548	1.690	1.652	1.618	1.588	1.562	1.540	1.522	1.508	1.498
	76	2.410	1.552	1.514	1.480	1.450	1.424	1.402	1.384	1.370	1.360
	80	2.276	1.418	1.380	1.346	1.316	1.290	1.268	1.250	1.236	1.226
	84	2.146	1.288	1.250	1.216	1.186	1.160	1.138	1.120	1.106	1.096

		Wholesale Price Period 2									
Inventory	20	0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	5.696	5.098	5.080	5.066	5.056	5.050	5.048	5.048	5.048	5.048
	52	3.018	2.420	2.402	2.388	2.378	2.372	2.370	2.370	2.370	2.370
	56	2.840	2.242	2.224	2.210	2.200	2.194	2.192	2.192	2.192	2.192
	60	2.666	2.068	2.050	2.036	2.026	2.020	2.018	2.018	2.018	2.018
	64	2.496	1.898	1.880	1.866	1.856	1.850	1.848	1.848	1.848	1.848
	68	2.330	1.732	1.714	1.700	1.690	1.684	1.682	1.682	1.682	1.682
	72	2.168	1.570	1.552	1.538	1.528	1.522	1.520	1.520	1.520	1.520
	76	2.010	1.412	1.394	1.380	1.370	1.364	1.362	1.362	1.362	1.362
	80	1.856	1.258	1.240	1.226	1.216	1.210	1.208	1.208	1.208	1.208
	84	1.706	1.108	1.090	1.076	1.066	1.060	1.058	1.058	1.058	1.058

Profit Table Buyer (HC):

Inventory	0	Wholesale Price Period 2									
		0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	1,300	1,344	1,380	1,408	1,428	1,440	1,444	1,440	1,428
	52	1,300	2,600	2,644	2,680	2,708	2,728	2,740	2,744	2,740	2,728
	56	1,344	2,644	2,688	2,724	2,752	2,772	2,784	2,788	2,784	2,772
	60	1,380	2,680	2,724	2,760	2,788	2,808	2,820	2,824	2,820	2,808
	64	1,408	2,708	2,752	2,788	2,816	2,836	2,848	2,852	2,848	2,836
	68	1,428	2,728	2,772	2,808	2,836	2,856	2,868	2,872	2,868	2,856
	72	1,440	2,740	2,784	2,820	2,848	2,868	2,880	2,884	2,880	2,868
	76	1,444	2,744	2,788	2,824	2,852	2,872	2,884	2,888	2,884	2,872
	80	1,440	2,740	2,784	2,820	2,848	2,868	2,880	2,884	2,880	2,868
	84	1,428	2,728	2,772	2,808	2,836	2,856	2,868	2,872	2,868	2,856

Inventory	5	Wholesale Price Period 2									
		0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	1,040	1,064	1,080	1,088	1,088	1,080	1,064	1,040	1,008
	52	1,560	2,600	2,624	2,640	2,648	2,648	2,640	2,624	2,600	2,568
	56	1,624	2,664	2,688	2,704	2,712	2,712	2,704	2,688	2,664	2,632
	60	1,680	2,720	2,744	2,760	2,768	2,768	2,760	2,744	2,720	2,688
	64	1,728	2,768	2,792	2,808	2,816	2,816	2,808	2,792	2,768	2,736
	68	1,768	2,808	2,832	2,848	2,856	2,856	2,848	2,832	2,808	2,776
	72	1,800	2,840	2,864	2,880	2,888	2,888	2,880	2,864	2,840	2,808
	76	1,824	2,864	2,888	2,904	2,912	2,912	2,904	2,888	2,864	2,832
	80	1,840	2,880	2,904	2,920	2,928	2,928	2,920	2,904	2,880	2,848
	84	1,848	2,888	2,912	2,928	2,936	2,936	2,928	2,912	2,888	2,856

Inventory	10	Wholesale Price Period 2									
		0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	780	784	780	768	748	720	684	640	588
	52	1,820	2,600	2,604	2,600	2,588	2,568	2,540	2,504	2,460	2,408
	56	1,904	2,684	2,688	2,684	2,672	2,652	2,624	2,588	2,544	2,492
	60	1,980	2,760	2,764	2,760	2,748	2,728	2,700	2,664	2,620	2,568
	64	2,048	2,828	2,832	2,828	2,816	2,796	2,768	2,732	2,688	2,636
	68	2,108	2,888	2,892	2,888	2,876	2,856	2,828	2,792	2,748	2,696
	72	2,160	2,940	2,944	2,940	2,928	2,908	2,880	2,844	2,800	2,748
	76	2,204	2,984	2,988	2,984	2,972	2,952	2,924	2,888	2,844	2,792
	80	2,240	3,020	3,024	3,020	3,008	2,988	2,960	2,924	2,880	2,828
	84	2,268	3,048	3,052	3,048	3,036	3,016	2,988	2,952	2,908	2,856

Inventory	15	Wholesale Price Period 2									
		0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	520	504	480	448	408	360	304	240	168
	52	2,080	2,600	2,584	2,560	2,528	2,488	2,440	2,384	2,320	2,248
	56	2,184	2,704	2,688	2,664	2,632	2,592	2,544	2,488	2,424	2,352
	60	2,280	2,800	2,784	2,760	2,728	2,688	2,640	2,584	2,520	2,448
	64	2,368	2,888	2,872	2,848	2,816	2,776	2,728	2,672	2,608	2,536
	68	2,448	2,968	2,952	2,928	2,896	2,856	2,808	2,752	2,688	2,616
	72	2,520	3,040	3,024	3,000	2,968	2,928	2,880	2,824	2,760	2,688
	76	2,584	3,104	3,088	3,064	3,032	2,992	2,944	2,888	2,824	2,752
	80	2,640	3,160	3,144	3,120	3,088	3,048	3,000	2,944	2,880	2,808
	84	2,688	3,208	3,192	3,168	3,136	3,096	3,048	2,992	2,928	2,856

Inventory	20	Wholesale Price Period 2									
		0	52	56	60	64	68	72	76	80	84
Wholesale Price Period 1	0	0	260	224	180	128	68	0	0	0	0
	52	2,340	2,600	2,564	2,520	2,468	2,408	2,340	2,340	2,340	2,340
	56	2,464	2,724	2,688	2,644	2,592	2,532	2,464	2,464	2,464	2,464
	60	2,580	2,840	2,804	2,760	2,708	2,648	2,580	2,580	2,580	2,580
	64	2,688	2,948	2,912	2,868	2,816	2,756	2,688	2,688	2,688	2,688
	68	2,788	3,048	3,012	2,968	2,916	2,856	2,788	2,788	2,788	2,788
	72	2,880	3,140	3,104	3,060	3,008	2,948	2,880	2,880	2,880	2,880
	76	2,964	3,224	3,188	3,144	3,092	3,032	2,964	2,964	2,964	2,964
	80	3,040	3,300	3,264	3,220	3,168	3,108	3,040	3,040	3,040	3,040
	84	3,108	3,368	3,332	3,288	3,236	3,176	3,108	3,108	3,108	3,108

Appendix B: Fairness Inventory Size

The buyer can use strategic inventory to reduce the absolute profit difference between herself and the supplier. The fairness inventory size that minimizes the profit difference is obtained by a comparison of the marginal profit of the supplier and the buyer with respect to the inventory size.

The marginal profit of the buyer can be obtained by calculating the first derivative of the buyer's profit function with respect to the inventory size. Under consideration of the automated determination of the selling quantities in both periods (see (3.38) and (3.33)) and under the assumption that the supplier will choose the optimal response wholesale price from (3.35) in the second period[65], according to (3.30) the relevant profit function of the buyer is:

$$\pi_B = \frac{5 \cdot a^2 - 8 \cdot a \cdot w_1 + 4 \cdot w_1^2 + 12 \cdot b \cdot a \cdot I - 16 \cdot b \cdot I \cdot \left(w_1 + h\right) - 12 \cdot b^2 \cdot I^2}{16 \cdot b} \tag{9.1}$$

Therefore, the derivation of the buyer's profit function (9.1) is:

$$\begin{aligned} \frac{\partial \pi_B}{\partial I} &= \frac{3}{4} \cdot a - \left(w_1 + h\right) - \frac{3}{2} \cdot b \cdot I \\ &= 114 - \left(w_1 + h\right) - 3 \cdot I. \end{aligned} \tag{9.2}$$

Hence, in the LC treatment (i.e., for $h = 4$), the buyer should only build up inventory if $w_1 < 110$. Further, we see that the advantage of building up inventory is larger, the lower w_1. Therefore, by setting a low wholesale price in the first-period, the supplier can influence the incentive of the buyer for building up inventory. This becomes visible from the optimality condition (setting the marginal profit in (9.2) equal to zero) so that the buyer's profit maximizing inventory decision is derived as $I_B(w_1) = (114 - w_1 - h)/3$. Further, the marginal profit of the inventory is diminishing.

The influence of inventory on the supplier's profit can be obtained similarly. Because of the automation of the buyer's selling quantities, the optimal response quantities again need to be inserted into the profit function (3.29) of the supplier. Under the assumption that the supplier will choose the optimal response wholesale price in the second period, the relevant profit function of the supplier is:

[65] The analysis of the observed wholesale price decision of the supplier shows that this assumption is fulfilled.

$$\pi_S = \frac{a^2 + 4 \cdot a \cdot w_1 - 4 \cdot w_1^2 + 8 \cdot b \cdot w_1 \cdot I - 4 \cdot b \cdot a \cdot I + 4 \cdot b^2 \cdot I^2}{8 \cdot b} \tag{9.3}$$

Therefore, the derivation of the supplier's profit function (9.3) is:

$$\begin{aligned} \frac{\partial \pi_S}{\partial I} &= w_1 - \frac{a}{2} + b \cdot I \\ &= w_1 - 76 + 2 \cdot I. \end{aligned} \tag{9.4}$$

If the buyer aims to minimize the profit difference by using strategic inventory, she should only lower her inventory size, if her marginal disadvantage of this action is lower than the corresponding marginal disadvantage of the supplier. The inventory size, for which the marginal profits from (9.2) and (9.4) are equal, therefore, is:

$$\begin{aligned} I &= \frac{5 \cdot a - 8 \cdot w_1 - 4 \cdot h}{10 \cdot b} \\ &= 38 - \tfrac{2}{5} \cdot w_1 - \tfrac{1}{5} \cdot h. \end{aligned} \tag{9.5}$$

As a negative inventory is not possible, the fairness inventory size is given by:

$$\begin{aligned} I &= \max\left\{0,\ \frac{5 \cdot a - 8 \cdot w_1 - 4 \cdot h}{10 \cdot b}\right\} \\ &= \max\left\{0,\ 38 - \tfrac{2}{5} \cdot w_1 - \tfrac{1}{5} \cdot h\right\}. \end{aligned} \tag{9.6}$$

Appendix C: Profit Comparison between Static and Dynamic Solution in the Horizontal Competition Model

Profit Comparison Supplier:

$$\pi_S^{HCM,Dynamic} \overset{?}{>} \pi_S^{HCM,Static}$$

$$\frac{\left(2N^4+8N^3+7N^2+1\right)a^2-\left(4N^2+4N\right)a\cdot h^{HCM}+\left(4N^4+8N^3+4N^2\right)\cdot\left(h^{HCM}\right)^2}{2N\left(2N^2+8N+7\right)\left(N+1\right)b}\overset{?}{>}\frac{N\cdot a^2}{2\left(N+1\right)b}$$

$$\left(2N^4+8N^3+7N^2+1\right)a^2-\left(4N^2+4N\right)a\cdot h^{HCM}+\left(4N^4+8N^3+4N^2\right)\cdot\left(h^{HCM}\right)^2\overset{?}{>}\left(2N^4+8N^3+7N^2\right)\cdot a^2$$

$$a^2-\left(4N^2+4N\right)a\cdot h^{HCM}+\left(4N^4+8N^3+4N^2\right)\cdot\left(h^{HCM}\right)^2\overset{?}{>}0$$

$$a^2-\left(4N^2+4N\right)a\cdot h^{HCM}+\left(4N^4+8N^3+4N^2\right)\cdot\left(h^{HCM}\right)^2\overset{?}{>}0 \qquad (9.7)$$

The left-hand side of (9.7) has a double zero spot at $\mathrm{h}^{HCM}=\frac{a}{2N\cdot\left(N+1\right)}$. Hence, the condition is either always violated or fulfilled. Inserting $h^{HCM}=0$, for example, shows that (9.7) is always fulfilled and that the supplier's profit is always larger in the dynamic solution in comparison of the static solution regardless of the number of buyers within the supply chain.

Profit Comparison (Single) Buyer:

$$\pi_B^{HCM,Dynamic} \overset{?}{>} \pi_B^{HCM,Static}$$

$$\frac{\left(4N^6+32N^5+96N^4+128N^3+63N^2-6N-7\right)a^2}{2N^2\left(2N^2+8N+7\right)^2\left(N+1\right)^2 b}-\frac{\left(8N^6+40N^5+72N^4+64N^3+38N^2+14N\right)ah^{HCM}}{2N^2\left(2N^2+8N+7\right)^2\left(N+1\right)^2 b}$$
$$+\frac{\left(24N^6+120N^5+224N^4+184N^3+56N^2\right)\left(h^{HCM}\right)^2}{2N^2\left(2N^2+8N+7\right)^2\left(N+1\right)^2 b}\overset{?}{>}\frac{a^2}{2\left(N+1\right)^2 b}$$

$$\left(4N^6+32N^5+96N^4+128N^3+63N^2-6N-7\right)a^2-\left(8N^6+40N^5+72N^4+64N^3+38N^2+14N\right)ah^{HCM}$$
$$+\left(24N^6+120N^5+224N^4+184N^3+56N^2\right)\left(h^{HCM}\right)^2 \overset{?}{>} N^2\left(2N^2+8N+7\right)^2 a^2$$

$$\left(4N^6+32N^5+96N^4+128N^3+63N^2-6N-7\right)a^2-\left(8N^6+40N^5+72N^4+64N^3+38N^2+14N\right)ah^{HCM}$$
$$+\left(24N^6+120N^5+224N^4+184N^3+56N^2\right)\left(h^{HCM}\right)^2 \overset{?}{>}\left(4N^6+32N^5+92N^4+112N^3+49N^2\right)a^2$$

$$\left(4N^4+16N^3+14N^2-6N-7\right)a^2-\left(8N^6+40N^5+72N^4+64N^3+38N^2+14N\right)ah^{HCM}$$
$$+\left(24N^6+120N^5+224N^4+184N^3+56N^2\right)\left(h^{HCM}\right)^2 \quad \overset{?}{>}0 \tag{9.8}$$

The left-hand side of (9.8) has zero spots at $(I.)$ $\mathrm{h}^{HCM}=\frac{a}{2N\cdot(N+1)}$ and at

$(II.)$ $\mathrm{h}^{HCM}=\frac{(4N^4+16N^3+14N^2-6N-7)\cdot a}{4N\cdot\left(3N^3+12N^2+16N+7\right)}$.

$(I.)$ does not need to be considered as the dynamic solution only applies for:

$$\mathrm{h}^{HCM}<\frac{a}{2N\cdot(N+1)}.$$

And $(II.)$ only needs to be considered if it applies within the dynamic solution:

$$\frac{(4N^4+16N^3+14N^2-6N-7)\cdot a}{4N\cdot\left(3N^3+12N^2+16N+7\right)}\overset{?}{<}\frac{a}{2N\cdot(N+1)}$$

$$(4N^5+20N^4+30N^3+8N^2-13N-7)\cdot a\overset{?}{<}\left(6N^3+24N^2+32N+14\right)\cdot a$$

$$(4N^5+20N^4+24N^3-16N^2-45N-21)\overset{?}{<}0 \tag{9.9}$$

For $N=1$ (i.e. in the standard model of Anand et al. (2009)) condition (9.9) is fulfilled and, therefore, there must exist both an interval of holding costs under which the buyer prefers the dynamic solution as well as an interval of holding costs under which the buyer prefers the static solution. Anand et al. (2009) have shown that the buyer prefers the dynamic solution if $h^{SM}<\frac{21}{152}a\approx 0.138a$. Increasing the number of buyers (e.g. $N=2$) shows that condition (9.9) is no longer fulfilled. Hence, the buyers will always prefer either the dynamic or the static solution regardless of the level of the holding costs (h^{HCM}). Inserting $N=2$ and $h^{HCM}=0$ into (9.8) shows that the left-hand side is larger and, therefore, that the buyers will always prefer the dynamic over the static solution, if there is more than one buyer within the supply chain (i.e. if $N\geq 2$).

Appendix D: Profit Comparison of the Static and Dynamic Solution in the Inventory Interest Rate Model

Profit Comparison Supplier:

$$\pi_S^{IIR,Dynamic} \overset{?}{>} \pi_S^{IIR,Static}$$

$$\frac{9a^2}{2\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)b} \overset{?}{>} \frac{a^2}{4b}$$

$$\frac{18a^2}{17+4i^{IIR}-4\left(i^{IIR}\right)^2} \overset{?}{>} a^2 \tag{9.10}$$

In order to test condition (9.10) it is important to consider whether its denominator is positive or negative:

$$17+4i^{IIR}-4\left(i^{IIR}\right)^2 \overset{?}{>} 0 \tag{9.11}$$

Condition (9.11) is fulfilled if $\frac{1}{2}-\frac{3\cdot\sqrt{2}}{2}<i^{IIR}<\frac{1}{2}+\frac{3\cdot\sqrt{2}}{2}$. As the dynamic solution is only feasible if $0<i^{IIR}<\frac{1}{2}$, condition (9.11) is always fulfilled and, therefore, only the case of a positive denominator in (9.10) needs to be considered:

$$18a^2 \overset{?}{>} \left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)a^2$$

$$\left(4\left(i^{IIR}\right)^2-4i^{IIR}+1\right)a^2 \overset{?}{>} 0 \tag{9.12}$$

Condition (9.12) is fulfilled if $i^{IIR} \neq 0.5$. Therefore, within the feasible range of the dynamic solution condition (9.12) is always fulfilled and the supplier always prefers the dynamic over the static solution.

Profit Comparison Buyer:

$$\pi_B^{IIR,Dynamic} \overset{?}{>} \pi_B^{IIR,Static}$$

$$\frac{\left(155+38i^{IIR}-60\left(i^{IIR}\right)^2+8\left(i^{IIR}\right)^3+32\left(i^{IIR}\right)^4\right)a^2}{4\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 b} \overset{?}{>} \frac{a^2}{8b}$$

$$\left(310+76i^{IIR}-120\left(i^{IIR}\right)^2+16\left(i^{IIR}\right)^3+64\left(i^{IIR}\right)^4\right)a^2 \overset{?}{>} \left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 a^2$$

$$\left(310+76i^{IIR}-120\left(i^{IIR}\right)^2+16\left(i^{IIR}\right)^3+64\left(i^{IIR}\right)^4\right) \overset{?}{>} \left(289+136i^{IIR}-120\left(i^{IIR}\right)^2-32\left(i^{IIR}\right)^3+16\left(i^{IIR}\right)^4\right)$$

$$\left(21-60i^{IIR}+48\left(i^{IIR}\right)^3+48\left(i^{IIR}\right)^4\right) \overset{?}{>} 0$$

$$\left(i^{IIR}-0.5\right)^2 \cdot \left(84+96i^{IIR}+48\left(i^{IIR}\right)^2\right) \overset{?}{>} 0 \qquad (9.13)$$

As condition (9.13) is fulfilled if $i^{IIR} \neq 0.5$, the buyer is always better off under the dynamic solution in comparison to the static solution.

Moreover, as both supplier and buyer are better off in the dynamic solution, the supply chain profit is also always larger in the dynamic solution.

<u>Appendix E:</u> Generality regarding the Parameter Choice in the Inventory Interest Rate Model

Varying the b parameter will influence the first-best, static and dynamic solutions of the standard model in the same way as in the corresponding solutions of the inventory interest model (see Table 2 and Table 12). If, for example, the parameter b is doubled, the first-period wholesale price of the first-best, static and dynamic solution will divide in half both in the standard as well as in the inventory interest rate model.

Varying the a parameter will influence the first-best, static and dynamic solution of the inventory interest rate model in the same way (see Table 12). If, for example, the parameter a is doubled, the first-period wholesale price of the first-best, static and dynamic solution will be doubled as well.

In the dynamic case of the standard model, however, the ratio between a and the holding cost parameter (h^{SM}) is the decisive factor. Hence, if a is doubled h^{SM} also needs to be doubled in order to get doubled result for the items displayed in Table 2. However, as h^{SM} gets replaced by $h^{SM} := i^{IIR} \cdot w_1^{IIR}\left(i^{IIR}\right) = i^{IIR} \cdot 9a \Big/ \left(17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2\right)$ in the graphical illustrations of section 6, the modified results of the standard model show the same behaviour as in the inventory interest rate if a is varied (e.g. if a is doubled, both the results of the interest rate model as well as the modified results of the standard model will be doubled in the first-best, static and dynamic case).

Appendix F: Transformation of the Holding Cost Parameters between Standard and Inventory Interest Rate Model

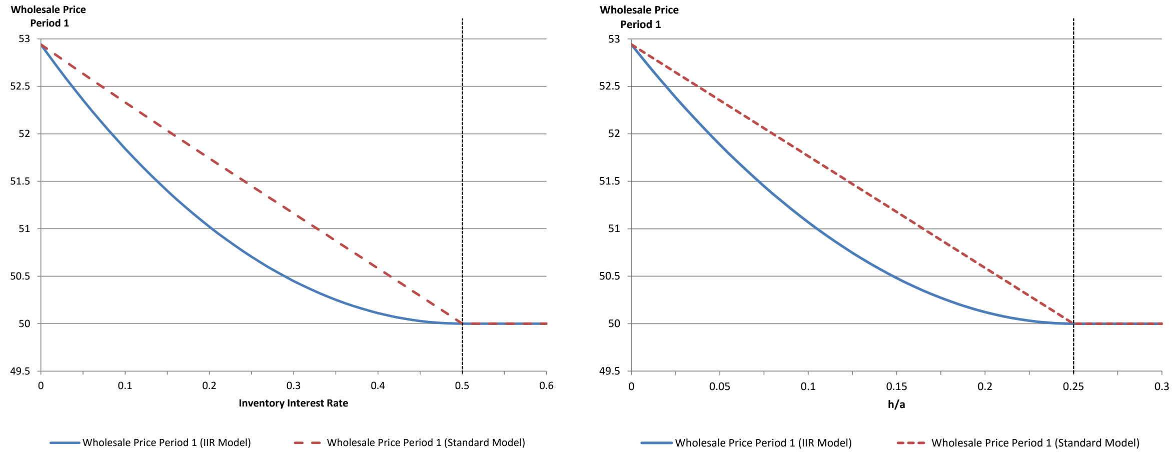


Figure 46: Error caused by the Direction of Transformation between Standard and Inventory Interest Rate Model

On the right hand side, the holding cost parameter of the standard model (h^{SM}) was transformed to an interest rate. The first-period wholesale prices (w_1^{SM} / w_1^{IIR}) were calculated by inserting h^{SM} into $w_1^{SM}\left(h^{SM}\right)$ and by inserting the transformed interest rate $\left(i^{IIR} = \dfrac{h^{SM}}{w_1^{SM}}\right)$ into $w_1^{IIR}(i^{IIR})$. On the left hand side, the transformation was made the other way around. Comparing both methods shows that no clear difference is visible as the created error is below 0.02%.

Appendix G: Profit Comparison between Inventory Interest Rate and Modified Standard Model (Dynamic Solutions)

In the following comparison, the holding cost parameter of the standard model gets calculated from the inventory interest rate of the IIR model:

$$h^{SM} := i^{IIR} \cdot w_1^{IIR} = i^{IIR} \cdot \frac{9a}{17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2} \tag{9.14}$$

As the static solution of the IIR and the standard model are identical, only the area in which the dynamic solution applies (i.e. $i^{IIR} < 0.5$) has to be considered. Moreover, $i^{IIR} = 0$ does not need to be considered, as the dynamic solutions of the IIR model and standard model are identical in this case.

Profit Comparison Supplier:

$$\begin{aligned} \pi_S^{IIR,dynamic} &\overset{?}{<} \pi_S^{SM,dynamic} \\ \pi_S^{IIR,dynamic} - \pi_S^{SM,dynamic} &\overset{?}{<} 0 \end{aligned} \tag{9.15}$$

$$\frac{9a^2}{2 \cdot \left(17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2\right) b} - \frac{9a^2 - 4ah^{SM} + 8\left(h^{SM}\right)^2}{34b} \overset{?}{<} 0 \tag{9.16}$$

Inserting (9.14) into (9.16):

$$\frac{9a^2}{2 \cdot \left(17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2\right) b} - \frac{9a^2 \cdot \left(16i^4 - 16i^3 - 64i^2 + 68i + 289\right)}{34 \cdot \left(4i^2 - 4i - 17\right)^2 b} \overset{?}{<} 0 \tag{9.17}$$

$$-\frac{18 \cdot a^2 \cdot \left(i^{IIR}\right)^2 \cdot \left(4\left(i^{IIR}\right)^2 - 4i^{IIR} + 1\right)}{17 \cdot \left(4\left(i^{IIR}\right)^2 - 4i^{IIR} - 17\right)^2 b} \overset{?}{<} 0 \tag{9.18}$$

For $0 < i^{IIR} < 0.5$, (9.18) can be simplified to:

$$-\left(4\left(i^{IIR}\right)^2 - 4i^{IIR} + 1\right) \overset{?}{<} 0 \tag{9.19}$$

As condition (9.19) is always fulfilled for $0 < i^{IIR} < 0.5$, the supplier's profit is smaller in the IIR model in comparison to the benchmark obtained by the modified standard model. Note, however, that the difference (9.18) is relatively small.

Profit Comparison Buyer:

$$\pi_B^{IIR,dynamic} \overset{?}{>} \pi_B^{SM,dynamic}$$
$$\pi_B^{IIR,dynamic} - \pi_B^{SM,dynamic} \overset{?}{>} 0 \tag{9.20}$$

$$\frac{\left(155 + 38i^{IIR} - 60\left(i^{IIR}\right)^2 + 8\left(i^{IIR}\right)^3 + 32\left(i^{IIR}\right)^4\right)a^2}{4 \cdot \left(17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2\right)^2 b} - \frac{155a^2 - 118ah^{SM} + 304\left(h^{SM}\right)^2}{1156b} \overset{?}{>} 0 \tag{9.21}$$

$$\frac{\left(155 + 38i^{IIR} - 60\left(i^{IIR}\right)^2 + 8\left(i^{IIR}\right)^3 + 32\left(i^{IIR}\right)^4\right)a^2}{4 \cdot \left(17 + 4i^{IIR} - 4\left(i^{IIR}\right)^2\right)^2 b}$$
$$- \frac{a^2\left(2480\left(i^{IIR}\right)^4 - 712\left(i^{IIR}\right)^3 + 1776\left(i^{IIR}\right)^2 + 3026i^{IIR} + 44795\right)}{1156\left(4\left(i^{IIR}\right)^2 - 4i^{IIR} - 17\right)^2 b} \overset{?}{>} 0 \tag{9.22}$$

$$\frac{9a^2\left(376\left(i^{IIR}\right)^4 + 404\left(i^{IIR}\right)^3 - 166\left(i^{IIR}\right)^2 - 325i^{IIR} - 365\right)}{578\left(4\left(i^{IIR}\right)^2 - 4i^{IIR} - 17\right)^2 b} \overset{?}{>} 0 \tag{9.23}$$

$$\frac{9a^2 i\left(188\left(i^{IIR}\right)^3 + 84\left(i^{IIR}\right)^2 - 531i^{IIR} + 221\right)}{289\left(4\left(i^{IIR}\right)^2 - 4i^{IIR} - 17\right)^2 b} \overset{?}{>} 0 \tag{9.24}$$

For $0 < i^{IIR} < 0.5$, (9.24) can be simplified to:

$$188\left(i^{IIR}\right)^3 + 84\left(i^{IIR}\right)^2 - 531i^{IIR} + 221 \overset{?}{>} 0 \tag{9.25}$$

As condition (9.25) is always fulfilled for $0 < i^{IIR} < 0.5$, the buyer's profit is larger in the IIR model in comparison to the benchmark obtained by the modified standard model.

Profit Comparison Supply Chain:

$$\frac{\left(461+110i^{IIR}-132\left(i^{IIR}\right)^2+8\left(i^{IIR}\right)^3+32\left(i^{IIR}\right)^4\right)a^2}{4\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 b}-\frac{461a^2-254ah^{SM}+576\left(h^{SM}\right)^2}{1156b}\overset{?}{>}0 \quad (9.26)$$

$$\begin{aligned}&\frac{\left(461+110i^{IIR}-132\left(i^{IIR}\right)^2+8\left(i^{IIR}\right)^3+32\left(i^{IIR}\right)^4\right)a^2}{4\cdot\left(17+4i^{IIR}-4\left(i^{IIR}\right)^2\right)^2 b}\\&-\frac{a^2\left(7376\left(i^{IIR}\right)^4-5608\left(i^{IIR}\right)^3-17808\left(i^{IIR}\right)^2+23834i^{IIR}+133229\right)}{1156\left(4\left(i^{IIR}\right)^2-4i^{IIR}-17\right)^2 b}\overset{?}{>}0\end{aligned} \quad (9.27)$$

$$\frac{9a^2i^{IIR}\left(52\left(i^{IIR}\right)^3+220\left(i^{IIR}\right)^2-565i^{IIR}+221\right)}{289\left(4\left(i^{IIR}\right)^2-4i^{IIR}-17\right)^2 b}\overset{?}{>}0 \quad (9.28)$$

For $0 < i^{IIR} < 0.5$, (9.28) can be simplified to:

$$52\left(i^{IIR}\right)^3+220\left(i^{IIR}\right)^2-565i^{IIR}+221\overset{?}{>}0 \quad (9.29)$$

As condition (9.29) always fulfilled for $0 < i^{IIR} < 0.5$, the overall supply chain profit is larger in the IIR model in comparison to the benchmark obtained by the modified standard model.

Appendix H: Generality regarding the Parameter Choice in the Discounted Cash Flow Model

Varying the a and b parameter will influence the first-best, static and dynamic solution of the discounted cash flow as well as in the inventory interest model in the same way (see Table 12 and Table 14 + Table 15). If, for example, the parameter $a\left(b\right)$ is doubled, the first-period wholesale price of the first-best, static and dynamic solution will divide in half (be doubled) both in the discounted cash flow as well as in the inventory interest rate model.

The same holds for the relationship between the standard and the inventory interest rate model as shown in Appendix E.

Appendix I: Profit Comparison Discounted Cash Flow Model

$$\begin{aligned}\Big[&\left(192\left(i^{DCF}\right)^3+480\left(i^{DCF}\right)^2+87i^{DCF}+42\right)a^2\\&+\left(320\left(i^{DCF}\right)^3+816\left(i^{DCF}\right)^2+24i^{DCF}-472\right)ah^{DCF}\\&+\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)\left(h^{DCF}\right)^2\Big]>0\end{aligned} \tag{9.30}$$

$$\begin{aligned}part1&:=\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)\\part2&:=\left(320\left(i^{DCF}\right)^3+816\left(i^{DCF}\right)^2+24i^{DCF}-472\right)a\\part3&:=\left(192\left(i^{DCF}\right)^3+480\left(i^{DCF}\right)^2+87i^{DCF}+42\right)a^2\end{aligned} \tag{9.31}$$

$$h_{1/2}^{DCF}=\frac{-part2\pm\sqrt{\left(part2\right)^2-4\cdot part1\cdot part3}}{2\cdot part1}$$

$$\begin{aligned}h_{1/2}^{DCF}=&-\frac{\left(320\left(i^{DCF}\right)^3+816\left(i^{DCF}\right)^2+24i^{DCF}-472\right)a}{2\cdot\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)}\\&\pm\frac{\sqrt{\begin{array}{l}\left(\left(192\left(i^{DCF}\right)^3+480\left(i^{DCF}\right)^2+87i^{DCF}+42\right)a^2\right)^2\\-4\cdot\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)\\\cdot\left(\left(320\left(i^{DCF}\right)^3+816\left(i^{DCF}\right)^2+24i^{DCF}-472\right)a\right)\end{array}}}{2\cdot\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)}\end{aligned} \tag{9.32}$$

$$\begin{aligned}h_{1/2}^{DCF}=&-\frac{\left(40\left(i^{DCF}\right)^2+62i^{DCF}-59\right)a}{4\cdot\left(12\left(i^{DCF}\right)^3+73\left(i^{DCF}\right)^2+137i^{DCF}+76\right)}\\&\pm\frac{\sqrt{\begin{array}{l}36864a^4\left(i^{DCF}\right)^6+184320a^4\left(i^{DCF}\right)^5+263808a^4\left(i^{DCF}\right)^4-245760a\left(i^{DCF}\right)^7\\+99648a^4\left(i^{DCF}\right)^3-2367488a\left(i^{DCF}\right)^6+47889a^4\left(i^{DCF}\right)^2-8758272a\left(i^{DCF}\right)^5\\+7308a^4\left(i^{DCF}\right)-15097344a\left(i^{DCF}\right)^4+1764a^4-10435072a\left(i^{DCF}\right)^3\\+2047488a\left(i^{DCF}\right)^2+6317568ai^{DCF}+2295808a\end{array}}}{2\cdot\left(192\left(i^{DCF}\right)^4+1360\left(i^{DCF}\right)^3+3360\left(i^{DCF}\right)^2+3408i^{DCF}+1216\right)}\end{aligned} \tag{9.33}$$

Appendix J: Comparison of the Dynamic and Static Solution of the Discounted Cash Flow Model

Combination of holding cost parameters (h^{DCF} , i^{DCF}) for which the dynamic supply chain profit is larger than the static profit:

$$\begin{aligned} \pi_{SC}^{DCF,dynamic} &\overset{?}{>} \pi_{SC}^{DCF,static} \\ \pi_{SC}^{DCF,dynamic} - \pi_{SC}^{DCF,static} &\overset{?}{>} 0 \end{aligned} \tag{9.34}$$

$$\left[\begin{array}{l} \dfrac{\left(461+680i^{DCF}+364\left(i^{DCF}\right)^2+64\left(i^{DCF}\right)^3\right)a^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ -\dfrac{\left(250+330i^{DCF}+60\left(i^{DCF}\right)^2-16\left(i^{DCF}\right)^3\right)ah}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ +\dfrac{\left(576+1660i^{DCF}+1704\left(i^{DCF}\right)^2+732\left(i^{DCF}\right)^3++122\left(i^{DCF}\right)^4\right)h^2}{4\cdot\left(1+i^{DCF}\right)\left(17+8i^{DCF}\right)^2 b} \\ -\dfrac{3\cdot\left(2+i^{DCF}\right)a^2}{16\cdot\left(1+i^{DCF}\right)b} \end{array}\right] \overset{?}{>} 0 \tag{9.35}$$

$$\left[\begin{array}{l} \left(110+221i^{DCF}+256\left(i^{DCF}\right)^2+64\left(i^{DCF}\right)^3\right)a^2 \\ -\left(1016+1320i^{DCF}+240\left(i^{DCF}\right)^2-64\left(i^{DCF}\right)^3\right)a\cdot h^{DCF} \\ +\left(2304+6640i^{DCF}+6816\left(i^{DCF}\right)^2+2928\left(i^{DCF}\right)^3+448\left(i^{DCF}\right)^4\right)\left(h^{DCF}\right)^2 \end{array}\right] \overset{?}{>} 0 \tag{9.36}$$