Paradoxes of the Multi-Chain Critical Paths as the Dissipative Structures

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Abstract: Parametric and structural uncertainties complicate the project management processes. The critical path is one of the pivotal parameters, which helps to control the project schedule and is used to determine the criticality of the tasks and activities that are the most decisive and should be treated during a project expediting or controlling. There may be a set of the critical paths in uncertain environment. Therefore, the main question is which of the critical paths to select. The aim of this paper is to answer to this question. We used Monte Carlo simulation to investigate the multiple critical paths. We revealed and explained several paradoxes that emerged as results of the multiple critical paths occurrence. They are inevitable late bias of the project duration under uncertainty, the tasks probability and their correlation effects, the impact of concurrent chains of the tasks on their criticality, multiplicity of the critical paths and especially multi-chain critical paths. We demonstrated that multiple critical paths are not negative effect. On the contrary, they play extraordinary useful role and are the reliable criterion of the project robustness and stability.

1 INTRODUCTION

Operational management is an ongoing process whereas project management has final deliverable and a finite timespan [1]. That is why timely completion of the project within the planned budget, the scope of the project results and their quality is of particular importance [2].

Unfortunately, the majority of the projects are deployed in uncertain environment [3]. Therefore, almost half of the projects are completed late, over budget, and the scope of planned services and deliverables is incomplete or does not meet the declared quality [4]. Wauters and Vanhoucke demonstrated that the uncertainty induces cost deviation in complex projects and leads to deadline overruns and, consequently, to losses and penalties [5].

One of the most critical parameters in the project schedule control is a critical path. It is a sequence of project network descendent activities. They adds up to the longest overall duration, regardless of whether that longest duration has a float or not. The critical path determines the shortest time possible to complete the project.

Deterministic approach consider a single critical path, which is sufficient to determine the project span. However, there may by several critical paths in a project schedule in uncertain environment. This peculiarity has been noted since the very inception of the critical path method [6], [7]. Nevertheless, the literature analysis has shown that experts try to find the most critical path neglecting the other minor critical paths. Other authors consider that multiple critical paths pose a threat to the project, as the most critical path is hidden among others and may appear in the future, when it is too late and a significant share of resources will be spent on reducing ‘faux or miner’ critical paths [8], [9]. Nazimko, Zakharova and Smith demonstrated that multiple critical paths in a project schedule as an oriented stochastic activity network are extremely useful information playing an important role in project schedule management under parametric and structural uncertainty. They showed in an example of a project expediting that the most critical path and near critical paths are equitable items or concepts that deserve equal treatment and attention.

Furthermore, Nazimko and Zakharova revealed multi-chain multiple critical paths, which emerge in the uncertain setting [10]. These authors demonstrated that multiple critical paths could bifurcate facilitating extra alternative multi-chain paths, which boost interaction among the tasks and create the synergetic nonlinear effects that multiply the efficiency of the schedule crashing and facilitate the fast-tracking process. The number of the single chain critical paths may be as many as the number of the tasks chains, whereas multi-chain critical paths quantity can exceed them two order of magnitude. Ignoring the multiple critical paths may lead to a 14-23% bias in the modeling results [10].

Despite such an important role of the multi-chain critical paths, their parameters have not been investigated thoroughly so far. Therefore, the aim of this paper was to explore parameters of these paths and their behaviour during the project deployment in an uncertain environment.

First, we described the method of stochastic simulation in this presentation and selected a relevant benchmark to investigate the multiple critical paths behaviour. In the next sections, we analysed the multiplicity of the critical paths, their multi-chain form, duration, effect of the tasks probability variation and correlation between the tasks. Then, we discussed some paradoxes associated with the multiple critical paths and limitations of the research scope and results, made conclusion and proposed practical recommendations.

2 METHODOLOGY OF THE MULTIPLE CRITICAL PATHS SIMULATION

2.1 Selection of a Computer Model

We used the Monte-Carlo Simulation method [11] to investigate the multiple critical paths, which emerge in a project schedule.

We presented a project schedule as an acyclic antisymmetrically oriented multigraph. Nodes in the graph stood for events, whereas arcs played the role of activities or tasks. As the activities duration varied, we simulated it using predetermined distributions. We employed field data, experience, or assigned the distributions by experts according to PMBoK [12]. Because numerical methods have been used, any popular type of distribution could be programmed, namely, uniform, normal, triangular, PERT, and arbitrary distribution that might be designed from discrete bins.

In addition, we took into account symmetric and asymmetric correlation between input parameters. The structural uncertainty has been modeled by assigning a probability to some activities according to [10]. When an activity disappeared eventually, a dummy arc with zero duration has been used.

An initial input record in a stochastic procedure presented every task. The record consisted of task number, task probability, and task position relatively a descendant (from one node of the multigraph to another). Next, the type of task duration distribution and its parameters were input.

Besides, we assigned correlation (yes or no) and the coefficient of correlation between the activities. For example, task A correlated with task B that means asymmetrical correlation when A depends on B. Symmetrical correlation provides other effects, thus this reciprocal correlation was indicated by the mutual dependence. To this end, we used the same correlation coefficient and indication of the companion both in the record of task A and the record of task B.

A current critical path has been detected during every run. When the critical path was structurally identical to previously identified critical paths, the number of such paths was incremented by unit. If there was no such critical path in the formerly identified set, then the critical paths' list has been upgraded.

The closeness of the activities correlation was less then unit because the duration of the activities was selected as a random value. Besides, the structure of the network was uncertain, hence the critical paths and their duration varied during the simulation. The model counted the number of the critical paths and determined the frequency of every structurally identical path at the end. As a result, a set of all possible critical paths has been identified because of a large number of runs.

Any type of the tasks duration distribution has lower and upper boundaries, whereas the dimension of the durations was integer and measured by days, weeks, etc. Thus, theoretically, the number of possible permutations and runs is limited. The real number of the permutations is even essentially less taking into account the correlations and physical restrictions. For example, ten thousands of runs are sufficient to discover almost all critical paths [7].

In addition, a limit rate of the critical paths number increment was assigned to check when the increment of the critical paths becomes negligible and simulation should be stopped.

2.2 Substantiation of a Benchmark

The critical paths diversity occurs especially in a schedule that consists of many serial and parallel chains of tasks.

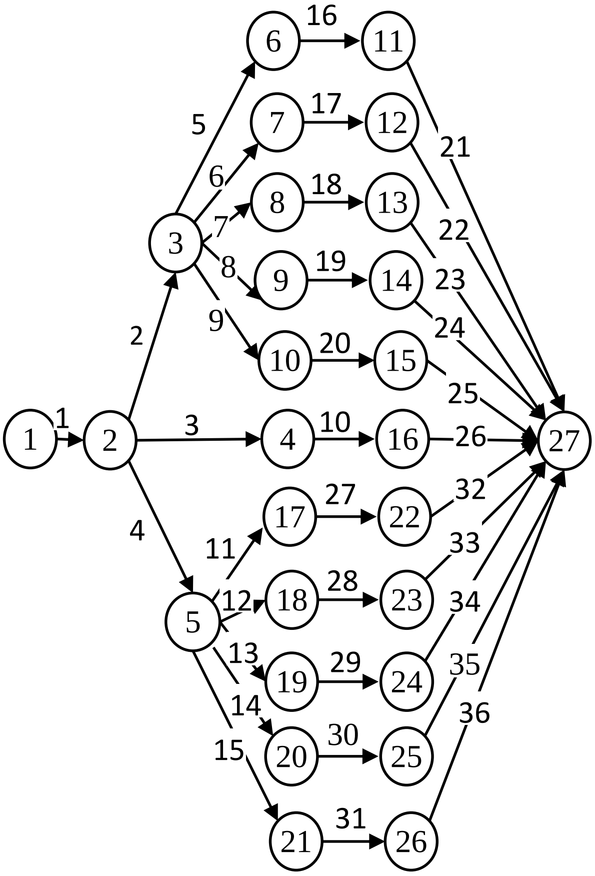

Figure 1: Project duration distribution.

That is why a project schedule composed by Tolentino Rena, R.A. [13] has been selected as the benchmark because it comprised a big number of serial and parallel chains of the tasks (Table 1). This schedule has been selected as the benchmark by several authors [10], [14].

The structure of the benchmark schedule is depicted in Figure 1. The benchmark schedule consisted of 36 tasks that were structured in 11 concurrent chains composed by 4-5 serial tasks. The tasks durations were presented by triangular distributions that have been characterized by minimum, maximum possible duration and the mode.

Table 1: Initial data of the benchmark project schedule composed by Tolentino Rena [13].

Task number	Probability	From	To	Type	Mode	Min	Max
1	1	1	2	3	10	5	15
2	1	2	3	3	20	10	30
3	1	2	4	3	30	20	40
4	1	2	5	3	20	10	30
5	1	3	6	3	15	10	20
6	1	3	7	3	15	10	20
7	1	3	8	3	15	10	20
8	1	3	9	3	15	10	20
9	1(0.5)	3	10	3	15	10	20
10	1	4	16	3	22	17	27
11	1	5	17	3	15	10	20
12	1	5	18	3	15	10	20
13	1	5	19	3	15	10	20
14	1	5	20	3	15	10	20
15	1	5	21	3	15	10	20
16	1	6	11	3	15	10	20
17	1	7	12	3	15	10	20
18	1	8	13	3	15	10	20
19	1	9	14	3	15	10	20
20	1	10	15	3	15	10	20
21	1	11	27	3	15	10	20
22	1	12	27	3	15	10	20
23	1	13	27	3	15	10	20
24	1	14	27	3	15	10	20
25	1	15	27	3	15	10	20
26	1	16	27	3	15	10	20
27	1	17	22	3	15	10	20
28	1	18	23	3	15	10	20
29	1	19	24	3	15	10	20
30	1	20	25	3	15	10	20
31	1	21	26	3	15	10	20
32	1	22	27	3	15	10	20
33	1	23	27	3	15	10	20
34	1	24	27	3	15	10	20
35	1	25	27	3	15	10	20
36	1	26	27	3	15	10	20

3 RESULTS OF SIMULATION

3.1 Multi-Chain Multiple Critical Paths

A specific feature of stochastic activity network is that it yields approximate or not exact result because the random nature. Put other words, the results of computer simulation are fluctuated relatively certain mathematical expectation which was 77 time units in this case. Seventy seven units duration has been computed and the crisp sum of the tasks' duration modes. The average duration of the project was 82.7±4.1 (Figure 2). Noticeably, this fluctuation biased to the right (82.7>77) or towards higher value relatively the crisp mathematical expectation.

The project may be completed during 84 time units with 95% confidence (Figure 2, bottom).

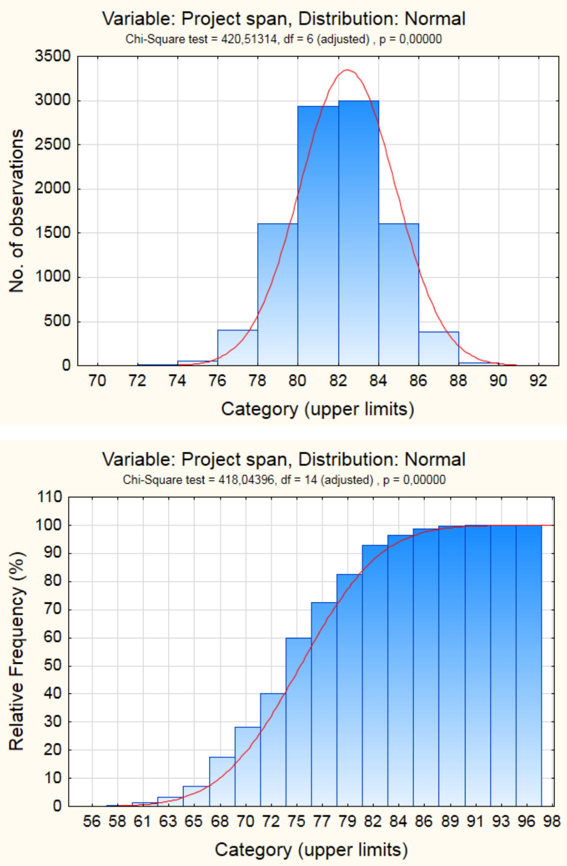

Figure 2: Project duration distribution.

Computer simulation produced 209 critical paths that is by order more than the number of the tasks chains in the project schedule. That is why we classified the critical paths as multiple. Eleven single-chain critical paths (Table 2) corresponded to eleven tasks sequences in the project schedule. Critical path 1-3-10-26 occurred the most frequently. The other 10 critical paths represented 10 parallel series of the tasks. The frequency of the every path variated in the range of 0.059-0.065. Thus, number of the single chain critical paths corresponded to the number of the task chains. The other one hundred and ninety six critical paths were multi-chain trails.

Table 2: Typical example of the multiple multi-chain critical paths.

#	Critical path	Frequency
7	1-3-10-26	0.207
2	1-2-7-18-23	0.059-0.065
4	1-4-12-28-33; 1-4-14-30-35	0.028-0.047
11	1-2-9-20-25; 1-3-10-26; 1-4-2-28-33	0.005-0.008
155	1-2-6-17-22; 1-2-9-20-25; 1-4-12-28-33; 1-4-15-31-36; 1-2-6-8-19-24; 1-2-6-17-21; 1-2-8-19-24	< 0.001
207	1-4-11-27-32; 1-4-12-28-33; 1-4-13-29-34-; 1-4-14-30-35	0.001
173	1-2-5-16-21; 1-2-7-18-23; 1-2-8-19-24; 1-4-14-30-35; 1-4-15-31-36;	0.001
203	1-2-6-17-22; 1-2-8-19-24; 1-2-9-20-25; 1-3-10-26; 1-4-11-27-32; 1-4-13-29-34	<0.001

Criticality of the major paths is presented in the Table 3. Sequential task 1 has maximal criticality that is natural. Despite the tasks 3 belongs to the most critical path, its criticality is almost two times less than those of tasks 2 and 4, belonging to the critical paths (Table 2) having lover frequency.

Table 3: Distribution of the tasks criticality.

Task	1	2,4	3,10,26	5-9	11-15
Criticality	1.0	**0.42**	**0.25**	0.09	0.09
Frequency of the critical paths, which contains this task	1.0	**0.059-0.065**	**0.207**	0.005-0.008	0.001

3.2 Duration of Multi-Chain Multiple Critical Paths

Another unusual feature of the multiple critical paths is their duration and composition. Let us recall that we identified the critical paths that go straight the same sequence of the tasks. However, similar critical paths may have different duration.

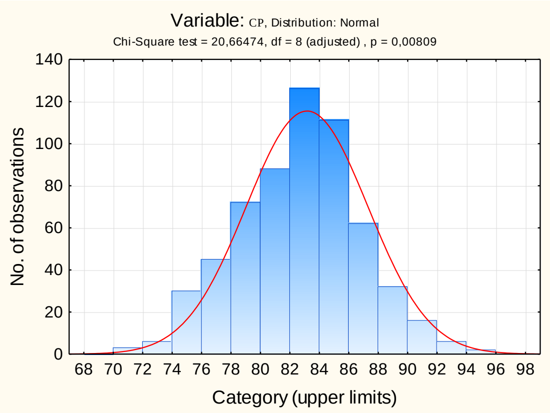

Figure 3: Distribution of duration for critical path 1-2-6-12-22-27.

For example, path 1-2-6-12-22-27 duration ranged from 68 time units up to 98, and average was 83.2±4.1 (Figure 3). Therefore, one should not only identify a relevant multiple critical path but also take into account the distribution of its duration. This particularity imposes additional restrictions and requirements on the possible range decisions.

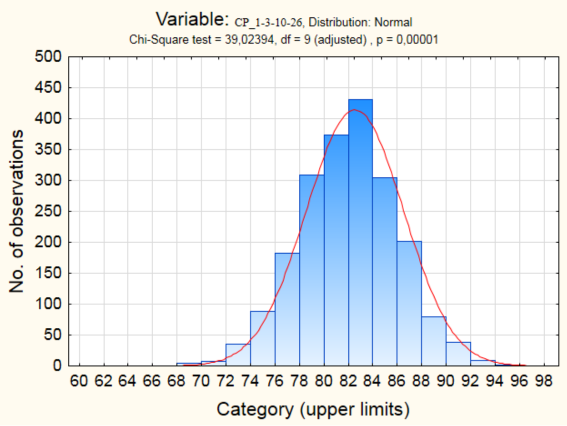

Figure 4: Critical path 1-3-10-26-27 duration distribution.

Figure 4 shows similar distribution of critical path 1-3-10-26-27. Expected value of path duration is 82.5±4.0 that is some more than former critical path. However, the distributions should be compared using certain criterion, for example Kolmogorov-Smirnov [15].

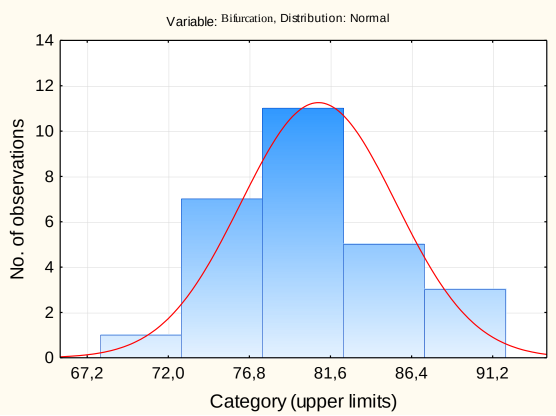

Figure 5: Critical path 1-3-10-26-27.

The duration of the bifurcated critical path 1-3-10-26;1-4-11-27-32 is presented in Figure 5. Range of its duration varied from 67 to 92 time units whereas the expected average was 80.9±4.6. This was another paradox: how may the multi-chain critical path have less duration in comparison the single-chain critical path?

3.3 Effect of a Task Probability and Tasks Correlation

The probability of the task 9 has been reduced down to 0.5 to simulate structural uncertainty (look in Table 1 position 9 in parenthesis). Table 4 demonstrates the main effect of tis modification. Previous frequencies of the critical paths are indicated in the parenthesis.

Table 4: Redistribution of the critical paths frequency after decrease of the task 9 probability.

#	Critical path	Frequency
7	1-3-10-26	0.219(0.207)
2	1-2-7-18-23	0.060-0.0692 (0.059-0.065)
21	1-2-**9**-20-25	0.028(0.062)

The single-chain critical path in which task 9 was arranged reduced the frequency more than twice whereas adjacent critical paths increased their incidences. All multi-chain critical paths or their branches which had task 9 dropped their frequencies essentially. Generally, redistribution of the frequencies occurred for the benefit of the neighboring chains of the schedule's tasks.

Reciprocal relation between the tasks was simulated indicating the correlation of the task 9 and task 24. The correlation coefficient was selected as 0.8.

Table 5: Redistribution of the critical paths frequency after introduction of the correlation.

#	Critical path	Frequency
7	1-3-10-26	0.201(0.207)
2	1-2-7-18-23	0.057-0.069 (0.059-0.065)
21	1-2-**9**-20-25	0.058(0.062)
11	1-2-8-18-**24**	0.07(0.065)

There was not apparent effect of the correlation between the tasks although some redistribution of the frequencies was registered. The frequency of the chain having task 9 slightly dropped whereas the frequency of the sequence where task 24 was located increased (Table 5).

3 DISCUSSION

The general effect of the uncertainty manifested first of all in the bias of the project duration to the more later deadliness. This bias of the tasks sequences and project duration is natural effect of the fundamental thermodynamic law, which postulates excess entropy production from the chaos and uncertainty [16]. Despite the task duration variation occurs symmetrically in initial data, the final results inevitably connect with the entropy production.

The next paradox was presented by illogical effect when the tasks that were located in the serial sequences having evidently less frequencies manifested essentially bigger criticality. The explanation of this paradox lays in the parallel structure of the schedule. The parallel chains of the tasks arrangements took over the flow of resources and information creating alternative channels. Such a parallel system played a role of an open dissipative structure that absorb the energy, material and information fluxes [10].

The most unexpected feature of the project schedule as a stochastic activity network was multiplicity of the critical paths and especially their multi-chain structure. The computer simulation has demonstrated that such structure induced essential bias and put forward the problem of a structural uncertainty that plays an important role in processes of the project management. This finding proved the importance of the critical paths multiplicity and even their versatile multi-chain forms. We demonstrated that all critical paths are important equally and should be concerned carefully. There is no 'the most critical' path or 'minor' path. The alternative paths may change each other which induces useful competitive process of a schedule expediting or enhancing.

We investigated effects of the tasks probability and their correlation demonstrating that these structural parameters may essentially impact the critical path frequency redistributing it between the tasks' sequences. These structural parameters may be involved more actively in the project management process to provide its sustainability and robustness.

4 CASE STUDY

Underground coalmines operate in highly uncertain geologic environment. Moreover, their activities exhibit evident characteristics of the project. For example, any panel extraction occurs in unique geologic conditions. Hence, an underground coal mine consistently takes shape within a structured program framework. There are strategic programs of coal extraction for a big block of coal resources. These programs have a substantial duration, often spanning 10 to 20 years. In contrast, tactical or current programs are generally organized for shorter intervals, typically covering a one-year period.

We analyzed a year program of a big Ukrainian coalmine "Pokrovs'ke". The program consisted of 118 tasks, namely extraction of 12 longwall panels and driving of 106 underground entries. Computer simulation registered 218 critical paths. The most critical tasks were development of Northern conveyor entry (horizon 780 m) and assembling of the equipment in Second northern panel (bock 10). These tasks were integrated in several critical paths of the year program that increased their frequency.

The reason was subjective. Technologists used a constant rate of the entry development. Furthermore, the coalmine management has stipulated a stringent requirement for the assembly of longwall equipment to be completed within a two-month timeframe. However, our examination of experimental data indicated that the actual average duration for this task is three months, with the possibility of its extending even further. Additionally, our findings revealed an exponential attenuation pattern in the driving of entries, correlating with an increase in their length. These distinctive characteristics were taken into consideration during program planning, enhancing its overall reliability.

To offset the delay in commencing the Second northern panel, a coal pillar was extracted from another operational panel. Additionally, a road-header underwent refurbishment to increase the Northern conveyor entry rate and align with the scheduled program.

We employ this Monte Carlo Simulation code to assess project schedules and programs within the construction and mining industries. The insights derived from the analyses have led to recommendations that significantly improve both planning processes and the ongoing control of risks.

5 CONCLUSIONS

Multiplicity of the critical paths generates structural uncertainty of the project schedule having casual variation of input parameters, first the tasks' uncertain durations. Furthermore, the critical paths can branch out which provides more effective dissipation of energy and information.

Having multiple critical paths is advantageous for effective project control, as it enhances the ability to manage and maintain the project's timeline within the scheduled limits.

Reducing the probability of a task from 1.0 to 0.5 can lead to a notable increase in the frequency of the critical path by a factor of 2.8. Furthermore, the correlation between the durations of tasks in distinct critical paths has the potential to redistribute their frequencies. This redistribution could result in an 8% shift, with one path experiencing an increase in frequency while the other undergoes a decrease.

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Results of this research may be used for the case of project management in uncertain environment and unlimited resources.

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