Application of the Time Method for Studying Linear Reproducing Systems for Assessing the Dynamic Accuracy of Devices Based on Magnetoelectric Systems

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Abstract: For the successful use of magnetoelectric measuring systems, an important point is the possibility of assessing their accuracy in a dynamic mode of operation. To obtain such estimates, methods based on frequency transformations and, accordingly, analysis of the composition of the spectral components of the currents of the interacting circuit and the magnetic field are traditionally used. At the same time, frequency methods have a number of limitations, in particular, due to the finiteness of the number of terms of the Fourier series in the analysis of periodic functions of time, as well as some other limitations when using the Fourier integral for non-periodic functions. In addition, there are certain limitations when using the well-known complex-spectral method and the method of typical effects. In the presented article, in addition to the indicated methods, it is proposed to consider the possibility of using a temporary research method to assess the dynamic properties of magnetoelectric systems. Also, as an example, the article presents an analysis of the dynamic properties of a magnetoelectric measuring system with electromagnetic damping, which can be extended to more complex measuring systems of this type.

1 INTRODUCTION

When assessing the dynamic accuracy of magnetoelectric systems (i.e., those whose operation is based on the interaction of a circuit with a current and a magnetic field), the frequency method for studying linear reproducing systems is usually used [1, 2]. This research method is based on the assumption that electric currents (initializing currents) flowing through the circuit can be represented as a set of constantly acting sinusoidal components.

It is easy to conclude that the validity of using the frequency method is limited to cases where the currents in the circuit are accurately described by periodic functions of time, as well as by functions of time expressed by the Fourier integral [3, 4].

At the same time, experience shows that in specific physical examples, the above time functions are not enough to accurately describe the currents in the circuit, and, consequently, to assess the accuracy of reproducing the effects of a specific device. The time functions used in such methods of studying magnetoelectric systems as the complex spectral method and the method of typical effects [5, 6] also turn out to be insufficient.

In this regard, of particular interest is the use of the time method for studying linear reproducing systems to evaluate the dynamic properties of magnetoelectric systems [7].

With this research method, to describe the impacts, time functions $\alpha_{in}(t)$ are used that satisfy the condition:

$$|\alpha_{in}^{\kappa}(t)| \begin{cases} =0; & at\ -\infty\leq k\leq+\infty, -\infty\leq k\leq+\infty \\ \leq M\Omega^{\kappa}; & at\ +0\leq t\leq\infty, -\infty\leq k\leq+\infty \end{cases} \quad (1)$$

where M and Ω are some given real positive numbers.

The set of time functions satisfying condition (1) will be referred to below as the class of functions $S(M, \Omega)$.

From a practical point of view, the determination of the parameters M and required for the time method for the class of impacts to be reproduced is associated, as a rule, with less difficulties than the determination of the spectral composition of these impacts, required in the frequency and complex-spectral methods, and the determination of the typical impact, required in the method of typical influences. It is characteristic that in the study of the reproducing properties of magnetoelectric systems, in most practical cases, the relation can be used to determine the parameter Ω:

$$\Omega \approx \frac{|i^{(1)}(t)|_{\max max}}{|i(t)|_{\max max}}.$$

where $|i(t)|_{max\,max}$ – the limiting value of the module of the currents to be investigated in the circuit at $-\infty \le t \le +\infty$;
$|i^{(1)}(t)|_{\max max}$ – is the limiting value of the modulus of the rate of change in time of the currents to be investigated in the circuit at $-\infty \le t \le +\infty$.

2 METHODS

The main purpose of magnetoelectric systems is, as is known, the conversion of an electric current flowing through a certain circuit, $i_\kappa(t)$, into a deviation of a pointer mechanically connected to this circuit with a current by the value $L(t)$**.** The required nature of this transformation is described either by the:

$$L_0(t) = K(0)i_k(t) \tag{2}$$

expressing the ideal tracking process, or by the

$$L_\tau(t) = K(0)i_k(t+\tau) \tag{3}$$

expressing the ideal registration process.

In (2) and (3): $L_0(t)$ and $L_\tau(t)$ are the functions of time describing the required reproduction of the impact $i_k(t)$: $K(0)$ is the sensitivity of the device with the magnetoelectric unit for direct current; τ – admissible time of displacement of registration of the investigated currents in the circuit.

Obviously, the actual reproduction of the impact $i_k(t)$, described by the time $L(t)$, function differs both from $L_0(t)$, and from $L_\tau(t)$. The mutual deviation of the time functions $L_0(t)$ and $L(t)$ is the instantaneous error of the tracking device (4):

$$\Delta_0(t) = L(t) - L_0(t) \tag{4}$$

and the mutual deviation of the time functions $L_\tau(t)$ and $L(t)$ is the instantaneous registration error (5)

$$\Delta_\tau(t) = L(t) - L_\tau(t)\,. \tag{5}$$

The functions $\Delta_0(t)$ and $\Delta_\tau(t)$ quite fully characterize the dynamic properties of the magnetoelectric system with respect to the current $i_k(t)$. When determining these functions, we will proceed from the assumption that the relationship between $L(t)$ and $i_\kappa(t)$ is expressed by a linear differential equation with constant coefficients.

In the classical theory of magnetoelectric systems [8], the relationship between $L(t)$ and $i_\kappa(t)$ is expressed by a linear inhomogeneous second-order differential (6):

$$L(t) + \frac{2s}{\omega_0} L^{(1)}(t) + \frac{1}{\omega_0^2} - L^{(2)}(t) = K(0)\; i_\kappa(t), \tag{6}$$

where s - the degree of calming of the moving system of the circuit with the investigated currents; ω_0 – natural frequency of the mobile system.

At the same time, a thorough study of the dynamic properties of magnetoelectric systems, carried out by R. R. Kharchenko and N. N. Evtikhiev, showed that to describe the relationship between $L(t)$ and $i_\kappa(t)$ it is necessary to use linear differential equations with orders higher than the second. Therefore, in this paper, to describe the relationship between $L(t)$ and $i_\kappa(t)$, we use the linear inhomogeneous differential:

$$L(t) + \sum_{k=1}^{n} b_k L^{(\kappa)}(t) = K(0) i_\kappa(t), \tag{7}$$

whose order (the number n) is not bounded from above.

The coefficients b_k in this case are real positive numbers determined by the design features of the used magnetoelectric device.

Consider the structure of the solution of the differential (7) for the case when the initializing currents $i_k(t) \in S(M, \Omega)$ and the transfer function of the device (8)

$$K(p) = K(0)\frac{1}{1 + \sum_{k=1}^{n} b_\kappa p^\kappa}, \tag{8}$$

is an analytic function of a complex variable inside $|p| < R_{min}$ and outside the circle $|p| \le R_{max}$. The relationship between coefficients b_k and numbers R_{min} and R_{max} is established by the well-known theorem from higher algebra on the limits of zeros of polynomials with real coefficients [9].

A) Initializing currents $i_k(t) \in S(M, \Omega)$, $\Omega < R_{min}$.

It can be proved that when solving the $\Omega < R_{min}$ differential (6), the function $L(t)$ can be represented as the sum of two components (9)

$$L(t) = L(t)_{forc} + L(t)_{fr}. \tag{9}$$

The first of these components is forced - $L(t)_{forc}$, due to the reaction of the moving system of the circuit with current to smooth changes in time of the current $i_k(t)$ and its derivatives $\{i_k{}^{(k)}(t)\}$ is determined by the (10):

$$L(t)_{forc} = \sum_{k=0}^{\infty} \frac{1}{\kappa!} K_{(0)}^{(k)}\, i_{\kappa}^{(k)}(t), \tag{10}$$

where

$$K^{(\kappa)}(0) = \lim_{p \to 0} \frac{d^{\kappa}}{dp^{\kappa}} K(p).$$

The second component is free - $L(t)_{fr}$, due to the reaction of the circuit system with current to abrupt changes in time of the current $i_k(t)$ and its derivatives ${i_k}^{(k)}(t)$, at $t = 0$, can be determined by the (11):

$$L(t)_{fr} = \sum_{k=1}^{N} \frac{1}{(s_{\kappa}-1)!} \lim_{p \to -\beta_{\kappa}} \frac{d^{\kappa}}{dp^{\kappa}} [K(p) I_{\kappa}(p)(p+\beta_{\kappa})^{sk}\, e^{pt}] \tag{11}$$

where N – the number of poles of the transfer function of the magnetoelectric system $K(\mathrm{p})$; β_k – k^{th} pole ${s_k}^{th}$ order $K(p)$; $I_k(p)$ – image (according to Laplace) of the initializing current $i_k(t)$.

From the possibility of division into components $L(t)$ follows the possibility of division into components $\Delta_\tau(t)$ and $\Delta_0(t)$. Obviously (12) – (14),

$$\Delta_0(t)_{forc} = \sum_{k=1}^{\infty} \frac{1}{\kappa!} K^{(\kappa)}(0)\; i_{\kappa}^{(\kappa)}(t), \tag{12}$$

$$\Delta_\tau(t)_{forc} = \sum_{k=2}^{\infty} \frac{1}{\kappa!} [K^{(\kappa)} -{}_{(0)} K(1)_{(0)}{}^{\kappa} K^{-k+1}{}_{(0)}]\, i_{\kappa}^{(\kappa)}(t), \tag{13}$$

$$\Delta_0(t)_{fr} = \Delta_\tau(t)_{fr} = L(t)_{fr}. \tag{14}$$

The above formulas allow us to estimate the limiting values $\left|\Delta_0(t)_{forc}\right|_{\max max}$, $\left|\Delta_\tau(t)_{forc}\right|_{\max max}$ and $\left|L(t)_{fr}\right|_{max\, max}$ for currents $i_c(t) \in S(M, \Omega), \Omega < R_{min}$ at $-\infty < t \le +\infty$.

It should be noted that for magnetoelectric systems, which are known to be stable systems, of greatest interest from a practical point of view are estimates of the limiting values $\left|\Delta_0(t)_{forc}\right|$ and $\left|\Delta_\tau(t)_{forc}\right|$.

To estimate the limiting values of $\left|\Delta_0(t)_{forc}\right|$ and $\left|\Delta_\tau(t)_{forc}\right|$, we use the inequalities

$$\left|i_c^{(k)}(t)\right| \le M, \Omega^k, 0 \le k \le \infty,$$

following from (1). The above inequalities allow us to write that for $-\infty < t \le +\infty$

$$|\Delta_0(t)_{forc}| \le K(0) M \sum_{k=1}^{\infty} \frac{1}{\kappa!} \left| \frac{K^{(\kappa)}(0)}{K(0)} \right| \Omega^{\kappa}, \tag{15}$$

$$|\Delta_\tau(t)_{forc}| \le K(0) M \sum_{k=1}^{\infty} \frac{1}{\kappa!} \left| \frac{K^{(\kappa)}(0)}{K(0)} - \left(\frac{K^{(1)}(0)}{K(0)} \right)^{\kappa} \right| \Omega^{\kappa}. \tag{16}$$

Analysis of estimate (15) shows that the minimum values $\left|\Delta_0(t)_{forc}\right|_{\max max}$ for currents $i_c(t) \in S(M, \Omega), \Omega < R_{min}$ occur at minimum values of the coefficients $\left\{\left|\frac{K^{(k)}(0)}{K(0)}\right|\right\}$ or, the same thing, at the minimum values of the coefficients b_k and ratio $\frac{\Omega}{R_{min}}$.

In turn, it follows from estimate (16) that for currents $i_c(t) \in S(M, \Omega), \Omega < R_{min}$, the minimum values $\left|\Delta_\tau(t)_{forc}\right|_{\max max}$ occur at the minimum values of the coefficients,

$$\left\{ \left| \frac{K^{(\kappa)}(0)}{K(0)} - \left(\frac{K^{(1)}(0)}{K(0)} \right)^{\kappa} \right| \right\},$$

which, in particular, is ensured when the following relations are performed

$$\begin{cases} b_2 = \frac{1}{2!} b_1^2; \\ \cdots\cdots\cdots\cdots; \\ b_n = \frac{1}{n!} b_1^n, \end{cases} \tag{17}$$

when for $2 \le k \le n$:

$$\left| \frac{K^{(\kappa)}(0)}{K(0)} - \left(\frac{K^{(1)}(0)}{K(0)} \right)^{\kappa} \right| = 0$$

For the differential (6) used in the classical theory of devices using the magnetoelectric principle, the minimization $\left|\Delta_\tau(t)_{forc}\right|_{\max max}$ conditions established by relations (17) are reduced to the equality

$$\frac{1}{\omega_0^2} = \frac{1}{2!} \left(\frac{2s}{\omega_0} \right)^2,$$

which matches $s = (\sqrt{2})^{-1}$ or $\frac{1}{\sqrt{2}}$.

It should be noted that at $n \ge 5$, magnetoelectric systems that satisfy conditions (17) are physically unfeasible, since for $n \ge 5$, the polynomial

$$b(p) = 1 + \sum_{k=1}^{n} \frac{1}{\kappa!} b^{\kappa}{}_1 p^{\kappa}$$

does not satisfy the Hurwitz stability conditions [10 - 12].

From a practical point of view, the use of inequalities (15) and (16) to assess the dynamic accuracy of a magnetoelectric system is inappropriate. This is explained, first of all, by the well-known difficulties of summing infinite series

$$\sum_{k=1}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)}\right|\Omega^{\kappa}$$

and

$$\sum_{k=2}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)}-\left(\frac{K^{(1)}(0)}{K(0)}\right)^{\kappa}\right|\Omega^{\kappa},$$

representing upper bounds for the modules of functions

$$\varepsilon_0(p)=\frac{K(p)}{K(0)}-1 \quad \text{and} \quad \varepsilon_\tau(p)=\frac{K(p)}{K(0)}-e^{\frac{K^{(1)}(0)}{K(0)}p}$$

respectively in a circle $|p| \le \Omega < R_{min}$.

In this regard, it is advisable to estimate the limiting values $\left|\Delta_0(t)_{forc}\right|$ and $\left|\Delta_\tau(t)_{forc}\right|$ using the inequalities

$$\left|\Delta_0(t)_{forc}\right| \le K(0)M\eta_i(\Omega), \qquad (18)$$

$$\left|\Delta_\tau(t)_{forc}\right| \le K(0)M\varphi_i(\Omega), \qquad (19)$$

where $\eta_i(\Omega)$ and $\varphi_i(\Omega)$ are rather simply calculated majorants of the series

$$\sum_{k=1}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)}\right|\Omega^{\kappa}$$

and

$$\sum_{k=2}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)}-\left(\frac{K^{(1)}(0)}{K(0)}\right)^{\kappa}\right|\Omega^{\kappa}.$$

It can be proved [7] that the function $\eta_i(\Omega)$ can be used as a majorant

$$\eta_1(\Omega)=\frac{\sum_{k=1}^{n} b_\kappa \Omega^{\kappa}}{1-\sum_{k=1}^{n} b_\kappa \Omega^{\kappa}},$$

and as a majorant $\varphi_i(\Omega)$ – the function

$$\varphi_1(\Omega)=\eta_1(\Omega)+e^{b_1\Omega}-1-2b_1\Omega.$$

It follows from inequalities (18) and (19) that for a magnetoelectric device characterized by differential (7), a sufficient condition for registering an initializing current $i_k(t) \in S(M,\Omega), \Omega < R_{min}$ with a reproduction scale $K(0)$, a time offset $\tau = \frac{K^{(1)}(0)}{K(0)} = -b_1$, and limit values $\left|\Delta_0(t)_{forc}\right|$ and $\left|\Delta_\tau(t)_{forc}\right|$, not exceeding for $-\infty < t \le +\infty$ some numbers $\bar{\Delta}_0$ and $\bar{\Delta}_\tau$, is the fulfillment of the relations

$$K(0)M\eta_i(\Omega) \le \bar{\Delta}_0 \qquad (20)$$

and

$$K(0)M\varphi_i(\Omega) \le \bar{\Delta}_\tau, \qquad (21)$$

at $0 \le \Omega < R_{min}$.

An analysis of the functions $\eta_1(\Omega)$ and $\varphi_1(\Omega)$ shows that, at sufficiently small values ΩR_{min}^{-1} the influence of the coefficients $b_k, 2 < k \le n$ on the nature of these functions is extremely insignificant.

This allows, when registering initializing currents $i_c(t) \in S(M,\Omega), \Omega < R_{min}$, to estimate the limit values $\left|\Delta_0(t)_{forc}\right|$ and $\left|\Delta_\tau(t)_{forc}\right|$, to use the relations (22), (23)

$$|\Delta_0(t)_{forc}| \le K(0)M\frac{2s\frac{\Omega}{\omega_0}+\frac{\Omega^2}{\omega_0^2}}{1-2s\frac{\Omega}{\omega_0}-\frac{\Omega^2}{\omega_0^2}}=K(0)M\varepsilon_0\left(\frac{\Omega}{\omega_0}\right) \qquad (22)$$

and

$$|\Delta_\tau(t)_{forc}| \le K(0)M\left[\varepsilon_0\left(\frac{\Omega}{\omega_0}\right)+e^{2s\frac{\Omega}{\omega_0}}-1-4s\frac{\Omega}{\omega_0}\right]= K(0)M\varepsilon_\tau\left(\frac{\Omega}{\omega_0}\right), \qquad (23)$$

derived from differential (6).

To estimate the attenuation intensity $L(t)_{fr}$ in the case under consideration, following relation can be used

$$|L(t)_{fr}| \le K(0)M\frac{e^{-s\omega_0 t}}{\left(1-\frac{\Omega}{\omega_0}\right)\sqrt{\left|1-s^2\right|}}=K(0)M\varepsilon\left(\frac{\Omega}{\omega_0},t\right). \qquad (24)$$

Graphs of the functions

$$\varepsilon_0\left(\frac{\Omega}{\omega_0}\right);\ \varepsilon_\tau\left(\frac{\Omega}{\omega_0}\right) \text{ and } \varepsilon\left(\frac{\Omega}{\omega_0},t\right)$$

for different values of s are shown in Figures 1-3.

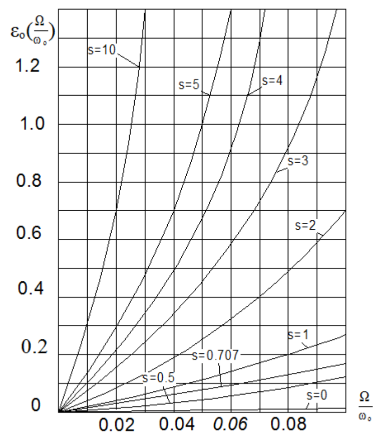

Figure 1: Graph of the function $\varepsilon_0\left(\frac{\Omega}{\omega_0}\right)$.

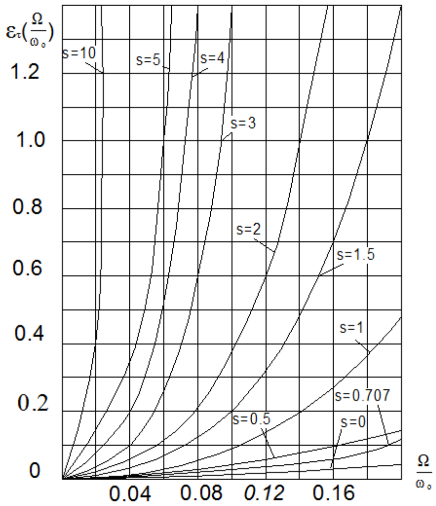

Figure 2: Graph of the function $\varepsilon_\tau\left(\frac{\Omega}{\omega_0}\right)$.

It follows from these graphs that the circuit with a current, characterized by the differential (6) at $K(0) = 2$ mm/mA, $\omega_0 = 500$ Hz and $s = 0{,}5$ will make it possible to display the initializing current $i_k(t) \in S(M_1, \Omega)$, $M = 100$mA, $\Omega = 25\ s^{-1}$ with a limit value $|\Delta_0(t)_{forc}|$, not exceeding $2 \times 100 \times 0{,}05 = 10\ mm$, and limit value $|\Delta_\tau(t)_{forc}|$ not exceeding $2 \times 100 \times 0{,}01 = 2\ mm$, (at $\tau = -0{,}002\ s$). The limiting majorant in this case is the function

$$K(0)M\varepsilon\left(\frac{\Omega}{\omega_0}, t\right) = 275e^{-25t}.$$

Note that currents $i_k(t) \in S(M, \Omega)$, $M = 100$ mA, $\Omega = 25\ s^{-1}$ include, in particular, currents $i_{1k}(t) = 100\sin 25t$; $i_{2k}(t) = 100e^{-25t}$, etc.

In cases where the amplitude-frequency characteristic of the magnetoelectric system $M^{(\omega)}$ and its phase-frequency characteristic $\varphi(\omega)$ are known, the following relations can be used to estimate the limit values $|\Delta_0(t)_{forc}|$ and $|\Delta_\tau(t)_{forc}|$, when displaying the initializing currents $i_c(t) \in S(M, \Omega), \Omega < R_{min} \le \omega_{kp}$

$$\Delta_0|(t)_{forc}| \cong M(0)M\left|\varphi^{(1)}(0)\right|\Omega, \qquad (25)$$

$$|\Delta_\tau(t)_{forc}| \cong M(0)M\left|\frac{M(\Omega)}{M(0)} - 1\right|. \qquad (26)$$

obtained by substituting into inequalities (20) and (17) the relations

$$\sum_{k=1}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)}\right|\Omega^{\kappa} > \frac{1}{1!}\left|\frac{K^{(1)}(0)}{K(0)}\right|\Omega,$$

$$\sum_{k=2}^{\infty}\frac{1}{\kappa!}\left|\frac{K^{(\kappa)}(0)}{K(0)} - \left(\frac{K^{(1)}(0)}{K(0)}\right)^{\kappa}\right|\Omega^{\kappa} > \frac{1}{2!}\left|\frac{K^{(2)}(0)}{K(0)} - \left(\frac{K^{(1)}(0)}{K(0)}\right)^{2}\right|\Omega^2,$$

as well as the relations proved in [8]

$$\frac{K^{(1)}(0)}{K(0)} = \varphi^{(1)}(0),$$

$$M(\Omega) \cong M(0) + \frac{1}{2!}M^{(2)}(0)\Omega^2,$$

$$\frac{1}{2!}M^{(2)}(0) = \frac{1}{2!}\left|K^{(2)}(0) - \frac{K^{(1)2}(0)}{K(0)}\right|.$$

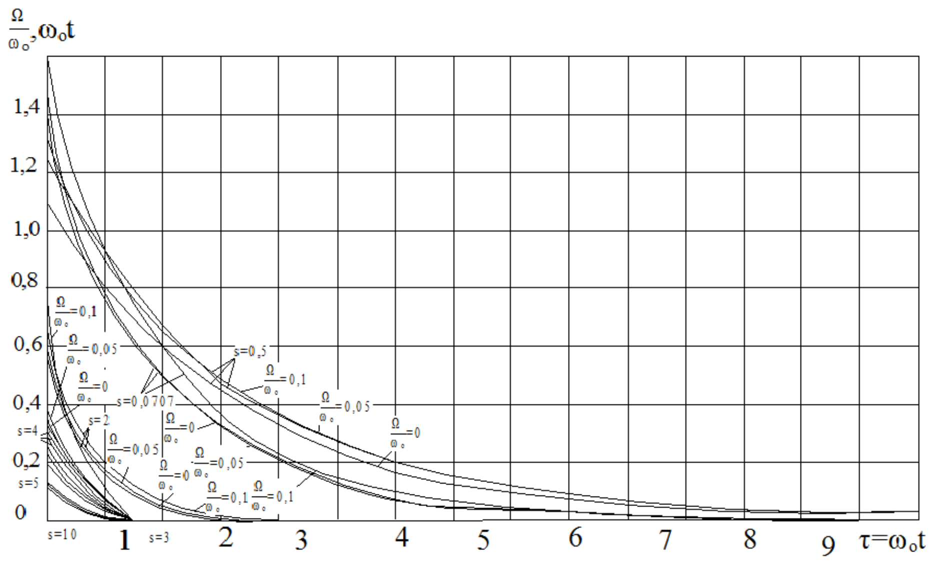

Figure 3: Graph of the function $\varepsilon\left(\frac{\Omega}{\omega_0}, t\right)$.

B) Initializing currents $i_k(t) \in S(M, \Omega)$, $\Omega > R_{max}$.

It can be proved that for $\Omega > R_{max}$, the representation of $L(t)$ as a sum of two components

$$L(t) = L(t)_{forc} + L(t)_{fr}.$$

is preserved. The possibility of determining $L(t)_{fr}$ by formula (11) also remains. At the same time, formula (10) is not suitable for determination $L(t)_{forc}$.

For $\Omega < R_{max}$, $L(t)_{forc}$ is defined this way:

$$L(t)_{forc} = \sum_{k=0}^{\infty} \frac{1}{K!} Q^{(K)}(0) i_c^{(-K)}(t),$$

where

$$Q^{(\kappa)}(0) = \lim_{z \to 0} \frac{d^{\kappa}}{dz^{\kappa}} K(z^{-1}),$$

and

$$i_{\kappa}^{(-\kappa)}(t) = \frac{1}{(k-1)!} \int_0^t i_{\kappa}(\tau)(t-\tau)^{\kappa-1} d_{\tau}$$

in case if

$$K(p) = K(0) \frac{1}{1 + \sum_{k=1}^{n} b_{\kappa} p^{\kappa}},$$

for $k \leq n, Q^{(k)}(0) = 0$ accordingly,

$$L(t)_{forc} = \sum_{\kappa=n+1}^{\infty} \frac{1}{\kappa!} Q^{(\kappa)}(0) i_{\kappa}^{(-\kappa)}(t). \qquad (27)$$

Analysis of formula (27) shows that in relation to the currents in the circuit $i_k(t) \in S(M, \Omega), \Omega > R_{min}$, the magnetoelectric system, characterized by the differential (7), manifests itself as an "opaque" system with a forced component of the "suppression" error of influences $\delta(t)_{forc}$, determined by the

$$\delta(t)_{forc} = L(t)_{forc}.$$

Formula (27) also implies the possibility of using the studied magnetoelectric system for (n + 1 + q) - a short integration of currents $i_k(t) \in S(M, \Omega), \Omega > R_{max}$ with a scale

$$\frac{1}{(n+1+q)!} Q^{(n+1+q)}(0)$$

and a forced component of the integration error $j(t)_{forc}$, determined by the

$$j(t)_{forc} = L(t)_{forc} - \frac{1}{(n+1+q)!} Q^{(n+1+q)}(0) i_c^{(n+1+q)}(t).$$

The evaluation of the limit values $|\delta(t)_{forc}|$ and $|j(t)_{forc}|$ for the currents in the circuit $i_k(t) \in S(M, \Omega), \Omega > R_{max}$ can be made by similar methods for evaluating the limit values $|\Delta_0(t)_{forc}|$ and $|\Delta_\tau(t)_{forc}|$ for the currents $i_k(t) \in S(M, \Omega)$, $\Omega < R_{min}$.

C) Initializing currents $i_k(t) \in S(M, \Omega)$, $R_{min} \leq \Omega \leq R_{max}$.

At $R_{min} \leq \Omega \leq R_{max}$, the division of $L(t)$ into forced and free components is generally impossible. This is explained by the fact that among the currents in the circuit $i_k(t) \in S(M, \Omega), R_{min} \leq \Omega \leq R_{max}$ there are those that are able to bring the moving system of the circuit of the magnetoelectric system into a state of generalized resonance, a phenomenon first described by S.P. Strelkov. A special case of this state is the well-known resonance that occurs when sinusoidal currents flow through the circuit of the magnetoelectric system, the frequency of which coincides with the natural frequencies of the moving system of the circuit with current.

3 RESULTS AND DISCUSSION

In the above analysis of the properties of the magnetoelectric system, it was assumed that the nature of the transfer function of the device is determined only by its design parameters and is completely independent from the parameters of the electrical circuit of the device.

However, this assumption is valid only in cases where the influence of EMF generated in a current-carrying circuit on the properties of the magnetoelectric system is practically imperceptible. It is justified, in particular, when analyzing the properties of magnetoelectric systems with single-turn circuits with oil and magneto-inductive damping. At the same time, for multi-turn circuits with electromagnetic damping, the above assumption is generally non-uniform.

As an example, let us analyze the properties of a magnetoelectric system with electromagnetic damping when using it to study the EMF of thermocouples.

Let $e_T(t)$ be the EMF of thermocouple to be studied; r_T is the ohmic resistance of the thermocouple; $e_k(t)$ – EMF generated by the circuit; r_k – ohmic resistance of the loop; r_∂ – damping loop resistance (shunt).

For the case under consideration, the relationship between $i_k(t)$, $e_T(t)$ and $e_k(t)$is expressed by the (28)

$$i_\kappa(t) = \frac{r_\partial e_T(t) + (r_T + r_\partial) e_\kappa(t)}{r_\kappa r_T + r_\kappa r_\partial + r_T r_\partial}. \tag{28}$$

Let's pretend that

$$e_c(t) = -k_{cur} L^{(1)}(t), \tag{29}$$

where k_{cur} – constructive constant of the loop with current, and

$$i_c(t) = \frac{1}{K(0)} \left\{ L(t) + \sum_{k=1}^{n} b_\kappa L^{(\kappa)}(t) \right\} \tag{30}$$

in accordance with the differential (7) without taking into account the effect of electromagnetic damping of the current loop due to the EMF generated by it. Substituting the expanded expressions for $e_k(t)$ (29) and $i_k(t)$ (30) into (28), we can write that

$$\frac{K(0) r_\partial e_T(t)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} = \tag{31}$$

$$= L(t) + \left(b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right) L^{(1)}(t) + \sum_{k=1}^{n} b_\kappa L^{(\kappa)}(t).$$

This means that the transfer function of the magnetoelectric system with respect to EMF of thermocouple is a fractional rational function

$$N(p) = \frac{K(0) r_\partial}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \times$$

$$\times \frac{1}{1 + \left(b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right) p + \sum_{k=2}^{n} b_\kappa p^\kappa}.$$

Let the studying EMF of thermocouple $i_k(t) \in S(M, \Omega), \Omega > R'_{min}$, where R'_{min} is the convergence radius of the series

$$\sum_{k=0}^{\infty} \frac{1}{\kappa!} N^{(\kappa)}(0) p^\kappa = N(p).$$

By analogy with the previous one, it can be argued that in the case under consideration $i_k(t) \in S(M, \Omega), \Omega < R'_{min}$, the effects will be reproduced by the magnetoelectric system with a reproduction scale

$$N(0) = \frac{K(0) r_\partial}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial},$$

displacement time

$$\tau = -\left(b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right)$$

and limit values $|\Delta_0(t)_{forc}|$ and $|\Delta_\tau(t)_{forc}|$, estimated with inequalities

$$|\Delta_0(t)_{forc}| \leq \frac{K(0) r_\partial M}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \eta'_1(\Omega) \tag{32}$$

and

$$|\Delta_\tau(t)_{forc}| \leq \frac{K(0) r_\partial M}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \varphi'_1(\Omega), \tag{33}$$

where

$$\eta'_1(\Omega) = \frac{\left| b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right| \Omega + \sum_{k=1}^{n} b_\kappa \Omega^\kappa}{1 - \left| b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right| \Omega - \sum_{k=1}^{n} b_\kappa \Omega^\kappa},$$

$$\varphi'_1(\Omega) = \eta'_1(\Omega) + e^{\eta_1^{\prime(1)}(0)\Omega} - 1 - 2\eta_1^{\prime(1)}(0).$$

When registering EMF of thermocouple $e_T(t) \in S(M, \Omega), \Omega \ll R'_{min}$, the condition for minimizing the limit value $|\Delta_0(t)_{forc}|$ is the fulfillment of the relation

$$b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} = 0 \tag{34}$$

and relations

$$b_k = 0 \text{ at } 2 \leq k \leq n. \tag{35}$$

The condition for minimizing the limiting value $|\Delta_\tau(t)_{forc}|$ in the case under consideration is the fulfillment of the relations

$$b_\kappa = \frac{1}{\kappa!} \left[b_1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial} \right]^\kappa, \tag{36}$$

at $2 \leq k \leq n$.

An analysis of the differential (31), inequalities (32) and (33), as well as the conditions for minimizing the limit values $|\Delta_0(t)_{forc}|$ and $|\Delta_\tau(t)_{forc}|$, shows that the use of the effect of electromagnetic damping of the current circuit, increasing in

$$\left[1 + \frac{k_{cur} K(0)(r_T + r_\partial)}{(r_\kappa r_\partial + r_\kappa r_T + r_T r_\partial) b_1} \right]$$

times the damping intensity $L(t)_{fr}$ at the same time leads to an increase in the displacement time τ and the limit value $|\Delta_0(t)_{forc}|$. From conditions (36) it follows, in particular, that for circuits with current, characterized by differential (6), the dynamic correction of the magnetoelectric system in order to reduce the limit value $|\Delta_\tau(t)_{forc}|$ takes place at

$$\frac{r_T r_\partial}{r_T + r_\partial} = \frac{k_{cur} K(0) \omega_0}{\sqrt{2}(1 - \sqrt{2}s)} - r_\kappa, \tag{37}$$

in cases where $r_\tau \gg r_\partial$ condition (37) reduces to the relation

$$r_\partial \approx \frac{k_{cur} K(0)\omega_0}{\sqrt{2}(1-\sqrt{2}s)} - r_\kappa. \quad (38)$$

With the resistance of the thermocouple $r_T = \frac{k_{cur} K(0)\omega_0}{\sqrt{2}(1-\sqrt{2}s)} - r_c$, the dynamic correction of the magnetoelectric system in order to reduce the limit value $|a_\tau(t)_{forc}|$is provided in the absence of r_∂ (i.e., at $r_\partial = \infty$ 1).

Similarly, the properties of more complex measuring devices, in which magnetoelectric systems are used as recording organs, can be investigated.

CONCLUSION

The possibility of using the time method for solving problems of dynamic accuracy associated with the design and operation of magnetoelectric systems, as well as various types of measuring devices, in which current-carrying circuits as part of the magnetoelectric system are used as recording organs, is shown.

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