On Fractal-Based Estimates of Subsidence Volumes for Various Types of Soils

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Abstract: In this paper we study the protracted degradation of the soil structure during technogenesis in loess and various types of silty and clay soils. We develop a new mathematical model of the degradation which is based on applying fractal theory to soils, and our modeling is mainly dealing with particle size distribution by volume. The values of the fractal dimension of the particle size distribution by volume were also calculated for various types of soils. The predicted values of the porosity coefficient in the state of complete decomposition of micro-aggregates were calculated according to the new model. The difference of the porosity coefficients in the initial (natural) and predicted states characterizes the value of the volume deformation of the soil. The particle size distribution of the samples was determined before and after several different sample preparation methods. The obtained results on volumetric deformation of loess and silty-clay soils do not contradict the known data on the deformation features of the deposits of the region formed in the epochs of interglacial and terrestrial glaciation.

1 INTRODUCTION

The forecast of soil deformation under external anthropogenic impact is a topical scientific problem. The amount of deformation depends on the properties and type of soils [1, 2]. During operation of the structure, the state, degree of aggregation and soil structure change [3]. The forecast of final deformations can be obtained using the methods of research of complex systems: neural networks, the method of group accounting of arguments and others [4, 5]. The theory of fractals has been successfully applied to describe structure and properties of soils [6, 7, 8] and to predict the mechanical behavior of the soil environment [9, 10]. In the presented paper, the forecast of final deformations is based on the application of fractal theory.

In modern publications devoted to the problems of deformation and destruction of various geomaterials, considerable attention is paid to the description of the geometric properties of the structures formed by particles and pores. Russell [11] described soil compaction using an example of a medium with particle and pore sizes that change during crushing. Using elements of fractal analysis Zhang and Zhang [12] showed that the ratio between volumetric deformation and the index of crushing is constant during crushing. Seblany et al. [13] performed an analysis of the distribution of pores in the granular material. The influence of the geometric properties of pores of shale on adsorption phenomena was performed in [14] using the Frenkel-Halsey-Hill model. Also the investigation of the influence of the phase composition of geomaterials on their strength and deformability are continued. Xue and others [15] investigated the effect of temperature and stress on electroosmosis in clays. Espitia et al. [16] showed that the Poisson's ratio in clays changes nonlinearly with increasing pressure and this has an effect on water-retaining properties. The effect of dispersion and cohesion on the behavior of soils is also very essential. In [17], internal and external factors of soil aggregates stability were studied, the author identified the types and explained the mechanism of soil aggregates stability.

The above brief review of publications shows that the search for new models of geomaterials and soils is carried out in various aspects, a common tendency is to study materials as complex dynamic systems.

The study of microaggregate composition allows to solve several problems of different types. It is well known that the number of aggregates, particles composing aggregates and free particles does not remain constant. In [18, 19] we obtained some estimations of subsidence volume for loess soils which are dealing with the fractal dimension of the particle size distribution. In the presented paper we develop a new fractal-based model which describes the process of the emergence of a new structure in a dispersed soil after the disintegration of microaggregates. This model is based on the fractal dimension of the particle size distribution by volume of particles, and it is more appropriate for estimations of subsidence volume. The theoretical conclusions are supported by results of experiments. The techniques of microaggregate analysis are described in [20].

The final deformations were studied as the result of the disintegration of microaggregates in dusty-clay soils of various types (loess-like loams and clays). Samples of natural density and moisture of loess-like loams and clays were examined. We studied Quaternary loess-like loams of eolian-deluvial and deluvial origin and Neogene-Quaternary clays of continental origin. The total number of experiments to determine the microaggregate composition was 200 for loess and 80 for clay. Samples selected from natural outcrops and from bores on the Dnipro raised platform and Donetsk folded structure of the Ukrainian array. The conditions of occurrence of the loess horizons of the Dnipro Upland in the area of the city of Dnipro are characterized with a map (Figure 1).

2 FRACTAL-BASED MODELING OF SOIL SUBSIDENCE

In [21] the particle size distribution $N_s(L > d_s)$ was defined as the number of particles being of any size L larger than d_s, where d_s runs over the real numbers. In the same way we can define the particle size distribution by volume $V_s(L > d_s)$ (and by mass $M_s(L > d_s)$) as the volume (mass) of particles being of any size L larger than d_s, where d_s runs over the real numbers. Certainly, $N_s(L > d_s)$, $V_s(L > d_s)$ and $M_s(L > d_s)$ are real functions.

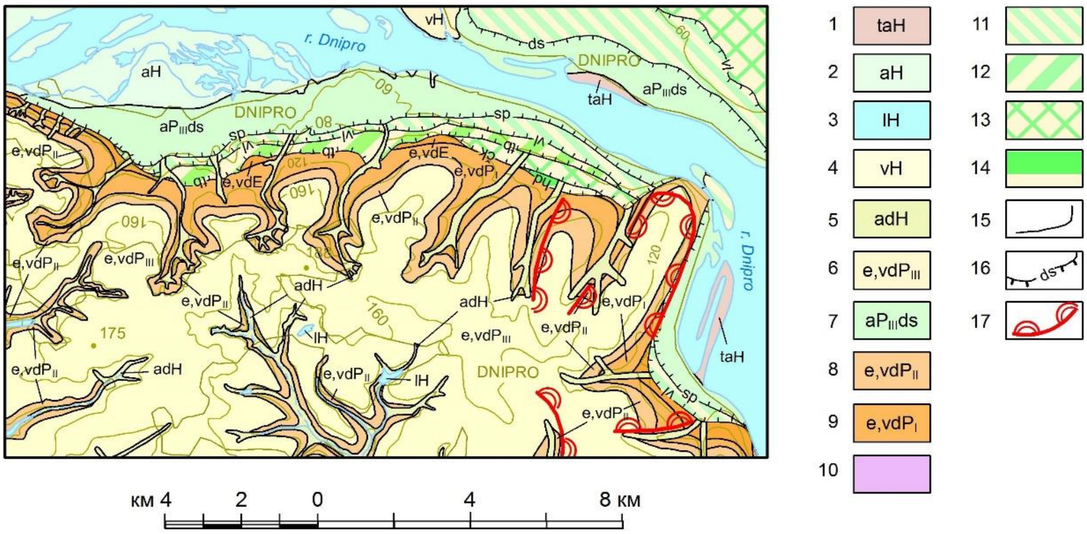


Figure 1: Geological map of Quaternary. Holocene: 1 technogenic sediments; washed; 2 alluvial; 3 limnetic; 4 aeolian; 5 alluvial-deluvial. Upper Quaternary: 6 eluvial, aeolian-deluvial sediments; 7 alluvium of the first terrace (Desnianska terrace); 8 eluvial; aeolian-deluvial sediments of Middle Pleistocene; 9 eluvial, aeolian-deluvial sediments of Lower Pleistocene; 10 pre-quaternary sediments. Terrace alluvium buried under subaerial sediments; 11 vilshanska; 12 trubizka; 13 cherkasska; 14 hadjibeiska; 15 geologic boundaries; 16 terrace inner margin and name; 17 landslide-prone slopes.

The particle size distribution by volume has fractal dimension DV_s if

$$V_s(L > d_s) \approx \gamma d_s^{-DV_s}, \tag{1}$$

where γ is a coefficient and the sign $\approx$ means "approximately". In our further equations we use the sign $=$ taking in mind (1) and realizing that the obtained formulas give us only some estimations of the real situation.

In fact (1) gives us an estimation of the particle size distribution by volume $V_s(L > d_s)$ which is based on a fractal topological invariant DV_s. As it was noted in [11], modeling the particle size distribution using fractals provides a good fit to experimental data (e.g. [20, 22, 23, 24, 25]).

Since in the presented paper we study volumetric characteristics of soil subsidence, the particle size distribution by volume is more convenient for us. Using the fractal dimension of the particle size distribution by volume essentially simplifies obtained formulas and facilitates calculations. The particle size distribution by volume can be easily obtained from the particle size distribution by mass, which appears as the result of the standard procedure of sieve analysis.

The particles forming the ground may have only a finite set of sizes. We denote these sizes

$$d_1, d_2, ..., d_{n-1}, d_n, \tag{2}$$

which are running from the biggest d_1 to the smallest d_n. Everywhere below we assume that the relation $\alpha = \alpha_j = d_{j+1} / d_j$ does not depend on j, where $1 \le j \le n$. This assumption corresponds to the idea of the self-similarity of fractal structures. In addition, all known mathematical fractals are constructed on this principle. Now, we define an "imaginary" particle size $d_{n+1} = \alpha d_n$ so that the relation $\alpha = d_{j+1} / d_j$ holds for all $1 \le j \le n$.

We will use the following denotations:

- A_j is the structure formed only by particles of one fixed size d_j;
- $V_s(A_j)$ is the volume of particles of size d_j which form the structure A_j;
- $V_p(A_j)$ is the volume of pores of the structure A_j;
- $V(A_j) = V_p(A_j) + V_s(A_j)$ is the volume of the whole structure A_j.

We will assume that the porous structure A_J, formed only by particles of size d_j, is similar to the porous structure A_i, formed only by particles of size d_i, for all $1 \le i, j \le n$. This property corresponds to the nature of self-similar fractal structures and is preserved in the process of subsidence and after it. Then, as the structure A_J is similar to A_i for all $1 \le i, j \le n$, we can assume that $V_p(A_j) / V_s(A_j) = V_p(A_i) / V_s(A_i) = \mathrm{k}_p$ for all $1 \le i, j \le n$, i.e. the coefficient of porosity of the structure k_p of A_J does not depend on j.

As it was mentioned in [12], one of the most common soil models is the granular medium model. We also assume that particles are spherical. It is well known that $k_p = (1 - s) / s$, where s denotes the shape factor, which for spheres becomes $s = \pi / 6$, as it was mentioned in Appendix A of [26]. Then below we assume that $k_p = (6 - \pi) / \pi$.

Equation (1) allows us to estimate the volume $V_s(A_s)$ of all particles of size d_s. We should note that an increment of the function $V_s(L > d_s)$ is only due to the particles of sizes from (2) (i.e. in points $d_1, d_2, ..., d_{n-1}, d_n$). Thus, $V_s(A_j) = V_s(L > d_{j+1}) - V_s(L > d_j)$ and using (1) we can calculate the volume of particles of size d_j for each d_j from (2) as the following

$$V_s(A_j) = \gamma d_j^{-DV_s} \beta, \tag{3}$$

where $\beta = (d_{j+1} / d_j)^{-DV_s} - 1$ and $1 \le j \le n$.

Then it follows from the definition of the coefficient of porosity k_p that

$$V_p(A_j) = \gamma \mathrm{k}_p d_j^{-DV_s} \beta, \tag{4}$$

where $1 \le j \le n$. Thus, (3) and (4) give the following

$$V(A_j) = \gamma (1 + \mathrm{k}_p) d_j^{-DV_s} \beta \tag{5}$$

where $1 \le j \le n$. It follows from (4), (5) that $V(A_{j+1}) > V_p(A_j)$.

We assume that particles of different sizes are distributed in the soil evenly and this property is preserved during the process of subsidence and after it. In the process of subsidence, the largest particles of the size d_1 will form a porous structure A_1 with the largest pores. The particles of the next size d_2 will be in these pores and they will form a porous structure A_2, whose pores will contain particles of the farther smaller size d_3, etc. to the particles of the smallest size d_n. We recall that

$$k' = V_p / V_s = (V - V_s) / V_s = (V / V_s) - 1$$

is the coefficient of porosity of the soil after subsidence. We have proved that

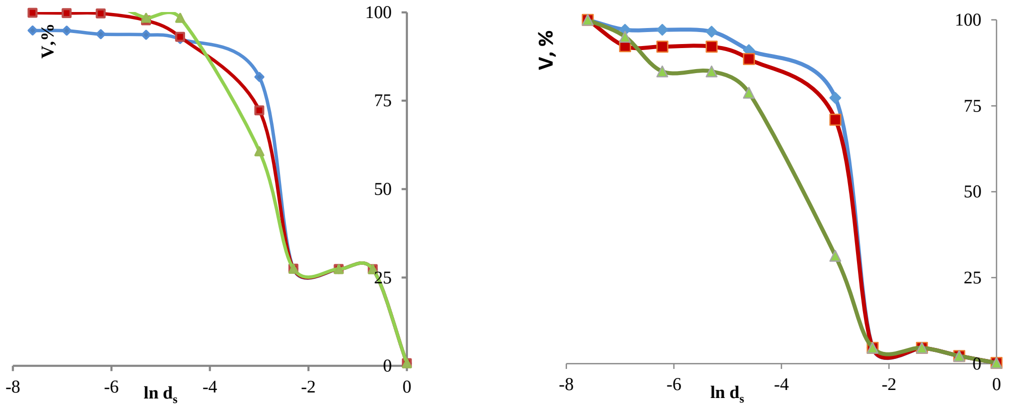
$$k' = \frac{k_p(1-\alpha^{DV_s})}{1-(\alpha^{DV_s})^n}. \qquad (6)$$

3 THE RESULTS OF EXPERIMENTAL STUDIES

The values of the particle size distribution by volume for soils of various origin and various preparation methods of samples were calculated. Typical curves of the granulometric composition of the loess of the Kryvyi Rih region and clays of the Western Donbass are presented in Figure 2.

The curves show the changes in the total volume of particles under different conditions of sample preparation for pipetting analysis. It follows from the graphs that the change in the microaggregate composition, depending on the preparation method of the sample for analysis, is manifested better in the clay of the Western Donbass than in the loess of the Kryvyi Rih region. Three different methods of sample preparation are used, therefore, in this case, different degrees of decomposition of microaggregates are achieved. In the first method, technogenic mechanical impact is simulated during mechanical shaking. The second and third methods characterize mainly the chemical impact that occurs in the conditions of large natural-technogenic systems, for example, in cities. Thus, one can get an idea of soil degradation under the conditions of mechanical and chemical technogenic impact.

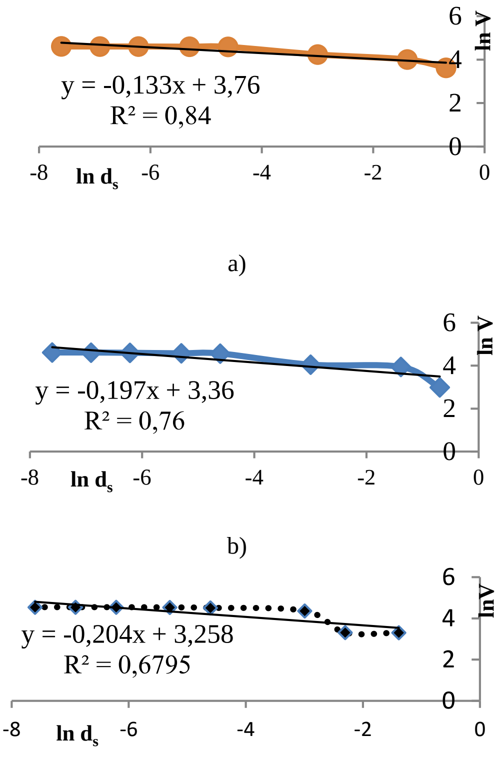
According to the results of particle size analysis, the values of the fractal dimension of the particle size distribution by volume were calculated for various preparation methods of samples for analysis. Examples are presented in Figure 3.



a)

b)

Figure 2: The graph of the particle size distribution by volume $V_s(L > d_s)$ (in %) with logarithmic X-axis for various preparation methods of samples: a) Kryvyi Rih, b) Western Donbass. The volume percentage $V, \%$ of particles whose size is bigger than the specified size d_s (in mm): ◆ with the aggregate method of preparation; ■ with the semi-dispersed method of preparation; ▲ with the dispersed method of sample preparation for pipetting analysis.



a)

b)

c)

Figure 3: The trend equation calculated by the results of particle size analysis of clay samples with the dispersed preparation: a) red-brown clay, b) green-gray clay, c) Kryvyi Rih loess. Denotations: $ln\, d_s$ - the logarithm of particle size d_s; lnV - logarithm of the volume percentage V of particles whose size is bigger than d_s; R^2 - the coefficient of determination.

The trend equation has the following form: $y = DV_s\, x + b$, where DV_s is the fractal dimension of the particle size distribution by volume. The value of DV_s is then define from the trend equation: a) Red-brown clay $DV_s = 0{,}133$, b) Green-gray clay $DV_s = 0{,}197$, c) Kryvyi Rih loess $DV_s = 0{,}204$. The porosity coefficient of red-brown clays will take values from 0.197 to 0.2. The porosity coefficient of green-gray clays will be in the range of 0.234 – 0.239. In loess, these values will vary from 0.372 to 0.380.

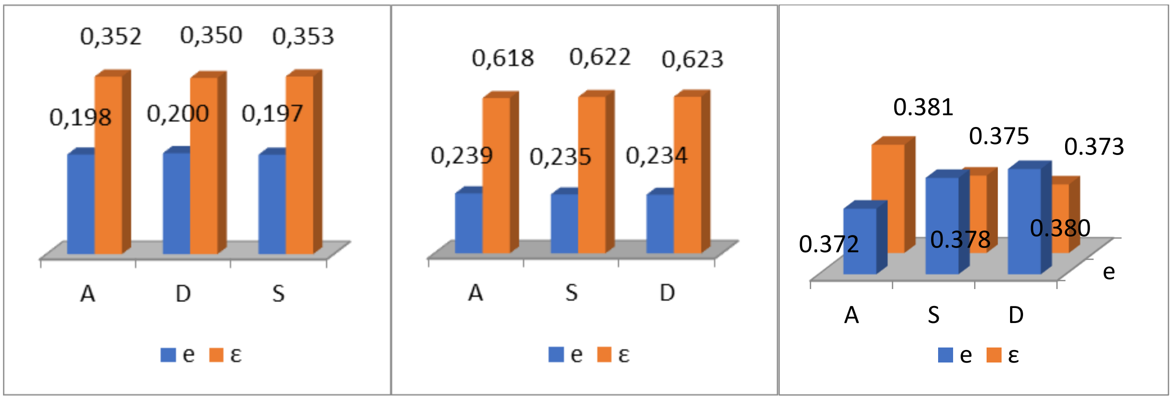
The tendencies of formation of the “new” structure in the studied soils are different. The disintegration of microaggregates will lead to some denser packing of particles of red-brown clay, the packing of particles of green-gray clay of Western Donbass will be relatively dense.

The loosest packaging is expected for particles of loess of the Kryvyi Rih structure. In previous studies, prognosed deformation values for the loess of the Middle Dnipro were calculated (Table 1). According to our calculations we have discovered that for all studied soil samples while disintegration of microaggregates the “new” porous structure is completely formed by particles of the smallest size, the particles of bigger sizes are presented in the "new" structure as separated inclusions. So, for each soil sample the porosity coefficient was calculated by (6) and then we calculated the average predicted porosity coefficient e for each type of soil and each method of sample preparation (Table 1).

The next Figure 4 illustrates gathered in Table 1 average predicted values of the porosity coefficients and the volumetric deformation of the final deformation with the complete disintegration of microaggregates. The deformations associated with the formation of a new structure during the destruction of microaggregates in loess are larger than in some clays, despite the large particle sizes.

Table 1: Average predicted values of the porosity coefficient and volumetric deformation of silty-clay soils of different types and genesis.

Soil Name Indicator	Values				
	The porosity coefficient of soil in natural state e_0.	Predicted values of the porosity coefficient e and the volumetric deformation $\varepsilon = e_0 - e$.	Aggregate method of sample preparation, A	Dispersed method of sample preparation, D	Semi-dispersed method of sample preparation for analysis, S
Red-brown clay	0,55	e	0,198	0,200	0,197
		ε	0,352	0,350	0,353
Green-gray clay	0,857	e	0,239	0,234	0,235
		ε	0,618	0,623	0,622
Kryvyi Rih loess	0,805	e	0,372	0,380	0,378
		ε	0,381	0,373	0,375



a) b) c)

Figure 4: Predicted values of the porosity coefficient and volumetric deformation during the decomposition of rock microaggregates of various ages and origins: a) red-brown clay, b) gray-green clay, c) Kryvyi Rih loess. Denotations: A - aggregate preparation; S - semi-dispersed preparation; D - dispersed preparation; e - the coefficient of porosity; ε - the volumetric deformation.

4 CONCLUSIONS

In this article, we present the results of theoretical and experimental studies of the degradation process of clay soils in Ukraine. Rocks of different time of formation and origin were studied. We consider the degradation of clay soils as a consequence of technogenic influence, leading to irreversible destruction of the structure. The final state of the soil is considered, in which the decay of all aggregates occurred. As a result of the performed experiments and the application of new theoretical models, the following conclusions can be drawn:

- the difference in the predicted values of the porosity coefficient of the soils, calculated by the new method, confirms the influence of the type, genesis and age of the dispersed soil on the density of compaction during the formation of the “new” structure;
- the developed techniques can be considered as the basis for creating a new classification of silty-dispersed soils by the values of the fractal dimension of their particle size distribution;
- for silty loess and clay soils, after the disintegration of microaggregates, the "new" porous structure is completely formed by particles of the smallest size, the particles of bigger sizes are presented in the “new” structure as separated inclusions.

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