Methodology for Evaluating the Impact of Sudden Failures on the Reliability Parameters of the Optical Cable Damaged Section

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Abstract: An optical cable is considered as a multicomponent system, which components are elementary sections of an optical cable. During operation, due to failures of these sections, they are replaced with new ones. The reason for the failures of elementary sections, which we will call local failures, are sudden failures. In addition to sudden failures, there are gradual failures that are a consequence of the fiber aging. There is a need to replace the optical cable as a whole. Such a failure is wear-out or global. The influence of sudden failures on the reliability measures of an optical cable in the conditions of its degradation is considered. To solve this problem, a Markov reliability model was developed, on the basis of which expressions were obtained for calculating the unavailability factor, the average time of operational and inoperable states during the degradation cycle of an optical cable. Methods for determining the probabilities of transitions from one state of degradation to another are proposed. It is shown that the unavailability factor of the optical cable and the average time of operational state on one degradation cycle increase with increasing intensity of sudden failures. The optical cable reliability model presented in this paper can be useful for predicting the operating time of an optical cable.

1 INTRODUCTION

The process of operation of the optical cable of the access network is accompanied by failures, which are the result of the rupture of the optical fiber. Failures follow at random intervals and are followed by the process of replacing the failed sections with new ones. During operation, the number of such replacements (repairs) is a random variable with the average time between MTBF failures (Mean Time Between Failure) and the average repair time (replacement of the site).

Thus, an optical cable can be considered as a multicomponent system consisting of sequentially connected cable sections (elements) with length l_e (elementary sections). The number of such sections on the length of optical cable is defined as L/l_e, where L is the total length of optical cable. The failure of one of the optical cable elements will call a local failure. The failures of elementary sections will be considered independent. After the failure of the elementary section, this section is repaired, which consists in replacing it with a new one. At the same time, the quality of repair can be considered perfect, i.e. after the repair, the section becomes "as good as new". After a certain number of local failures and repair of failed sections, there is a moment when it is necessary to start replacing the optical cable as a whole, since the cost of its repair becomes higher than the cost of replacement. Replacing the optical cable with a new one is a consequence of a failure, which will call global. This failure characterizes the service life of the optical cable and, of course, this period differs from the warranty period, which corresponds to about 25 years.

Global failure is a consequence of fiber degradation, i.e. it is a wear-out failure. This failure is caused by the natural processes of aging, wear, corrosion and fatigue in compliance with all the rules and (or) standards of design, manufacture and operation [1]. Considering the wear-out failure as an optical fiber breakage, i.e. from a mechanical point, the strength of the optical fiber depends mainly on the appearance of microcracks on its surface [2].

Since the change in the depth of the crack is a non-decreasing function depending on the applied load, this process can be represented as a Markov process of pure reproduction or, if we consider the decrease in the strength of the optical fiber, as a process of pure doom.

Figure 1 shows the states of damage accumulation that occur due to the growth of the crack depth *a*. Let the first state be the initial one. It corresponds to the start time from the beginning of the operation of the optical fiber. The probability of finding a microcrack in the initial state is assumed to be 1. The state *b* is absorbing and at $t \to \infty$ the probability of finding a microcrack in the state *b* tends to 1.

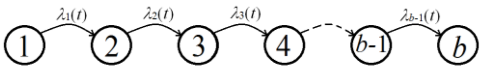

Figure 1: This caption has one line so it is centered.

The use of Markov processes for modeling the degradation phenomenon was first proposed and justified in [3-7]. The approach proposed in these works has found wide application. As an example, there are the works [8-10], where the theory of Markov processes is effectively used to describe the processes of systems.

Thus, in [8] the problem of maintenance and repair is considered from the position of degradation processes management. In [9], an optimal strategy for maintenance and repair is proposed in the conditions of degradation described by the Markov process of pure doom. Researches of degradation of thin-walled pipeline systems with defects are presented in [10].

In this article, unlike the works in which the processes of maintenance and repair in conditions of degradation are considered in general terms, the task is to obtain formulas for assessing the impact of sudden failures on reliability parameters in conditions of degradation of fiber.

2 CONCEPTUAL MODEL OF THE PROCESS

Over time, the optical cable degrades. At the same time, transitions between states occur. The state of degradation is determined by the depth of the microcrack. The depth of the microcrack varies from state to state according to the linear law: $a_j = a_0 + j \cdot \Delta a$, where Δa is the increment of the depth of the microcrack during the transition from one state to another. In a certain limiting state the depth of the microcrack reaches a value at which a wear-out failure occurs.

After the onset of wear-out failure, the optical cable section is restored by replacing it.

The operating time of an optical fiber is divided into the same time intervals, which is called the operating intervals. At each operating interval a sudden failure may occur, which is detected at the moment of its occurrence, that is, the failure is explisit. After such a failure is detected, the optical cable section is restored by replacing it.

When forming the model, the following conditions are accepted.

1) The operating time of the optical fiber is divided into the same operating intervals *T*.
2) Each operating intervals is characterized by a state of degradation, which is determined by the depth of the microcrack.
3) When the depth of the microcrack increases by a certain amount, the transition to the next state of degradation occurs.
4) Sudden failure during the operation interval occurs with the same intensity and does not depend on the state of degradation.
5) The initial state of the degradation cycle is a crack-free state.

3 MATHEMATICAL MODEL OF THE PROCESS

The mathematical model is based on the theory of Markov and semi-Markov processes. Under the accepted conditions, a state-transitions diagram can be drawn up. To describe the approach to model development, the number of degradation states is assumed to be five, that is, the crack length corresponding to the wear-out failure is divided into five parts and each part is a characteristic of the degradation state. In real conditions, the number of states taken into account can reach several tens or even hundreds.

The degradation cycle is the duration of operation from the initial state to the inoperable state, which is the result of a wear-out failure. The degradation cycle includes being in states of degradation and in states of recovery after sudden and wear-out failures.

The state-transitions diagram is shown on Figure 2.

There are following signs on the diagram Figure 2: D_j − the state of degradation; p – the probability of transition to the next state of degradation at one operating interval; λ − sudden failure rate; R – restoration of the optical fiber section up state.

In the given model, transitions between states occur both in discrete time ($D_j \rightarrow D_{j+1}$) and in continuous time ($D_j \rightarrow$ R). The dotted arrow indicates the last transition to recovery in a state of wear-out failure. These transitions can be described together using a semi-Markov process. Therefore, the main model is based on the theory of semi-Markov processes.

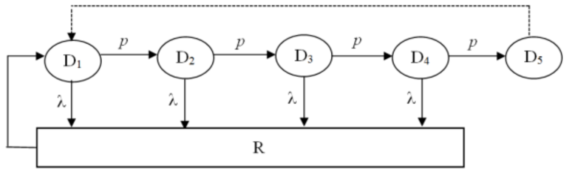

Figure 2: The state-transitions diagram of one section with transitions in discrete and continuous time.

In the states D1, D2, D3, D4, there is an increase in the depth of the crack in the optical fiber, and in the state D5 there is a wear-out failure. After replacing the optical cable due to wear-out failure, the degradation cycle ends. The initial state of the degradation process is D1.

Let's introduce the probabilities of transitions between the semi-Markov process states, which in [11] are called probabilities of passage: $\hat{p}_{ij}$ is the probability that a transition to the j-th state will occur, provided that there is an exit from the i-th state.

In continuous time, the transition to the recovery state occurs during the operation interval. The probability of this transition is $q = 1 - \exp(-\lambda T)$. For sufficiently small values of $\lambda \cdot T$ ($\lambda \cdot T << 1$), $q \approx \lambda T$ can be assumed.

The probabilities of passage are determined by the initial probabilities p and q, which do not depend on the state of degradation:

$$\hat{p} = \frac{p}{p+q}, \ \ \hat{q} = \frac{q}{p+q}. \tag{1}$$

The state-transitions diagram of the semi-Markov process is shown on Figure 3.

The diagram Figure 3 shows: $\hat{p}$ is the probability of transition from one state of degradation to another; $\hat{q}$ is the probability of sudden failure during the operation interval. $\hat{p}$ and $\hat{q}$ are defined from (1).

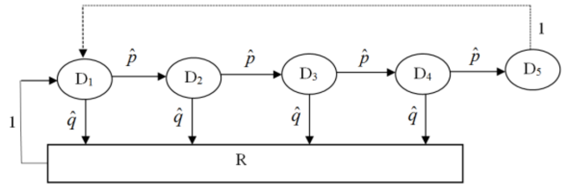

Figure 3: The state-transitions diagram of the semi-Markov process.

Matrix of transition probabilities of a semi-Markov process is:

$$\hat{P} = \begin{pmatrix} 0 & \hat{p} & 0 & 0 & 0 & \hat{q} \\ 0 & 0 & \hat{p} & 0 & 0 & \hat{q} \\ 0 & 0 & 0 & \hat{p} & 0 & \hat{q} \\ 0 & 0 & 0 & 0 & \hat{p} & \hat{q} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let's form a subset of states where transitions occur on one degradation cycle. Let U = {D_1, D_2, D_3, D_4, R} be a subset of states in which there is no wear-out failure. Then the matrix of transition probabilities on the subset U is:

$$\hat{P}_{UU} = \begin{pmatrix} 0 & \hat{p} & 0 & 0 & \hat{q} \\ 0 & 0 & \hat{p} & 0 & \hat{q} \\ 0 & 0 & 0 & \hat{p} & \hat{q} \\ 0 & 0 & 0 & 0 & \hat{q} \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

The matrix (2) is used to calculate the matrix of relative frequencies of the subset U [11]:

$$\hat{N}_U = \left(\hat{n}_{ij}\right) = \left(E - \hat{P}_{UU}\right)^{-1}, \tag{3}$$

where $\hat{n}_{ij}$ is the average number of hits in the j-th state during one degradation period at the initial i-th state; E is a unit matrix.

Since the state D_1 is assumed to be initial, all information about the degradation process is contained in the first row of the matrix $\hat{N}_U$ from (3):

$$\hat{n}_1 = \begin{pmatrix} \frac{1}{\hat{p}^4} & \frac{1}{\hat{p}^3} & \frac{1}{\hat{p}^2} & \frac{1}{\hat{p}} & \frac{1-\hat{p}^4}{\hat{p}^4} \end{pmatrix}. \tag{4}$$

To determine the duration of being in states, convert string (4) into a string whose elements are the durations of states in units of time. After entering the states D1, D2, D3, D4, the process is in each of them for an average of 1/(*p* + *q*) operating intervals. The average time spent in the state R after entering it is 1/μ, where μ is the recovery rate.

After making simple transformations of (4), get a string of average durations of states:

$$\bar{t_1} = T \cdot \left(\frac{(p+q)^3}{p^4} \quad \frac{(p+q)^2}{p^3} \quad \frac{p+q}{p^2} \quad \frac{1}{p} \quad \frac{(p+q)^4}{\mu \cdot T \cdot p^4} \right). \quad (5)$$

The average up state duration t_u is obtained by summing the elements of row (5), which correspond to the states D1, D2, D3, D4. The average down state duration t_d is equal to the element of row (5), which corresponds to the state R:

$$t_u = \frac{(p+q)^4 - p^4}{q \cdot p^4} \cdot T, \ \ t_d = \frac{(p+q)^4}{\mu \cdot p^4}. \quad (6)$$

The average degradation cycle duration of an optical cable elementary section is determined by the sum of t_u and t_d from (6):

$$t_{dc} = \frac{T}{p^4} \cdot \left[\frac{(p+q)^4 - p^4}{q} + \frac{(p+q)^4}{\mu \cdot T} \right].$$

The obtained results can be generalized to the degradation process with an arbitrary number of states *n*. Formulas for calculating reliability measures for one degradation cycle are aggregated in Table 1.

Table 1: The expressions for calculating reliability measures.

Reliability measures	Expression for calculating
Average up state time per degradation cycle	$t_u = \frac{(p+q)^{n-1} - p^{n-1}}{q \cdot p^{n-1}} \cdot T$
Average down state time per degradation cycle	$t_d = \frac{(p+q)^{n-1}}{\mu \cdot p^{n-1}}$
Average degradation cycle duration	$t_{dc} = t_u + t_d$
Availability factor per degradation cycle	$F_{av} = \frac{t_u}{t_u + t_d}$
Unavailability factor per degradation cycle	$F_{un} = \frac{t_d}{t_u + t_d}$
Average wear-out failure frequency	$\omega_{dc} = \frac{1}{t_{dc}}$
Average number of sudden failures per degradation cycle	$n_f = \frac{(p+q)^{n-1} - p^{n-1}}{p^{n-1}}$
Average sudden failures frequency	$\omega_f = \frac{n_f}{t_{dc}}$

4 METHODOLOGY FOR CALCULATING RELIABILITY MEASURES

To carry out calculations, it is necessary to set the value of p and in the future for the selected value of *p*, changing the probability of sudden failures *q*, to assess the impact of sudden failures on the reliability measures of a degraded optical cable. The probabilities of transitions from state to state as a result of the optical cable degradation actually determine the degradation cycle time.

The option is considered when the cause of failures are sudden and wear-out failures. Sudden failures follow with rate λ. For a given value λ, find the probability *q*. The reason for the wear-out failure is the presence of a microcrack, the depth of which only increases over time.

There are three stages of the development of microcracks. The first stage is the stage of microcrack nucleation; the second is the stage of microcrack depth growth; the third stage lasts for seconds and ends with the rupture of the optical fiber. The second stage lasts for years, and it mainly determines the optical cable service life.

To describe the second stage of microcrack development under the influence of cyclic loading, Paris and Erdogan [12] proposed an equation called “Paris Law”:

$$\frac{da}{dN} = C \cdot \Delta K^m, \quad (7)$$

where $\frac{da}{dN}$ is the rate of microcrack growth, [mm/cycle]; *C* is a constant determined by the material and depending on the frequency of the applied voltage change, $\left[\text{mm} / \left(\text{MPa}\sqrt{m} \right)^m \right]$; *m* is a constant determined by the material; ΔK is a stress intensity factor, $\left[\text{MPa}\sqrt{m} \right]$.

The range of the stress intensity factor can be calculated based on the values of the maximum (K_{max}) and minimum (K_{min}) stress intensity for the load cycle: $\Delta K = K_{max} - K_{min}$.

Using (7), it is possible to obtain an expression for the rate of decrease in the strength of an optical fiber depending on the depth of the crack [13].

By setting the value of the minimum required strength of the fiber, it is possible to determine the service life of the fiber at a known stress value. It should be noted that if determining the stress applied to the fiber in the future does not cause much difficulty, then to determine the strength of the fiber, it is necessary to know the depth of the crack on the surface of the fiber. Determining the crack depth, in relation to optical fiber, seems to be quite a difficult task due to the lack of non-destructive methods for controlling the strength of optical fiber. Therefore, the prediction of the service life of an optical cable is usually carried out using the results of fiber testing for strength. So, in [14], an expression is proposed to determine the probability of fiber failure:

$$F = 1 - \exp\left[\frac{-L \cdot m_S \cdot B \cdot S_t^{n_g - 2}}{L_p \cdot (n_g - 2) \cdot \sigma_a^{n_g} \cdot t_a}\right], \qquad (8)$$

where F is the probability of a wear-out failure; L is the length of the elementary fiber section (equal to L_p); L_p is the length of the fiber that has been tested for rupture; σ_a is the load applied during degradation; m_s is the parameter of the Weibull distribution (equal to 2 - 5); B, n_g are the parameters characterizing the strength of fused quartz glass (n_g = 15-20); t_a is the γ- percentage service life, defined as the calendar duration from the start of operation of the optical cable, during which it will not reach the limit state with a given probability (1-F); S_t – is the fiber strength.

5 EXPERIMANTAL RESULTS

For certain values of the parameters included in (8), the service life of the optical cable t_a can be found for F. Let, as follows from [15], this service life is 30 years. Let's divide this service life into intervals of 6 years and calculate the dependence of $t_u(p)$ at $q \to 0$. Find the value of p^* at which $t_u \approx t_a$ and, finally, investigate the dependence of the reliability measures given in Table 1 from q.

The uptime probability (1-F) during the service life is chosen sufficiently large. So, according to [16], the γ-percent service life is chosen to be 0.95.

As a result of calculations, the value $p^* = 0.95$ was obtained.

Figure 4 shows the dependence of the average up state time on the degradation cycle from the probability of the optical cable transition to the degradation state $t_u(p)$ at the values of the sudden failures probability $q = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$.

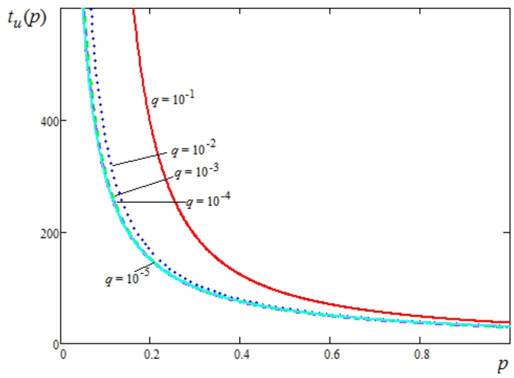

Figure 4: Dependence $t_u(p)$ at different values of the sudden failures probability.

Figure 5 shows the dependence of the unavailability factor $F_{un}(q)$ from the sudden failures probability at $q \in [10^{-6}; 10^{-1}]$. Figure 5 is constructed for the recovery rate μ = 1/3 of an hour [17].

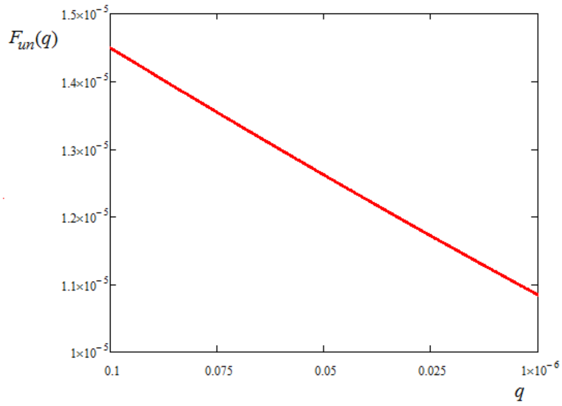

Figure 5: Dependence $F_{un}(q)$.

6 CONCLUSIONS

The average operating time for one degradation cycle increases with growth q due to an increase in the intensity of replacement of optical cable sections. At the same time, the unavailability factor of the optical cable increases with the growth q due to the increase in the average time of down state on one degradation cycle.

The above model allows calculating a number of reliability measures based on the initial characteristics, which are the probability of transition to the next state of degradation, the sudden failure rate, the operation interval duration, the recovery rate after sudden failures.

Based on the approach presented in the article, a methodology for collecting and processing statistical information about the reliability of an optical cable can be developed. During the observation, the following statistical parameters can be recorded: the number of wear-out failures, the number of sudden failures, and the recovery time after failures. According to these parameters, the initial characteristics of the degradation process can be calculated.

The data of the technical condition monitoring system can serve as a source of reliable statistical information.

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